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RESEARCH PAPER

Elastic moduli for a rectangular fibers array arrangement in a two phases composite

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Abstract

Determining the elastic constants for composites with fibers is a continuous concern of researchers, being studied and analyzed different types of materials, with different topologies and geometries. In the work, these constants are determined for a composite reinforced with cylindrical fibers with a rectangular packing. The obtained results are applied for the calculation of these constants for a composite used in engineering applications.

Keywords: elastic moduli**;** cylindrical fibers; two phases' composites; rectangular array; Young's modulus;

1. Introduction

Determining the elastic constants for any type of material used in engineering applications is an important objective for designers. The methods of obtaining them are theoretical, using different calculation formulas obtained in certain assumptions by researchers or experimental methods, the safest for this purpose. For composite materials, in particular for those reinforced with fibers, the way to obtain these formulas is presented in a rich literature. We mention that for many of these methods it is necessary to determine the stress and strain field, so definitely a difficult numerical calculation. Other frequently used methods are the variational ones that offer upper and lower limits of the modulus of elasticity or other elastic constants (bulk modulus, shear modulus, Poisson's ratio, etc.). The accuracy of the estimates in this approach is given by the difference between the two limits. And it can be quite bad for certain concentrations of the phases of the composite material [1]. They are obtained for particular cases of boundary conditions and are, in general, rather imprecise [2, 3]. Another type of approach, using micromechanical models, gives better values for the elastic constants but requires the determination of the stress and strain field for the studied materials, in certain loading cases [4-6]. In engineering practice, composites reinforced with cylindrical fibers are frequently used and, as a consequence, numerous studies on elastic properties have been carried out [7-10]. A series of theoretical results are presented in [11-16]. In all cases, the experimental methods of obtaining the results represent the most suitable solution for obtaining credible values, but the use of these methods involves significant costs with equipment and time. This is the reason why, in the first phase of a project, it is necessary to have some estimates that can be obtained easily, quickly and with a satisfactory degree of precision.

A method that allows the determination of the homogenized elastic coefficients using a repeatable unit of the material, called representative volume, is presented in [17], being applied to a composite material reinforced with short fibers. The method can be successfully used for other topologies of the fiber structure. Among the methods

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used, the Finite Element Method (FEM) can be distinguished, which has proven to offer very good results for solving such problems [18, 19].

FEM has the advantage of being able to introduce other factors that could influence the value of the elastic constants, such as temperature or humidity. The study of a polymer composite reinforced with unidirectional graphite fibers is done in [20-22].

The experimental methods ensure values obtained with high precision for materials currently used in engineering such as brass, copper, Plexiglas and PVC [23-26]. An important application in practice is the use of composites in the manufacture of pipes. They are made of rubber reinforced with metal braids. The use of pipes on a very large scale justifies the interest in the study of these elements and the materials from which they are made [27]. Wood is a composite material with fibers and in this case, FEM is used for the mechanical identification of some types of wood [28-32].

And for the analysis of reinforced concrete MEF proved to be a useful and suitable method for the analysis of the elastic constants of the homogenized material [33, 34]. Experimental checks have demonstrated in this case the accuracy of the results obtained. The study of polymer composites reinforced with glass fibers has experienced a strong development in the last period, due to the numerous engineering applications where they are found. The viscoelastic behaviour of these materials makes the study more difficult [35]. The time factor in the description of viscoelastic materials is introduced in [34]. Other types of materials are studied in [36-41].

In the work, the elastic constants are determined for a polymeric material reinforced with cylindrical, parallel fibers placed in a rectangular arrangement. The method is simple and the sought values can be obtained with a very good precision for applications. The method based on which these elastic constants are calculated can also be applied to other situations encountered in practice.

2. Models and Methods

The effective viscoelastic behaviour of a two phases composite body is dependent upon the elastic/viscoelastic property of the constituent material.

A number of approaches are presented in the literature for the prediction of the bounds on elastic/viscoelastic parameter of multi (or two) phase composite. Most of the works which have been done to estimate the bounds on the effective elastic/viscoelastic property of fiber reinforced composite, assume both of the phases to possess isotropic material behaviour. Little work has been presented in the literature for those cases where for instance the reinforcing phase has anisotropic or transversally isotropic properties. An example of this could be a graphite/epoxy composite where the matrix material is considered to be isotropic but the fiber show anisotropic behaviour. To get a better insight into such problems, let us briefly review some of the methods encountered in the literature. First however some hypothesis for estimation of the bounds on elastic/viscoelastic moduli, when both phases are isotropic, will be presented.

The problem considered for the computation of the bounds on elastic/ viscoelastic moduli is that of parallel fibers which are long enough so that end effects can be neglected. The material may be represented by a cylindrical specimen whose cross section is very large in comparison the fiber cross section. The longitudinal axis of the specimen coincides with the fiber direction and since the end effects are neglected, the fiber may be assumed to run continuously through the entire length of the specimen.

