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RESEARCH PAPER

Behavior of functionally graded semiconducting rod with internal heat source under a thermal shock

Praveen Ailawalia [a], Marin Marin [b, c,*], Harish Nagar [a]

[a] *Department of Mathematics, University Institute of Science Chandigarh University, Gharaun-Mohali Punjab, India*
[b] *Department of Mathematics and Computer Science, Transilvania University of Brasov, 500036 Brasov, Romania*
[c] *Academy of Romanian Scientists, Ilfov Street, 3, 050045 Bucharest, Romania*

Abstract

The research article investigates the behavior of a functionally graded semiconducting rod with internal heat source of length l under the thermal shock. A sudden heat source is applied to the left boundary of the finite rod. The equations of motion are solved analytically and the analytical expressions of displacement, carrier density, temperature distribution and stresses are obtained. The numerical values of these expressions are calculated and presented graphically to show the effect of non-homogeneity parameter on the components. The variations of the parameters are shown for different theories of thermoelasticity namely Modified Green-Lindsay(MGL) theory, Green-Lindsay(GL) theory, Lord-Shulman(LS) theory and Coupled(CT) theory.

Keywords: Semiconducting, Functionally Graded, heat source, Carrier density, internal heat source.

1. Introduction

The thermal effect in mechanics has been the centre of study for researchers in the past due to its wide applications in many branches of engineering. Initially, the classical theory was modified by Biot [1] by considering the theory of coupled thermos-elasticity. Later on many theories of thermos-elasticity were developed [2-6]. A lot of problems related to wave propagation in thermoelastic media in reference to these theories have been discussed in the past decades. Most recently, Marin and coworkers [7-9] studied some problems in thermos-elasticity. Codarcea-Munteanu et al. [10] discussed linear thermos-elasticity without energy dissipation in the context of a porous micropolar media. Scutaru et al. [11] showed that the existence of symmetries can lead to the disappearance of some terms of the forced vibration response of a mechanical system. Vlase et al. [12] developed a model that uses vector methods to analyze the forces that appear in the single cylinder of an IC engine.

Functionally graded materials [13-15] which regulates the thermal variation was developed in Japan 1984 using powder metallurgy which makes FGM a significant thermal barrier. The thermo-mechanical analysis of FG cylinders and plates studied by Reddy and Chin [16]. Sankar and Tzeng [17] studied the thermal stresses in functionally graded beams. Some prominent work in the field of functionally graded thermoelastic materials has been done by Mallik and Kanoria [18], Abbas and Zenkour [19], Ghunghas et.al. [20], Kalkal et al. [21], Barak and Dhankar [22], and Sheokand et al. [23] in recent times.

During the past few years, semiconductor and its applications in different engineering fields have caught the eyes of researchers. The light energy changes the properties and temperature of the semiconducting material. This temperature gradient helps for free carrier density to appear in the medium. Gordan et al. [24] discovered electronic

* Corresponding author. Tel.: +0-000-000-0000 ; fax: +0-000-000-0000 .
E-mail address: author@institute.xxx

deformations to photothermal spectroscopy. Photothermal methods are being applied for measuring physical quantities such as temperature, electric effects of semiconducting material [25, 26]. Besides this many researchers have investigated problems related to semiconductors [27-31]. The study of semiconducting medium has caught the attention of researchers in the last five years. Many research problems [32-44] in this field have been published.

In the present paper, the deformation in a non-homogeneous photothermal semiconducting rod with an internal heat source of finite length is discussed. The rod is heated at the left end. Displacement, carrier density, temperature field and stresses are calculated and presented graphically. The effect of non-homegeneity parameter is shown for different theories of thermoelasticity.

2. Basic Equations

We shall consider a thin semi-infinite non-homogeneous photothermal semiconducting rod with internal heat source occupying the region $x \geqslant 0$. The basic equations and stress relations for the homogeneous semiconducting solid are given by Mandelis et al. [30] and Todorovic [31].

