Journal of Computational Applied Mechanics 2024, 55 (1): 26-38

DOI: 10.22059/JCAMECH.2024.370519.934

RESEARCH PAPER

Stability Analysis of a Laminated Composite Micro scaled beam embedded in elastic medium using modified coupled stress theory

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Abstract

This paper presents size dependent stability analysis a cantilever micro laminated beam embedded in elastic medium by using the modified coupled stress theory which includes the length scale parameter. The micro beam subjected to compressive load is considered as three composite laminas and embedded in elastic medium which is modelled in the Winkler foundation model. In the obtaining of the governing equations, the energy principle is used. In the solution of the buckling problem, the energy based Ritz method is implemented with algebraic polynomials. In order to accuracy obtained expressions and used method, a comparative study is performed. Many parametric studies are presented in order to investigate the buckling of laminated micro beams. For this purpose, effects of stacking sequence of laminas, geometric parameters, length scale parameter, fiber orientation angle, the parameter of elastic medium on critical buckling loads of laminated micro beams are investigated.

Keywords: Micro Scaled Beam; Laminated Composites; Buckling; Elastic Medium; Modified Coupled Stress Theory

1. Main text

Using micro/nano scaled structures in engineering projects is increasing in the process of time for instance microactuators, ultra-thin film, micro- and nano-electro mechanical systems, biosensors. In researching of mechanical behavior of micro/nano scaled materials, experimental study is still difficult in this day and age. Therefore, computational modelling and theoretical approach of these structures gain importance in the design and investigation stages. In the computational modelling of these type structures, molecular dynamics simulation and nonlocal continuum models are preferred. The couple stress theory is a type of nonlocal continuum models which consist of size effect for micro/nano scales. In the literature, a lot of researchers investigated and developed the nonlocal continuum theory in the analysis nano/micro structures (Mindlin [1,2], Eringen [3,4], Toupin [5], Lam et al. [6], Yang et al. [7], Park and Gao [8]). Yang et al. [7] investigated strain energy formulations of the Modified Couple Stress Theory (MCST). A lot researches used MCST to mechanical analysis of micro/nano structural elements [9-42].

In last decade, mechanical analysis of micro/nano composite structures is an interesting topic and investigated by a lot of researches. In last years, laminated micro/nano structures which one type of composite structures are used a

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lot of studies about analytical and computational solutions by some researches. For example, Chen and Li [43] used MCST for investigation of vibration of layered micro-scaled beams. Roque et al. [44] used meshless method for static analysis of laminated micro beams based MCST. Buckling behaviour of layered micro scaled beams is investigated via MCST with various beam theories for by Abadi and Daneshmehr [45]. Mohammadabadi et al. [46] used differential quadrature method for thermal buckling behaviour of laminated microbeams. Vibration of nonuniform micro scaled beams is investigated by Hosseini Hashemi and Bakhshi Khaniki [47]. Feng et al. [48] presented nonlinear static analysis of layered Timoshenko nano beams. Dong et al. [49] analysed hygro-thermal post-buckling of layered micro scaled beams based MCST. Nguyen et al. [50] implemented Ritz method for static, vibration and stability of layered beams based MCST. Jouneghani et al. [51] investigated vibration of curved shells by using MCST. Khaniki et al. [52] used differential quadrature method for buckling analysis of nonuniform beams by using nonlocal strain gradient beams. Lal and Dangi [53] presented vibration responses of functionally graded Timoshenko nanobeams using nonlocal theory. Bhattacharya and Das [54] presented dynamics of tapered micro scaled beam with functionally graded material under thermal effect by using MCST. Jouneghani et al. [55] presented an investigation about dynamics of layered beams via MCST and Ritz method. Priyanka and Pitchaimani [56] used Ritz method for static stability and vibration of layered microbeams. Akbaş [57, 58] used Ritz method for d dynamically analysis of layered micro scaled beams under moving load based MCST.