It is also assumed that the specimen is statistically both homogeneous and transversally isotropic. The problem to be considered is that of predicting the bound on elastic/viscoelastic property of such a specimen in terms of its geometric and the elastic/viscoelastic moduli of its constituents. In order to demonstrate the difficulties in obtaining the bounds on characteristic values of a composite with analytical procedure, let us first assume that the two constituents are linear and isotropic. Let's K_{23}^{+} and G_{23}^{+} denote the upper bounds of the bulk and shear moduli respectively, while K_{23}^{-} and G_{23}^{-} represent the corresponding lower bounds. Note that here and is the remaining of the present section the upper and the lower bound on the characteristic parameters are represented by "+" and "-" upper indices as in E_{11}^{+}, E_{11}^{-}, ν_{12}^{+}, ν_{12}^{-}.

There are different approaches in the literature which allows the prediction of the above bounds. Using Hill's theory [24-27] one may obtain the bounds on the bulk modules K_{23}, the longitudinal Young's modulus E_{11}, and the Poisson's ratio $\nu_1 = \nu_{12} = \nu_{13}$. These bounds are determined in terms of the volume ratios of fiber v_f and matrix v_m:

$$K_{23}^{+}=\frac{v_f K_f\left(K_m+G_m\right)+v_m K_m\left(K_f+G_f\right)}{v_f\left(K_m+G_f\right)+v_m\left(K_f+G_f\right)} \quad ; \tag{1}$$

$$K_{23}^{-}=\frac{v_f K_f\left(K_m+G_m\right)+v_m K_m\left(K_f+G_m\right)}{v_f\left(K_m+G_m\right)+v_m\left(K_f+G_m\right)} \quad ; \tag{2}$$

Note that in this relation K_λ is the plane-strain bulk modulus and G_λ is the shear modulus, where as before $\lambda = f, m$ is to represent the fiber and matrix phases, respectively. These bound should be more impressive than those suggested by Voigt (V) and Reuss (R) expressed as:

$$K_{23}^{+}=K_V=v_f K_f+v_m K_m \quad ; \tag{3}$$

and

$$K_{23}^{-}=K_R=\frac{1}{\dfrac{v_f}{K_f}+\dfrac{v_m}{K_m}} \quad ; \tag{4}$$

for the bulk modulus and

$$G_{23}^{+}=G_V=v_f G_f+v_m G_m \quad ; \tag{5}$$

and

$$G_{23}^{-}=G_R=\frac{1}{\dfrac{v_f}{G_f}+\dfrac{v_m}{G_m}} \quad , \tag{6}$$

for the shear modulus. This means that the bulk and shear moduli follow the relations:

$$K_{23}^{-}=K_{23}=K_{23}^{+} \quad , \tag{7}$$

and

$$G_{23}^{-}=G_{23}=G_{23}^{+} \quad , \tag{8}$$

respectively.

For the longitudinal Young's modulus E11, the results obtained by Hill can be written as:

$$E_{11}^{+}=E_{1mix}+\frac{4\left(\nu_f-\nu_m\right)^2 v_m v_f}{\dfrac{v_f}{K_m}+\dfrac{v_m}{K_f}+\dfrac{1}{G_f}} \quad , \tag{9}$$

$$E_{11}^{-}=E_{1mix}+\frac{4\left(\nu_f-\nu_m\right)^2 v_m v_f}{\dfrac{v_f}{K_m}+\dfrac{v_m}{K_f}+\dfrac{1}{G_m}} \quad , \tag{10}$$

where E_{1mix} is to represents the modulus determined according to the law of mixture, viz.

$$E_{1mix}=v_f E_f+v_m E_m \ . \tag{11}$$

It is interesting to note that if the Poisson's ratio of the two constituents is the same, the upper and lower bounds are identical which in turn means that the law of mixtures can be used for the exact estimation of the Young's modulus.

The bounds on the Poisson's ratio have been shown to be:

$$\nu_1^+ = \frac{\nu_{1mix} + (\nu_f - \nu_m)\left(\frac{1}{K_m} - \frac{1}{K_f}\right) v_m v_f}{\frac{v_f}{K_m} + \frac{v_m}{K_f} + \frac{1}{G_f}} \quad ; \tag{12}$$

$$\nu_1^- = \frac{\nu_{1mix} + (\nu_f - \nu_m)\left(\frac{1}{K_m} - \frac{1}{K_f}\right) v_m v_f}{\frac{v_f}{K_m} + \frac{v_m}{K_f} + \frac{1}{G_m}} \quad . \tag{13}$$