$$\sigma_{ji,j} = \rho \ddot{u}_i, \tag{1}$$

$$D_e \nabla^2 N(\vec{r},t) - \frac{1}{\tau} N(\vec{r},t) + \kappa T(\vec{r},t) - \frac{\partial N(\vec{r},t)}{\partial t} = 0, \tag{2}$$

$$K\nabla^2 T(\vec{r},t) + \frac{E_g}{\tau} N(\vec{r},t) - \gamma_T T_0 \left(1 + \eta_1 \tau_0 \frac{\partial}{\partial t}\right) \nabla \cdot \frac{\partial \vec{u}(\vec{r},t)}{\partial t}$$
$$-\rho C_e \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial T(\vec{r},t)}{\partial t} = Q, \tag{3}$$

$$\sigma_{ij} = \left(1 + \eta_2 \tau_1 \frac{\partial}{\partial t}\right)\left(2\mu e_{ij} + \lambda \delta_{ij} e_{k,k}\right) - \delta_{ij}\left[\gamma_T \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + \gamma_N N\right],$$
$$e_{ij} = \frac{1}{2}\left(u_{j,i} + u_{i,j}\right), \tag{4}$$

where λ,μ represent material constants, σ_{ij} is stress tensor, ρ density, N carrier density, E_g energy gap of semiconductor, C_e specific heat at constant strain, δ_n difference of deformation potential of conduction and valence band, D_e carrier diffusion coefficient, K coefficient of thermal conductivity, α_t coefficient of linear thermal expansion, $\kappa=((\partial N_0)/\partial t)(T/\tau)$ is the heating activation coupling parameter, N_0 is equilibrium carrier concentration at temperature T, τ photogenerated carrier lifetime, T thermodynamic temperature, Q is internal heat source, $\gamma_T=(3\lambda+2\mu)\alpha_t$, $\gamma_N=(3\lambda+2\mu)\alpha_n$ in semiconduting medium.

Here τ_0, τ_1 denote thermal relaxation times such that $\tau_1 \geqslant \tau_0 \geqslant 0$, η_1, η_2 are the parameters taken to reduce the above system to

1. MGL model as $\eta_1=\eta_2=1$,
2. GL model as $\eta_1=\eta_2=0$,
3. LS model as $\eta_1=1,\eta_2=0,\tau_1=0$,
4. CTE model as $\eta_1=0,\eta_2=1,\tau_0=\tau_1=0$.

For a functionally graded medium, the parameters $\lambda,\mu,\gamma_T,\gamma_N,\rho,k,E_g,Q$ are assumed to be space dependent, unlike homogeneous medium where they are considered to be constants. Hence, we change $\lambda,\mu,\gamma_T,\gamma_N,\rho,k,E_g$ to $\lambda_0 f(\vec{x}),\mu_0 f(\vec{x})$, $\gamma_{T0} f(\vec{x}),\gamma_{N0} f(\vec{x}),\rho_0 f(\vec{x}),k_0 f(\vec{x}),E_{g0} f(\vec{x}),Q_0 f(\vec{x})$ respectively, where $\lambda_0,\mu_0,\gamma_{T0}$, $\gamma_{N0},\rho_0,k,E_{g0},Q_0$ are treated as constants and $f(\vec{x})$ depends on the space variables (x,y,z). It is also supposed that the material properties vary only along the x axis. Hence, we can take $f(\vec{x})=f(x)$.

Using the above parameters, equation of motion together with the constitutive relation, the energy equation in a non-homogeneous medium is given by:

$$\sigma_{ji,j} = f(\vec{x})\rho_0 \ddot{u}_i \tag{5}$$

$$\sigma_{ij} = f(\vec{x})\left(1 + \eta_2 \tau_1 \frac{\partial}{\partial t}\right)\left(2\mu e_{ij} + \lambda \delta_{ij} e_{k,k}\right) - \delta_{ij} f(\vec{x})\left[\gamma_T \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + \gamma_N N\right], \tag{6}$$

With $$\gamma_{T0} = (3\lambda_0 + 2\mu_0)\alpha_t, \gamma_{N0} = (3\lambda_0 + 2\mu_0)\alpha_n$$

3. Problem defined

For two-dimensional problem, the equation of motion (5) with the help of equation (6) in component form reduces to:

$$f(x)\left[\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)(\lambda_0+2\mu_0)\frac{\partial^2 u}{\partial x^2}-\gamma_{T0}\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}-\gamma_{N0}\frac{\partial N}{\partial x}\right]+$$
$$\frac{\partial}{\partial x}f(x)\left[\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)(\lambda_0+2\mu_0)\frac{\partial u}{\partial x}-\gamma_{T0}\left(1+\tau_1\frac{\partial}{\partial t}\right)T-\gamma_{N0}N\right]=\rho_0 f(x)\frac{\partial^2 u}{\partial t^2}\ . \quad (8)$$