The topic of this study was investigated for moving load analysis by the author [57, 58]. Effects of elastic medium or foundation on the stability analysis of laminated micro beams have not been investigated. This study aims to fill this blank for stability analysis of laminated micro beams embedded in elastic medium by using MCST and Ritz method. Contribution to literature of this study is to investigate and obtaining formulations and Ritz solution procedures of buckling-stability analysis of laminated micro beams embedded in elastic medium based on MCST. The elastic medium is modelled in the Winkler foundation model. Effects of geometric parameters, length scale constant, parameter of the elastic medium, stacking sequence of laminas and fiber angle on the critical buckling loads of the laminated micro scaled beam are obtained and discussed.

2. Formulations

A cantilever laminated micro scaled beam made of three identical laminas under compressive force (P) at the free end embedded in elastic medium is shown with the Cartesian coordinates X_1, X_2 and X_3 in figure 1. The length, width and height of the laminated micro beam are indicated as L, b and h, respectively. The elastic medium is considered as the Winkler model and its elastic parameter is indicated as k_w.

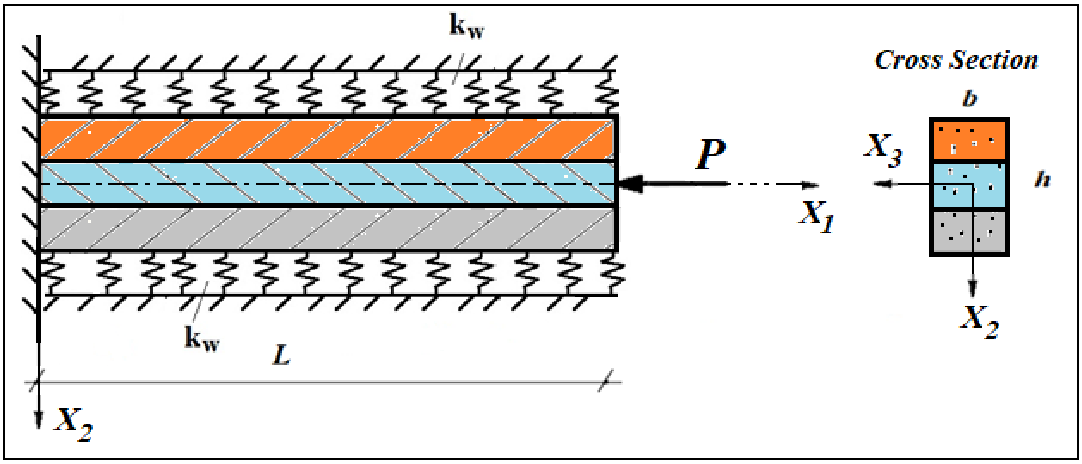

Fig 1: A layered micro scaled beam embedded in elastic medium under a compressive load.

By using MCST, strain energy (U_i) is expressed as follows: (Yang et al. [7]);

$$U_i = \int_V (\boldsymbol{\sigma}:\boldsymbol{\varepsilon} + \boldsymbol{m}:\boldsymbol{\chi})\, dV \tag{1}$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, $\boldsymbol{m}$, $\boldsymbol{\chi}$ indicate stress tensor, strain tensor, the deviatoric part of the couple stress tensor, symmetric curvature tensor, respectively [8];

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon} \tag{2}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}] \tag{3}$$

$$\boldsymbol{m} = 2l^2 G(y)\,\boldsymbol{\chi} \tag{4}$$

$$\boldsymbol{\chi} = \frac{1}{2}[\nabla\theta + (\nabla\theta)^T] \tag{5}$$

where l , θ , λ, μ indicate material length scale parameter, rotation vector and Lamé constants;

$$\theta = \frac{1}{2}\mathrm{curl}\; u \tag{6}$$

$$\lambda = \frac{Ev}{(1+v)(1-2v)},\quad G = \frac{E}{2(1+v)} \tag{7}$$

For n th layer, Displacement-strain and constitutive relations for Euler-Bernoulli beam theory are presented as follows;

$$\varepsilon_{X_1} = \frac{\partial u_1}{\partial X_1} - X_2\frac{\partial^2 u_2}{\partial {X_1}^2} \tag{8}$$