Note that in this relation:

$$\nu_{1mix} = v_f \nu_f + v_m \nu_m \quad . \tag{14}$$

An example of the bounds on K_{23} and ν_1 is shown in Fig. 1 and 2 for a composite with the indicated properties. In Hashin [17] following bounds are given:

$$K_{23}^+ = K_f + \frac{\nu_m}{\frac{1}{K_m - K_f} + \frac{\nu_f}{K_f + G_f}} \quad ; \tag{15}$$

$$K_{23}^- = K_m + \frac{\nu_f}{\frac{1}{K_f - K_m} + \frac{\nu_m}{K_m + G_m}} \quad , \tag{16}$$

for the bulk modulus K_{23} and

$$G_{23}^+ = G_f + \frac{\nu_m}{\frac{1}{G_m - G_f} + \frac{\nu_f (K_f + 2G_m))}{2G_f (K_f + G_f)}} \quad ; \tag{17}$$

$$G_{23}^- = G_m + \frac{\nu_f}{\frac{1}{G_f - G_m} + \frac{\nu_m (K_m + 2G_m)}{2G_m (K_m + G_m)}} \quad , \tag{18}$$

for the share modulus G_{23} and

$$G_1^+ = G_f + \frac{\nu_m}{\frac{1}{G_m - G_f} + \frac{\nu_f}{2G_f}} \quad ; \tag{19}$$

$$G_1^- = G_m + \frac{\nu_f}{\frac{1}{G_g - G_m} + \frac{\nu_m}{2G_m}} \quad , \tag{20}$$

for the axial shear modulus $G_1 = G_{12} = G_{13}$.

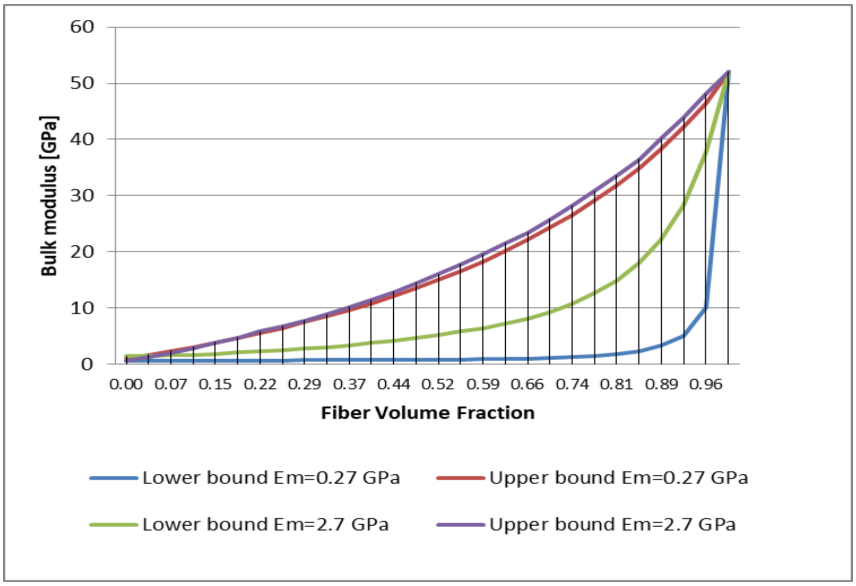

Fig.1: variation of upper and lower bounds on K_{23} with v_f [24]

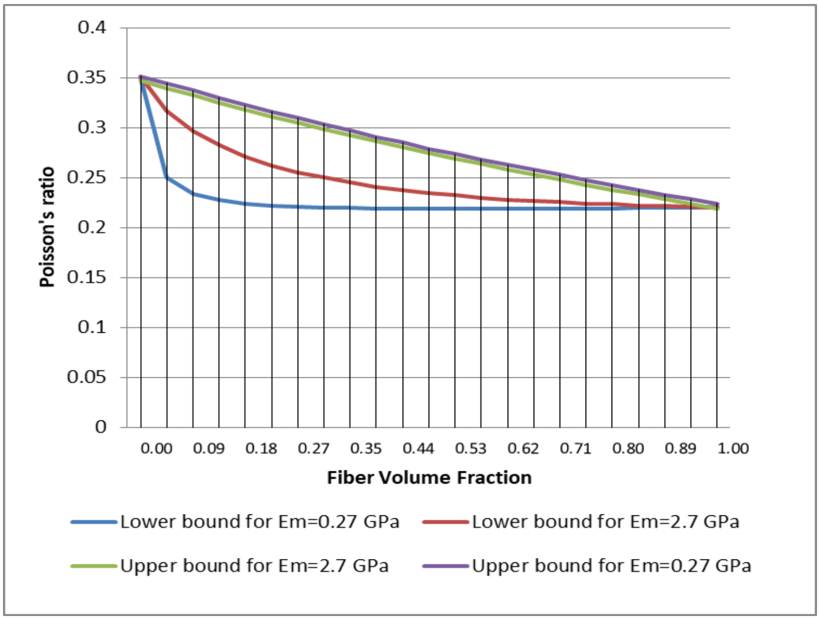

Fig.2: variation of upper and lower bounds on ν_1 with v_f [24]

The results reveals that there exists a rather large difference between the bounds and as a consequence this formulation of the bounds cannot be of much interest from engineering point of view. Attention is drawn to the effect that using Hill's [23-26] and Hashin's [17-20] theories not all the five effective model required for the characterization of the transversally isotropic composite can be obtained. This is necessary to establish all the relations required to obtain the compliance matrix and hence the time dependent strain in a creep experiment. Another limitation of the above theories is that the two phases in the composite are considered isotropic. Recall that is the computation of the above bounds, the concentration of the two phases and not the shape of the reinforcing phase is considered. Therefore, it should be expected that in some cases these bounds may be very different.