The Energy equation (7) reduces to

$$K_0\left[f(x)\nabla^2 T+\frac{\partial}{\partial x}f(x)\frac{\partial T}{\partial x}\right]=\rho_0 C_e f(x)\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}$$
$$-f(x)\frac{E_{g0}}{\tau}N+f(x)\gamma_{T0}T_0\left(1+\eta_1\tau_0\frac{\partial}{\partial t}\right)\nabla\cdot\frac{\partial\vec{u}}{\partial t}+Q_0 f(x). \quad (9)$$

The stress components arising from equation (6) reduces to:

$$\sigma_{xx}=f(x)\left[\left(1+\eta_2\tau_1\frac{\partial}{\partial t}\right)(\lambda_0+2\mu_0)\frac{\partial u}{\partial x}-\gamma_{T0}\left(1+\tau_1\frac{\partial}{\partial t}\right)T-\gamma_{N0}N\right]. \quad (10)$$

4. Non-Homogeneity

Assuming f(x)=e^(-nx), where n is dimensionless parameter. This indicates that the physical properties of non-homogeneous medium vary exponentially in x direction. For the convenience of numerical calculation, following dimensionless parameters are introduced:

$$x'=\frac{1}{c_1t^*}x,\ u'=\frac{1}{c_1t^*}u,\ t'=\frac{t}{t^*},\ Q_0'=Q_0,$$
$$\sigma_{xx}'=\frac{\sigma_{xx}}{\mu_0},\ T'=\frac{\gamma_{T0}}{(\lambda_0+2\mu_0)}T,\ N'=\frac{\gamma_{N0}}{(\lambda_0+2\mu_0)}N\ , \quad (11)$$

where,

$$c_1^2=\frac{(\lambda_0+2\mu_0)}{\rho_0},\ t^*=\frac{\kappa_0}{\rho_0C_ec_1^2}$$

Using (11) in equations (2), (8)-(10), the equations takes the following form:

$$\left[\left(1+\frac{\eta_2\tau_1}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial x^2}-\left(1+\frac{\tau_1}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}-\frac{\partial N}{\partial x}\right]-nc_1t^*\left[\left(1+\frac{\eta_2\tau_1}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}-\left(1+\frac{\tau_1}{t^*}\frac{\partial}{\partial t}\right)T-N=\frac{\partial^2 u}{\partial t^2},\right. \quad (12)$$

$$\frac{\partial^2 T}{\partial x^2}-nc_1t^*\frac{\partial T}{\partial x}-b_4\left(1+\frac{\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t}-\epsilon_1\left(1+\frac{\eta_1\tau_0}{t^*}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\frac{\partial u}{\partial x}+\epsilon_2 N-Q_0^*=0, \quad (13)$$

$$\frac{\partial^2 N}{\partial x^2}-b_1N+\epsilon_3T-b_2\frac{\partial N}{\partial t}=0, \quad (14)$$

where,

$$b_1=\frac{c_1^2t^{*2}}{\tau D_e},\ b_2=\frac{c_1^2t^*}{D_e},\ \epsilon_3=\frac{\kappa c_1^2t^{*2}\gamma_{N0}}{D_e\gamma_{T0}}$$

$$b_4=\frac{\rho_0C_ec_1^2t^{*2}}{\kappa_0},\ \epsilon_1=\frac{\gamma_{T0}^2T_0t^*}{\rho_0\kappa_0},\ \epsilon_2=\frac{E_{g0}c_1^2t^{*2}\gamma_{T0}}{\gamma_{N0}\tau\kappa_0},\ Q_0^*=\frac{c_1^2t^{*2}}{\kappa_0}Q_0. \quad (15)$$

5. Problem Solution

The solution of the parameters is considered in the form of modes as:

$$\{u,N,T\}=[\bar{u},\bar{N},\bar{T}](x)\exp(\omega t), \quad (16)$$

where ω is complex frequency.

On substituting (16) in (12)-(14), the following equations are obtained:

$$\left(H_1\frac{d^2}{dx^2}-na_1H_1\frac{d}{dx}-\omega^2\right)\bar{u}+\left(na_1-\frac{d}{dx}\right)(H_2\bar{T}+\bar{N})=0, \quad (17)$$

$$-\epsilon_1H_4\frac{d\bar{u}}{dx}+\left(\frac{d^2}{dx^2}-na_1-b_4H_3\right)\bar{T}+\epsilon_2\bar{N}=Q_0^*, \quad (18)$$

$$\left(\frac{d^2}{dx^2}-b_1-b_2\omega\right)\bar{N}+\epsilon_3\bar{T}=0, \quad (19)$$

where,

$$H_1=1+\frac{\eta_2\tau_1}{t^*}\omega,\ H_2=1+\frac{\tau_1}{t^*}\omega, H_3=\omega\left(1+\frac{\tau_0}{t^*}\omega\right)$$

$$H_4=\omega\left(1+\eta_1\frac{\tau_0}{t^*}\omega\right),\ a_1=c_1t^*. \quad (20)$$