$$\sigma_{X_1}^n = Q_{11}^n\left[\frac{\partial u_1}{\partial X_1} - X_2\frac{\partial^2 u_2}{\partial {X_1}^2}\right] \tag{9}$$

$$\chi_{X_1X_3} = -\frac{1}{2}\left(\frac{\partial^2 u_2}{\partial {X_1}^2}\right) \tag{10}$$

$$m_{X_1X_3}^n = \left(l_k^2\,\breve{Q}_{44}^n + l_k^2\breve{Q}_{55}^n\right)\left[-\frac{1}{2}\left(\frac{\partial^2 u_2}{\partial {X_1}^2}\right)\right] \tag{11}$$

In equations 8-11, u_1 indicate horizontal displacement and u_2 indicate vertical direction. For each layer, Q_{11}^n, $\breve{Q}_{44}^n$, $\breve{Q}_{55}^n$ indicate transformed reduced stiffness constants [43];

$$Q_{11}^n = p^4C_{11}^n + r^4C_{22}^n + 2\,p^2r^2(C_{12}^n + 2C_{66}^n) \tag{12}$$

$$l_n^2\breve{Q}_{44}^n = p^4l_{kb}^2C_{44}^n + r^4l_{km}^2C_{55}^n + \; p^2r^2(l_{kb}^2C_{44}^n + l_{km}^2C_{55}^n) \tag{13}$$

$$l_n^2\breve{Q}_{55}^n = r^4l_{kb}^2C_{44}^n + p^4l_{km}^2C_{55}^n + \; p^2r^2(l_{kb}^2C_{44}^n + l_{km}^2C_{55}^n) \tag{14}$$

In equations (12-14), $p = cos\theta^n$, $r = sin\theta^n$, θ^n indicates angle fiber orientation respect to the X_1 direction for nth layer. For kth layer, l_{km} and l_{kb} indicate length scale constant of matrix and fiber, respectively. In this study, it is considered as $l_{kb} = l_{km} = l_n$. C_{11}^n, C_{44}^n, C_{55}^n , C_{66}^n are elastic stiffness components;

$$C_{11}^n = \frac{E_1^n}{(1-\vartheta_{12}^n)^2}\;,\; C_{12}^n = \frac{\vartheta_{12}^n\,E_2^n}{(1-\vartheta_{12}^n\,\vartheta_{21}^n)},\; C_{22}^n = \frac{E_2^n}{(1-\vartheta_{21}^n)^2},\; C_{44}^n = G_{13}^n\;,\; C_{55}^n = G_{23}^n,\; C_{66}^n = G_{12}^n \tag{15}$$

where G_{13}^n, G_{23}^n and G_{12}^n indicate shear modules. ϑ_{12}^n and ϑ_{21}^n indicate Poisson ratios. Substituting strain and stress expressions to energy expressions, strain energy (U_i) and potential energy of the external loads (U_e) are expressed as following;

$$U_i = \frac{1}{2}\int_0^L \left[A^0 \left(\frac{\partial u_1}{\partial X_1}\right)^2 - 2A^1 \left(\frac{\partial u_1}{\partial X_1}\right)\left(\frac{\partial^2 u_2}{\partial {X_1}^2}\right) + (A^2 + C^0)\left(\frac{\partial^2 u_2}{\partial {X_1}^2}\right)^2 + k_w (u_2)^2 \right] dX_1 \tag{16}$$

$$U_e = -\frac{1}{2}\int_0^L P \left(\frac{\partial u_2}{\partial X_1}\right)^2 \, dX_1 \tag{17}$$

where

$$A^0, A^1, A^2 = \sum_{\mathrm{n}=1}^{\mathrm{nl}} \int_{X_{2n}}^{X_1\ n+1} b\, Q_{11}^n X_2^i \, dX_2 \qquad i=1,2,3 \tag{18a}$$

$$C^0 = \sum_{\mathrm{n}=1}^{\mathrm{nl}} \int_{X_{2n}}^{X_1\ n+1} b\, l_n^2 \left(\breve{Q}_{44}^n + \breve{Q}_{55}^n\right) dX_2 \tag{18b}$$