Obviously by taking also the geometry of the fiber parking into account, more accurate bounds can be expected then in those cases when only the volume ratios of the constituents are considered. For a reinforced composite with cylindrical fibers, the upper and lower bounds on the effective elastic moduli have been calculated for a geometry of hexagonal fiber array (Fig. 3) by Hashin [20]. For illustrative purpose two of these bounds are presented in Figures 6 and 7 for two different ratios of fiber to matrix moduli.

Now an attempt will be made to apply Hashin's approach [20] to determine the upper and lower bounds on the bulk modulus for a rectangular fiber arrangement. To the authors knowledge this theory has not been applied to the quadratic array and is done for the first time in the present work. The procedure is applied in the elastic domain. This can be taken as an approximation of the viscoelastic behavior at an infinitesimally small-time *t*. By using the time dependent function of the elastic constant of the matrix constituent, the long-time material behaviour of the composite can be determined. Note also that both phases of the composite are considered to be isotropic. The Young's modulus and Poisson's ratio of the phases E_λ and ν_λ ($\lambda = m = f$) as well as geometrical parameters r_0, r_f and r_m are known (see the results in Fig.4 and 5).

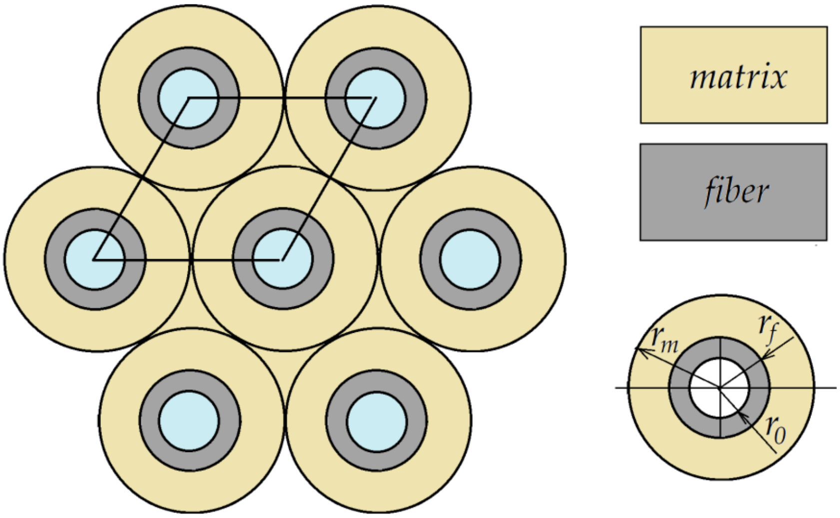

Fig.3: The hexagonal array

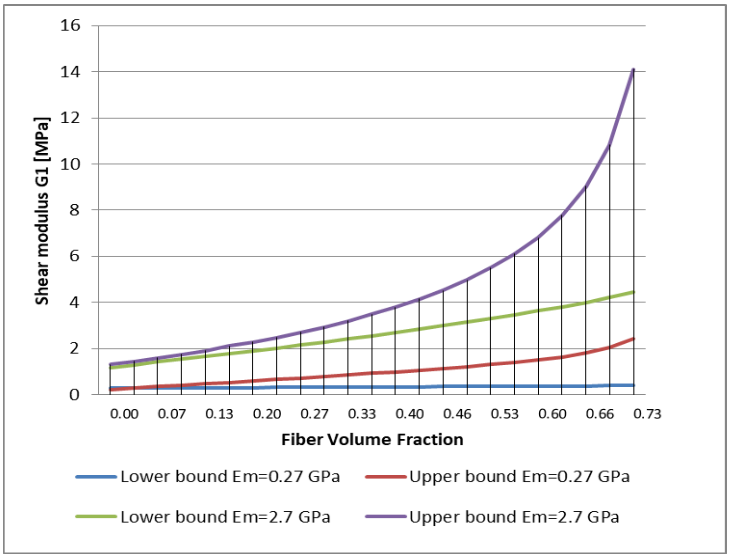

Fig.4: the upper and lower bounds on G_1 for hexagonal array, E_f = 72.4; ν_m =0.35; ν_f =0.22 (after [21])