Solving equations (17)-(19), we obtain a sixth order differential equation given by,

$$\left[\frac{d^6}{dx^6}+ng_1\frac{d^5}{dx^5}+g_2\frac{d^4}{dx^4}+ng_3\frac{d^3}{dx^3}+g_4\frac{d^2}{dx^2}+ng_5\frac{d}{dx}+g_6\right](\bar{N},\bar{u},\bar{T})=Q_0^* \quad (21)$$

where,

$$g_1 = -a_1,\ g_2 = -\frac{f_1 + H_1(b_1 + b_2\omega)}{H_1},\ g_3 = \frac{f_2 + a_1 H_1(b_1 + b_2\omega)}{H_1},$$
$$g_4 = \frac{f_3 + f_1(b_1 + b_2\omega) + \epsilon_3 f_4}{H_1},\ g_5 = \frac{f_5\epsilon_3 - f_2(b_1 + b_2\omega)}{H_1},$$
$$g_6 = \frac{f_6\epsilon_3 - f_3(b_1 + b_2\omega)}{H_1},\ f_1 = \epsilon_1 H_2 H_4 + n a_1 H_1 + b_4 H_1 H_3 + \omega^2,$$
$$f_2 = a_1\epsilon_1 H_2 H_4 + n a_1^2 H_1 + a_1 b_4 H_1 H_3,\ f_3 = \omega^2(n a_1 + b_4 H_3),$$
$$f_4 = \epsilon_1 H_4 - \epsilon_2 H_1,\ f_5 = a_1(\epsilon_2 H_1 - \epsilon_1 H_4),\ f_6 = \omega^2\epsilon_2. \tag{22}$$

The solution of equation (21) is given by

$$\bar{N} = E_1 e^{-k_1 x} + E_2 e^{k_1 x} + E_3 e^{-k_2 x} + E_4 e^{k_2 x} + E_5 e^{-k_3 x} + E_6 e^{k_3 x}, \tag{23}$$
$$\bar{T} = E_1^* e^{-k_1 x} + E_2^* e^{k_1 x} + E_3^* e^{-k_2 x} + E_4^* e^{k_2 x} + E_5^* e^{-k_3 x} + E_6^* e^{k_3 x}, \tag{24}$$
$$\bar{u} = E_1^{**} e^{-k_1 x} + E_2^{**} e^{k_1 x} + E_3^{**} e^{-k_2 x} + E_4^{**} e^{k_2 x} + E_5^{**} e^{-k_3 x} + E_6^{**} e^{k_3 x}, \tag{25}$$

where $k_i^2 (i = 1,2,3)$ are the roots of the equation (21) and $E_i^*(\omega), E_i^{**}(\omega)(i = 1,2,3)$ depends on ω, and are given by

$$E_1^* = E_4^* = X_{11}E_1,\ E_2^* = E_5^* = X_{12}E_2,\ E_3^* = E_6^* = X_{13}E_3,$$
$$E_\zeta^{**} = X_{2\zeta}E_\zeta, (\zeta = 1,2 \ldots 6),$$

where

$$X_{11} = \frac{b_1 + b_2\omega - k_1^2}{\epsilon_3},\ X_{12} = \frac{b_1 + b_2\omega - k_2^2}{\epsilon_3},\ X_{13} = \frac{b_1 + b_2\omega - k_3^2}{\epsilon_3},$$
$$X_{21} = \frac{(na_1 + k_1)(X_{11}H_2 + 1)}{\omega^2 - H_1 k_1(na_1 + k_1)},\ X_{22} = \frac{(na_1 - k_1)(X_{11}H_2 + 1)}{\omega^2 + H_1 k_1(na_1 - k_1)},$$
$$X_{23} = \frac{(na_1 + k_2)(X_{12}H_2 + 1)}{\omega^2 - H_1 k_2(na_1 + k_2)},\ X_{24} = \frac{(na_1 - k_2)(X_{12}H_2 + 1)}{\omega^2 + H_1 k_2(na_1 - k_2)},$$
$$X_{25} = \frac{(na_1 + k_3)(X_{13}H_2 + 1)}{\omega^2 - H_1 k_3(na_1 + k_3)},\ X_{26} = \frac{(na_1 - k_3)(X_{13}H_2 + 1)}{\omega^2 + H_1 k_3(na_1 - k_3)}.$$
$$h_1 = \frac{(b_1 + b_2\omega)Q_0^*}{\epsilon_3 g_6},\ h_2 = \frac{[na_1 H_2(b_1 + b_2\omega) - \epsilon_3]Q_0^*}{\omega^2\epsilon_3 g_6}. \tag{26}$$