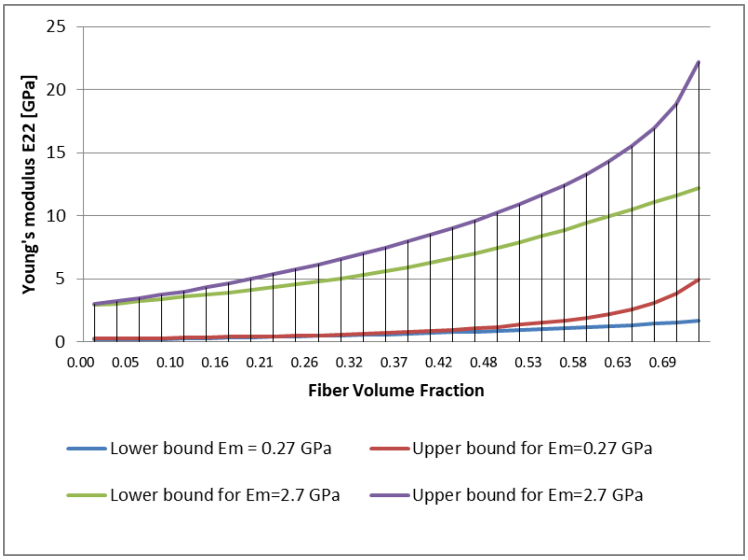

Fig. 5: the upper and lower bounds on E_{22} for hexagonal array, E_f = 72.4; ν_m =0.35; ν_f =0.22 (after [21])

3. Rectangular array

For a fiber reinforced cylindrical specimen with infinitely long circular fibers running in x_1 direction and distributed in the x_2x_3 plane - plane of isotropy - following problem can be considered (Fig.6):

The boundary conditions imposed on the composite specimen can be expressed as:

$$u_i^0(S) = \bar{\varepsilon}_{ij} x_j \quad , \tag{21}$$

where $u_i^0(S)$ are displacement component or as:

$$T_i^0(S) = \bar{\sigma}_{ij} n_j \quad , \tag{22}$$

with $T_i^0(S)$ as the stress-vector components. In both of the above conditions, S is to represent the entire bounding surface of the specimen, x_j are the coordinates of any point on the surface and n_j are the components of the outward normal to $\bar{\Gamma}$ representing the contour of the unit cell.

Note that if boundary condition (11) is applied to a macroscopically homogeneous material the average strains over the specimen can be shown as to be $\bar{\varepsilon}_{ij}$ and for (12) the average stresses to be $\bar{\sigma}_{ij}$ [42, 43]. Hashin refers to cylindrical sub-regions of the composite specimen having the same property as those of the entire composite as the" representative volume element" (RVE). For the rectangular array considered in the current investigation the same analogy can be used for a repeating unit cell (RUC) which is a rectangular prism consisting of one central fiber and the corresponding matrix material as shown in Figure 8.

The constitutive relation for a composite specimen assuming a homogeneous body can be expressed as either of the following relations:

$$\bar{\sigma}_{ij} = C_{ijkh} \bar{\varepsilon}_{kh} \quad , \tag{23}$$

$$\bar{\varepsilon}_{ij} = S_{ijkh} \bar{\sigma}_{kh} \quad . \tag{24}$$

The various notations applying to C_{ijkh} will be explained in Appendix C. In these equations $\bar{\sigma}_{ij}$ and $\bar{\varepsilon}_{ij}$ are to represent the average stresses and strains in the RUC which are assumed to be the same as those existing in the bulk material. As mentioned earlier, the number of independent elastic moduli is reduced to five for a transversely isotropic specimen.

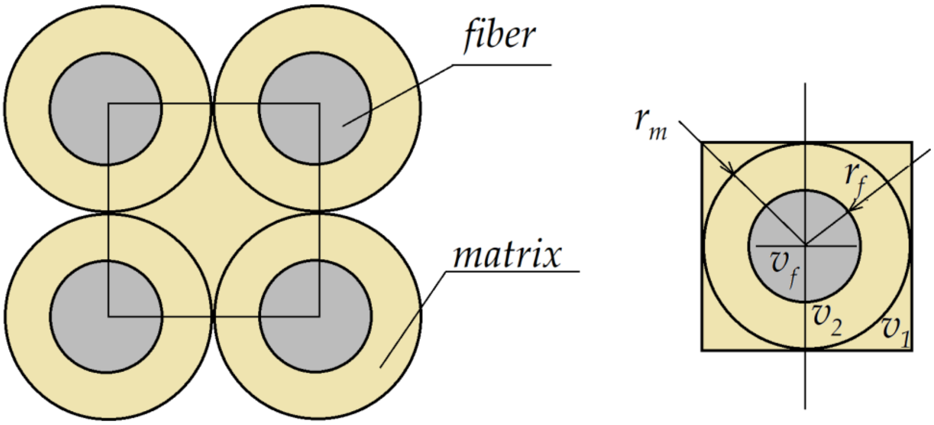

Fig.6: the rectangular fiber packing arrangement

Depending on the problem being investigated, either $\bar{\sigma}_{ij}$ or the average strain $\bar{\varepsilon}_{ij}$ are sought while applying conditions (21) or (22) respectively. When applying either of the above boundary conditions the strain energy W stored in the RUC is:

$$W = \frac{1}{2}\bar{\sigma}_{ij}\bar{\varepsilon}_{ij} \; . \tag{25}$$

By considering boundary conditions (21) and making use of Eq. (23) this last equation can be written as:

$$W^e = \frac{1}{2}C_{ijkh}\bar{\varepsilon}_{ij}\bar{\varepsilon}_{kh} \; . \tag{26}$$

Conversely when boundary condition (22) is imposed, it follows together with Eq. (24) that:

$$W^e = \frac{1}{2}S_{ijkh}\bar{\sigma}_{ij}\bar{\sigma}_{kh} \; , \tag{27}$$

where C_{ijkh} and S_{ijkh} are the effective elastic moduli and compliances respectively. As mentioned earlier in the present study the fiber have a square parking geometry which give the composite specimen quadratic symmetry. Therefore, the plan transverse to the longitudinal axis of the composite is taken as the plan of isotropy.

The stress strain Eq. (13) for the above transversally isotropic material may be written in terms of the five elastic moduli in the form presented in appendix C. Now the upper and the lower bound on the bulk modulus for the square array presented in Figure 8 will be determined. In this figure r_f denote in a cylindrical surface the radius of the circular fibers surrounded by the circular matrix mantle with the radius r_m . By applying Eq. (21) to the boundary of the above cylindrical surface and using the principle of minimum potential energy throughout the composite following relation can be written:

$$U^{\varepsilon} \le \tilde{U}^{\varepsilon} \; . \tag{28}$$

In this equation $\tilde{U}^{\varepsilon}$ is the strain energy for the displacement field (21) and U^{ε} is the actual strain energy Eq.(15). This relation supplies one of the conditions to be satisfied by the combination of the elastic constant, viz.the upper bound. Similar arguments can be made for the stress energy $\tilde{U}^{\sigma}$ and the corresponding actual stress energy U^{σ}, Eq.(16). By applying the stress field (22) throughout the composite and using the principle of minimum complementary energy it follows that:

$$U^{\sigma} \le \tilde{U}^{\sigma} \; . \tag{29}$$

This provides a new bound for the constants. Let us know for the sake of illustration, apply the foregoing procedures to determine the bounds on the bulk modulus:

$$K_{23} = \frac{1}{2}\left(C_{22} + C_{33)}\right) \quad . \tag{30}$$

For a particular plane-strain field, where

$$\bar{\varepsilon}_{22} = \bar{\varepsilon}_{33} = \bar{\varepsilon} \ , \tag{31}$$

with other strain component being identical to equal to zero, the boundary conditions (21) become:

$$u_1^0 = 0 \quad ; \quad u_2^0 = \bar{\varepsilon}x_2 \quad ; \quad u_3^0 = \bar{\varepsilon}x_3 \quad . \tag{32}$$

In this case the strain-energy is:

$$W = \frac{1}{2}C_{22}\bar{\varepsilon}_{22}^2 + \frac{1}{2}C_{22}\bar{\varepsilon}_{33}^2 + C_{23}\bar{\varepsilon}_{22}\bar{\varepsilon}_{33} = 2K_{23}\bar{\varepsilon}^2. \tag{33}$$

The boundary condition (32) expressed it in cylindrical coordinate system are:

$$u_1^0 = u_x^0 = 0 \quad ; \quad u_r^0 = \bar{\varepsilon}\, r \quad ; \quad u_\theta^0 = 0 \quad (r = r) \quad . \tag{34}$$

This axially symmetric plane-strain problem is well introduced in the literature for which the following general solution may be written as:

$$u_r = A_\lambda r + \frac{B_\lambda}{r} \quad ; \tag{35}$$

and:

$$\sigma_r = 2K_\lambda A_\lambda - 2G_\lambda \frac{B_\lambda}{r^2} \quad , \tag{36}$$

where u_r and σ_r are radial displacement and radial stress respectively. In this equation K_λ is the plane-strain bulk modulus and G_λ is the share modulus. It should be pointed out again that for the fiber $\lambda = f$ and for the matrix $\lambda = m$. And this is only for notational purposes and does not imply some summation of tensors. Now, one should seek to such solution for $r_0 \le r \le r_f$ and $r_f \le r \le r_m$ with the corresponding elastic constants. These two sets of equations contain for constant A_f, A_m, B_f and B_m which required four conditions for their evaluation. One condition is provided by substituting $r = r_m$ in the relation $u_r^0 = \bar{\varepsilon}\, r$ given above. The other three are obtained by requiring continuity of u_r and σ_r at the interfacial location $r = r_f$ and $\sigma_r = 0$ at $r = r_0$. According to Hashin [20] the radial stress at $r = r_m$ can be shown to be:

$$\sigma_r^{(m)}\Big|_{r_m} = 2\bar{\varepsilon}K_m m_K \quad , \tag{37}$$

where:

$$m_K = \frac{\Phi\left(1-a^2\right)\left(1+2\nu_m b^2\right) + \left(1+\frac{a^2}{2\nu_f}\right)\left(1-b^2\right)2\nu_m}{\Phi\left(1-a^2\right)\left(1-b^2\right) + \left(1+\frac{a^2}{2\nu_f}\right)\left(b^2+2\nu_m\right)} \quad , \tag{38}$$

with:

$$a = \frac{r_0}{r_f} \quad ; \quad b = \frac{r_f}{r_m} \quad ; \quad \Phi = \frac{K_f}{K_m} \quad . \tag{39}$$