6. Boundary Conditions

We consider a functionally graded semiconducting rod under photothermal theory with its boundary $0 \le x \le l$. The rod has a uniform temperature T_0. At the left end of the rod, a heat source is applied. Hence, the boundary conditions for this particular problem are,

$$(i) u(x,t) = 0, \text{ at x } = 0, l,$$
$$(ii) N(x,t) = 0, \text{ at x } = 0, l,$$
$$(iii) T(0,t) = P_0 \exp(\omega t),$$
$$(iv) T(l,t) = 0. \tag{27}$$

Using (16), (23)-(25) in (27), we get a system of six equations given by:

$$X_{21}E_1 + X_{22}E_2 + X_{23}E_3 + X_{24}E_4 + X_{25}E_5 + X_{26}E_6 = 0, \tag{28}$$

$$X_{21}e^{-k_1 l}E_1 + X_{22}e^{k_1 l}E_2 + X_{23}e^{-k_2 l}E_3 + X_{24}e^{k_2 l}E_4 + X_{25}e^{-k_3 l}E_5 + X_{26}e^{k_3 l}E_6 = 0, \tag{29}$$

$$X_{11}E_1 + X_{12}E_2 + X_{13}E_3 + X_{11}E_4 + X_{12}E_5 + X_{13}E_6 = P_0, \tag{30}$$

$$X_{11}e^{-k_1 l}E_1 + X_{12}e^{k_1 l}E_2 + X_{13}e^{-k_2 l}E_3 + X_{11}e^{k_2 l}E_4 + X_{12}e^{-k_3 l}E_5 + X_{13}e^{k_3 l}E_6 = 0, \tag{31}$$

$$e^{-k_1 l}E_1 + e^{k_1 l}E_2 + e^{-k_2 l}E_3 + e^{k_2 l}E_4 + e^{-k_3 l}E_5 + e^{k_3 l}E_6 = 0. \tag{32}$$

The values of constants $E_\zeta(\zeta = 1, \dots 6)$ are evaluated using MATLAB software.

Using the expressions of $\bar{u}, \bar{N}$ and $\bar{T}$ given by (23)-(25) in the expressions (16), the displacement component, carrier density, temperature field and stress in functionally graded semiconducting medium are obtained.

7. Validation of Results

To validate the theoretical results, Silicon (Si) material is taken as the semiconducting solid for which the constants are (Song et al. [30]),

$\lambda_0 = 3.64 \times 10^{10}\ \mathrm{N/m^2}$, $\mu_0 = 5.46 \times 10^{10}\ \mathrm{N/m^2}$, $\rho_0 = 2330\mathrm{Kg/m^3}$, $\mathrm{T_0} = 800\ \mathrm{K}$,
$\tau = 5 \times 10^{-5}\ \mathrm{s}$, $\mathrm{D}_e = 2.5 \times 10^{-3}\ \mathrm{m^2/s}$, $\mathrm{E}_{g0} = 1.11\ \mathrm{V}$,
$\alpha_t = 4.14 \times 10^{-6}/\mathrm{K}$, $\kappa = 150\ \mathrm{W/mK}$, $\alpha_n = -9 \times 10^{-31}\ \mathrm{m^3}$,
$C_e = 695\ \mathrm{J/KgK}$,

The other constants are assumed as $Q_0 = P_0 = 1.0, \omega = \omega_0 + \iota\varsigma, \omega_0 = -0.2, \varsigma = 0.1$.

The numerical results are obtained for displacement, force stress, carrier density and temperature distribution for $l = 1.0, \tau_0 = 0.075, \tau_1 = 0.1$ against horizontal distance x. The graphical results are shown for Modified Green-Lindsay (MGL) theory, Green-Lindsay (GL) theory, Lord-Shulman (LS) theory and Coupled (CT) theory for homogeneous $(n = 0)$ and non-homogeneous $(n \neq 0)$ thermoelastic medium.

8. Discussions

Figure 1 shows the variation of displacement against horizontal distance for two values of non-homogeneity parameter n namely 0 and 1 in the context of different models of generalized thermoelasticity. The variation is linear and coincides for both values of n in the case of the MGL model. A similar variation is observed for the CTE model. Also, the value of displacement is maximum for the MGL model. Further, the variation is linear for the LS model but the value of displacement is greater for $n = 0$. For the GL model, linear variation is observed initially for both values of n which is followed by exponential increase. Further, the magnitude of displacement is more for $n = 0$. Figure 2 shows the variation of normal stress against horizontal distance. This variation is almost linear and coincides for all models and for both values of n except in the case of the GL model for $n = 0$. For this case, the variation is first linear in nature and then increases exponentially to attain the maximum value of normal stress.

Figure 3 shows the variation of temperature against horizontal distance. This variation is linear and coincides for MGL, LS and CTE models in the case of both values of n. The exceptional behavior is observed for the GL model where variation is linear initially but as horizontal distance increases temperature decreases exponentially for both values of n attaining the minimum value. Figure 4 shows the variation of carrier density against horizontal distance. The behavior of carrier density is identical as observed in the case of normal stress.

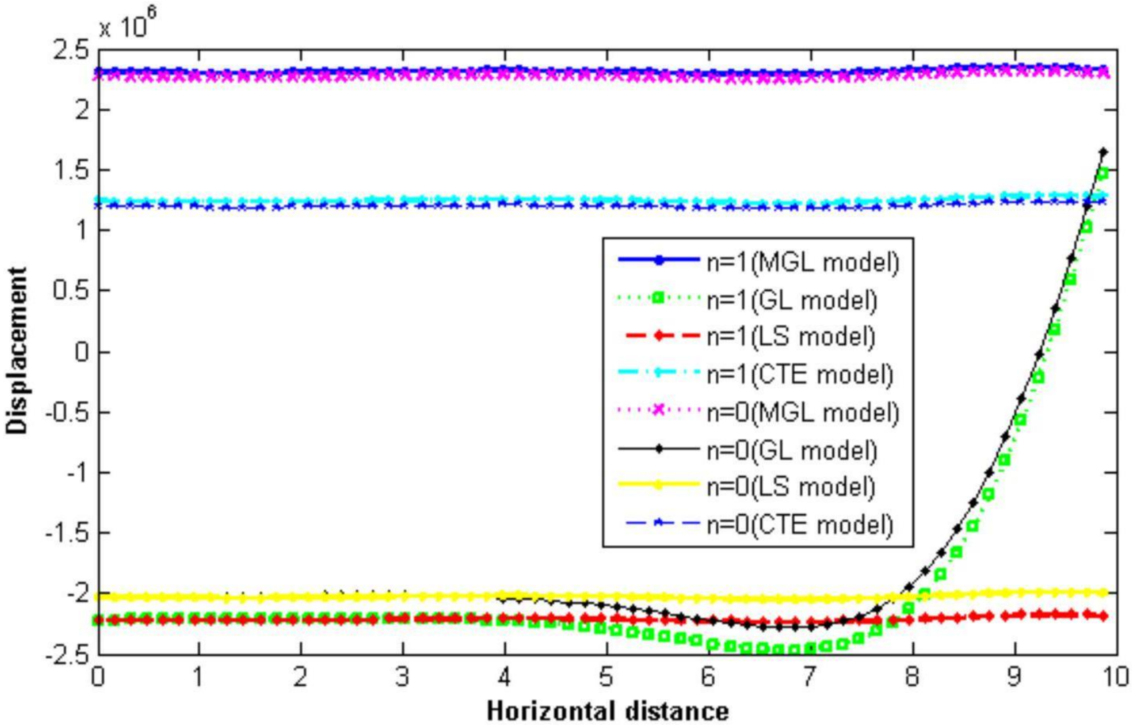

Figure 1: Variation of displacement u with horizontal distance x

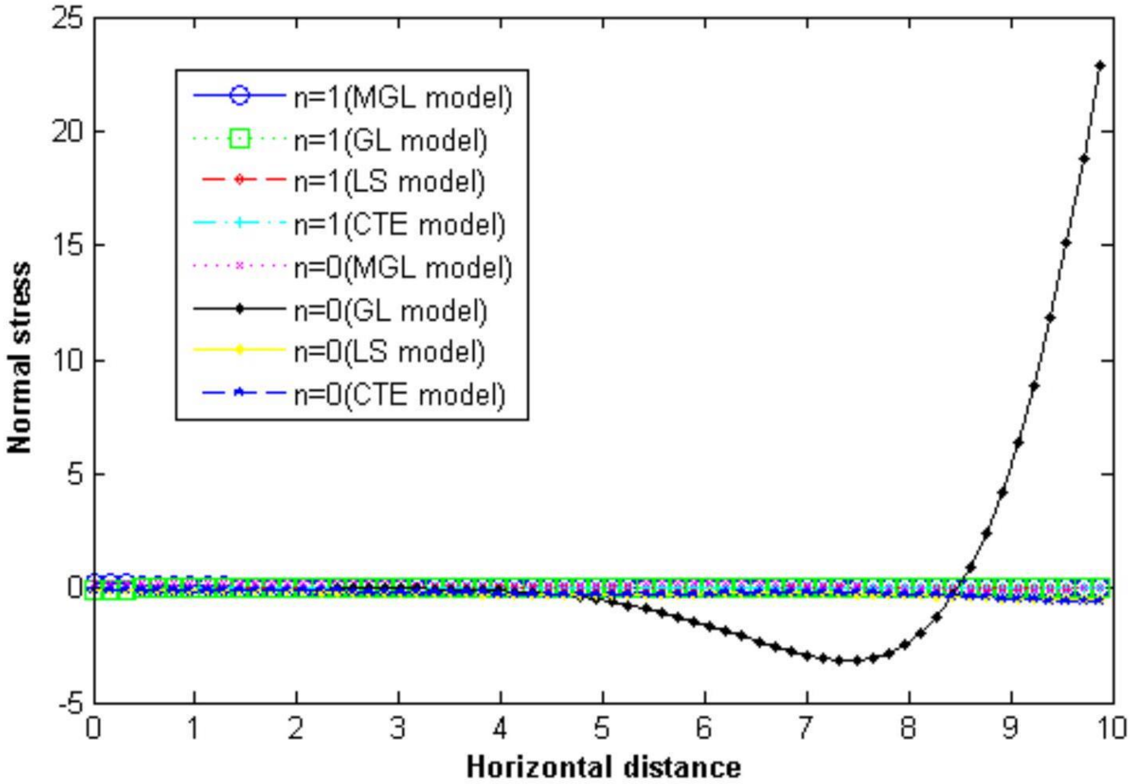