For the fiber packing arrangement considered in the present study where $r_0 = 0$, the expression for m_K simplifies to:

$$m_K = \frac{\Phi\left(1+2\nu_m b^2\right)+\left(1-b^2\right)2\nu_m}{\Phi\left(1-b^2\right)+\left(b^2+2\nu_m\right)} \quad . \tag{40}$$

The strain energy stored in a composite cylinder is given by:

$$U_{cc}^{\varepsilon} = \frac{1}{2}\sigma_r^{(m)}\Big|_{r_m} u_r^{(m)}\Big|_{r_m} 2\pi r_m l \ , \tag{41}$$

where l is the length of the cylinder. This expression can be written in a simplified form as:

$$U_{cc}^{\varepsilon} = 2K_m m_K \bar{\varepsilon}\, V_{cc} \quad , \tag{42}$$

where Eqs.(35) and (37) have been used. Here $V_{cc} = \pi r_m^2 l$ is the volume of the composite cylinder. The "strain energy" $\tilde{U}^{\varepsilon}$ stored in the entire composite is now given by:

$$\tilde{U}^{\varepsilon} = 2K_m m_K \bar{\varepsilon}^2\, V_1 + 2K_m \bar{\varepsilon}^2\, V_2 \quad , \tag{43}$$

where V_1 represents the volume occupied by all composite cylinders and V_2 is the remaining volume. On the other hand, the actual strain energy can be written as:

$$U^{\varepsilon} = 2K_{23}\bar{\varepsilon}^2\, V \ , \tag{44}$$

where V is the sum of the above two volumes. The upper bound K_{23}^{+} can be obtained by using the expression (17) togheter the Eqs. (43) and (44), i.e.

$$K_{23}^{+} = K_m\left(m_K v_1 + v_2\right) \quad . \tag{45}$$

with: $v_1 = \dfrac{V_1}{V}$; $v_2 = \dfrac{V_2}{V}$; $v_1 + v_2 = 1$. For the geometry of the square array, it can be shown that:

$$V = (2r_m)^2 = 4r_m^2 \quad ; \quad V_1 = \pi\, r_m^m \quad ; \tag{46}$$

$$v_t = \frac{\pi\, r_f^2}{V} = \frac{\pi\, r_f^2}{4r_m^2} \quad ; \quad v_m = 1 - v_t \quad ; \tag{47}$$

$$v_1 = \frac{\pi}{4} = 0.785 \quad ; \quad b^2 = \frac{v_t}{v_{1\,t}} \quad . \tag{48}$$

where v_t represents the fractional volume of the fiber in the composite.

In order to obtain the lower bound K_{23}^{-}, the boundary condition (22) is used with:

$$\bar{\sigma}_{22} = \bar{\sigma}_{33} = \bar{\sigma} \ . \tag{49}$$

And all other stress components equal to zero except $\bar{\sigma}_{11}$ which is necessary to prevent $\bar{\varepsilon}_{11}$. A procedure similar to that presented to obtain the upper bound should be followed now while applying the concept of "stress energy" $\tilde{U}^{\sigma}$ as well as the inequality (39). It can be shown that:

$$W^{\sigma} = \frac{\bar{\sigma}^2}{2K_{23}} \quad , \tag{50}$$

which results in the following expression for the lower bound on the bulk modulus:

$$K_{23}^{-} = \frac{K_m}{\dfrac{v_1}{m_K} + v_2} \quad . \tag{51}$$

In Figures 7 and 8 the bounds computed with (45) and (51) in the composite for a square packing geometry together with those obtained by Hashin [20] for a hexagonal arrangement are presented. The bounds of Hashin are more impressive because of the more efficient fiber arrangement compared with that of the square fiber packing. Figures 7 and 8 highlight that for the fiber concentration used in the present investigation ($\nu_f = 60\%$) there exists a rather difference between the bounds presented, thus the information supplied by these bounds can not be descriptive of the true behavior of the material.

4. Conclusions

In the work, the authors propose a method for calculating the elastic constants for a composite material reinforced with cylindrical fibers, in which the fibers are arranged in a rectangular network. The advantage of the method is to provide fast and relatively accurate results for the design stage of a product. In this way the costs are reduced and the results used are sufficient for the first phase of the design. It is only necessary to know the values of the elastic constants for the constituent phases of the composite and the percentage of fiber and matrix that make up the material. The speed with which this method can be used and with which the results are obtained and its simplicity represent an important argument for its application for a quick and precise estimation of the mechanical properties. For the case presented in the paper, the results are similar to other results obtained with other methods, numerically or experimentally [44]. The presented model can be improved if this is desired, especially in the case of the appearance of new parameters that describe the material situation. For example, if the effect of temperature or humidity must be taken into account. The method can also be used for these cases by refining the presented model. There are therefore possibilities for the development of research in the future.

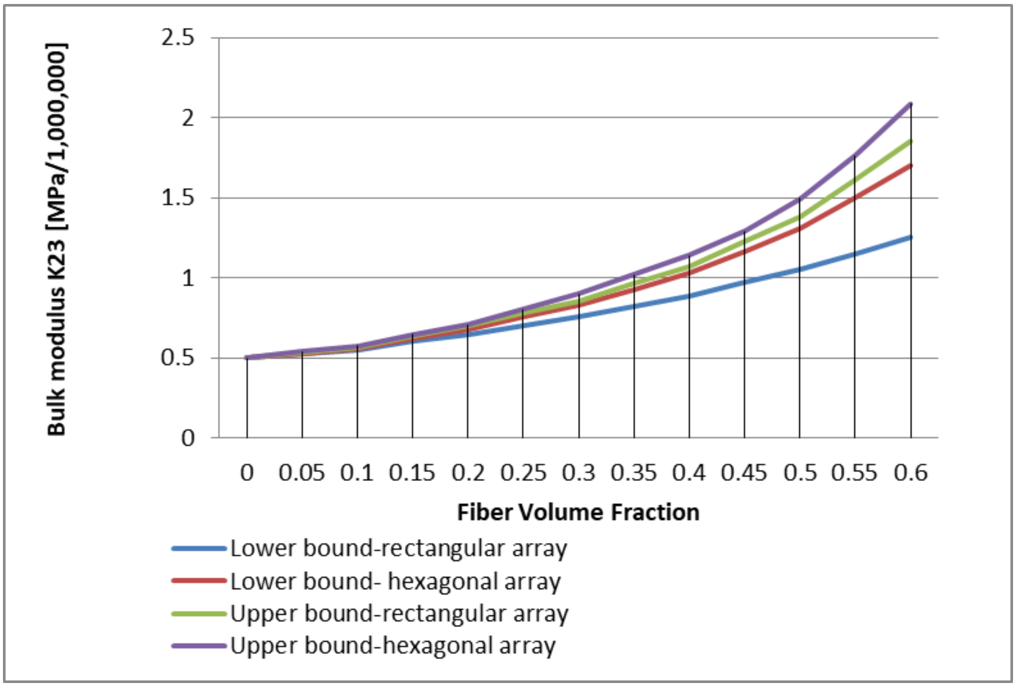

Fig. 7: the upper and lower bounds on K_{23} for hexagonal and rectangular array

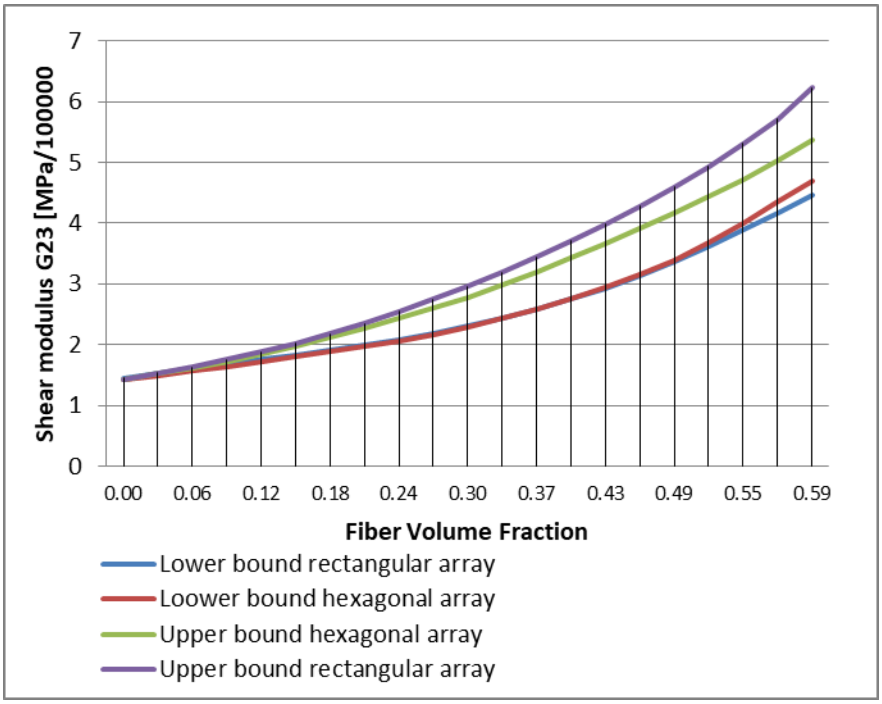

Fig.8. the upper and lower bounds on G_{23} for hexagonal and rectangular array

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