Figure 2: Variation of force stress σ_{xx} with horizontal distance x

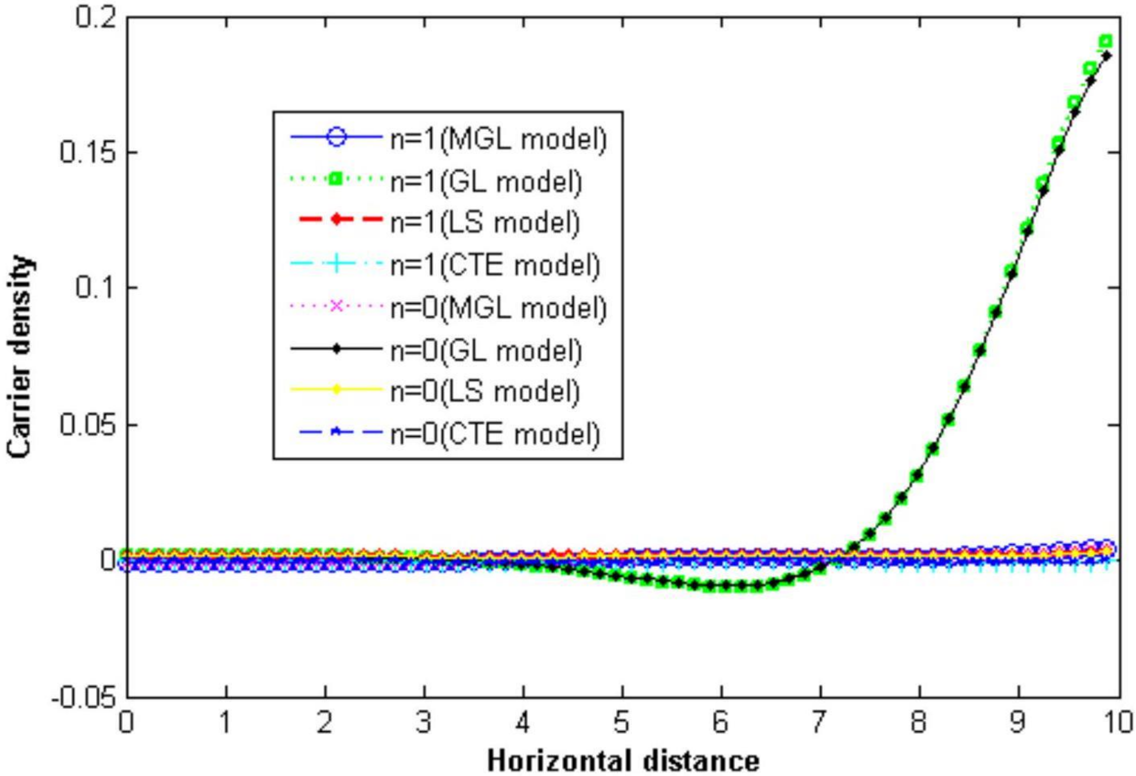

Figure 3: Variation of Carrier density N with horizontal distance x

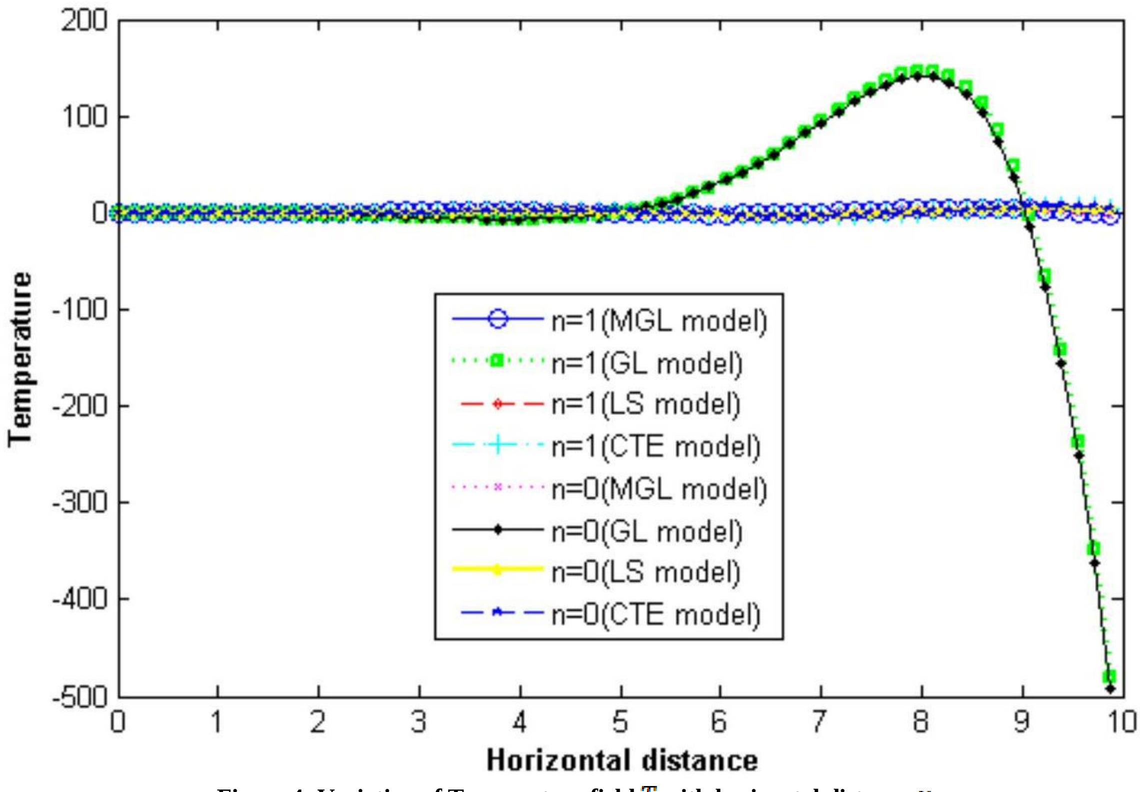

Figure 4: Variation of Temperature field T with horizontal distance x

9. Conclusion

The analytical and graphical results of the present investigation conclude that:

1. All physical quantities show linear variation for MGL, CTE and LS models except the GL model.

2. The maximum values of carrier density and normal stress are observed for the GL model.

3. The minimum value of temperature is observed for the GL model.

4. The displacement is most affected by values of non-homogeneity parameter in the context of GL and LS models whereas normal stress is greatly affected by values of non-homogeneity parameter in the context of GL model.

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