Total potential energy is presented as:

$$\Pi = U_{\mathrm{i}} + U_e \tag{19}$$

Ritz method is implemented in solution of buckling problem with algebraic polynomials as a series of *m* terms as following.

$$u_1(X_2) = \sum_{k=1}^{m} a_k \varphi_k(X_2) \tag{20}$$

$$u_2(X_2) = \sum_{k=1}^{m} b_k \beta_k(X_2) \tag{21}$$

where a_k and b_k indicate unknown coefficients, and $\varphi_k(X_2)$ and $\beta_k(X_2)$ indicate coordinate functions and presented for cantilever beam as follows:

$$\varphi_k(X_2) = {X_2}^k \tag{22}$$

$$\beta_k(X_2) = {X_2}^{(k+1)} \tag{23}$$

where *j* indicates the number of polynomials. Based on the minimum total potential energy principle, unknown coefficients can be obtained by the conditions:

$$\frac{\partial \Pi}{\partial a_k} = \frac{\partial \Pi}{\partial b_k} = 0 \tag{24}$$

Implementation differentiation of Π in respect to a_k and b_k gives eigenvalue equations for the buckling problem:

$$\{[K] - P[K]^G\}\begin{Bmatrix} a_k \\ b_k \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{25}$$

where $[K]$ and $[K]^G$ are the stiffness and geometric matrixes, respectively :

$$[K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix}, \quad [K]^G = \begin{bmatrix} [K_{11}]^G & [K_{12}]^G \\ [K_{21}]^G & [K_{22}]^G \end{bmatrix} \tag{26}$$

where

$$[K_{11}] = \sum_{i=1}^{k}\sum_{j=1}^{k} \int_0^{\mathrm{L}} A^0 \frac{\partial \varphi_i}{\partial X_1}\frac{\partial \varphi_j}{\partial X_1} dX_1, \tag{27}$$

$$[K_{12}] = [K_{21}]^T = -\sum_{i=1}^{k}\sum_{j=1}^{k} \int_0^{\mathrm{L}} A^1 \frac{\partial \varphi_i}{\partial X_1}\frac{\partial^2 \beta_j}{\partial {X_1}^2} dX_1, \tag{28}$$

$$[K_{22}] = \sum_{i=1}^{k}\sum_{j=1}^{k}\int_0^L (A^1 + C^0)\frac{\partial^2 \beta_i}{\partial X_1{}^2}\frac{\partial^2 \beta_j}{\partial X_1{}^2}\,dX_1, \tag{29}$$

$$[K_{11}]^G = [K_{12}]^G = [K_{21}]^G = 0, \tag{30}$$

$$[K_{22}]^G = \sum_{i=1}^{k}\sum_{j=1}^{k}\int_0^L \frac{\partial \beta_i}{\partial X_1}\frac{\partial \beta_j}{\partial X_1}\,dX_1, \tag{31}$$

The critical buckling loads of the laminated composite beam can be obtained with solving in the eigenvalue equations (Eq.25). Obtained expressions of the problem yields to classical theory (CT) if length scale constant l=0.

3. Analysis Results and Discussion

In the numerical results, effects of geometric parameters (*L/h*), length scale parameter, fiber orientation angle, the parameter of the elastic medium, stacking sequence of laminas on the critical buckling loads of the laminated micro beams are presented. Used parameters are presented as; E_2 = 6.9 *GPa*, E_1 =25 E_2, G_{12} = G_{13}=0.5E_2 , G_{23}=0.2E_2 , ν_{12}=ν_{13}=ν_{23}=0.25 [43], b=h=10μm, l_n=10^{-6} m, the length of the micro beam is changed according to the ratio of *L/h*. The number of the series term is taken as 10 in the Ritz solution.

For accuracy the obtained expressions and used method, a comparison study is implemented in table 1. Critical buckling loads of laminated microbeam for *0/90/0* stacking sequence with pinned-rolled supports are obtained and compared with the results of Abadi and Daneshmehr [45]. As can be seen from table 1 that the presented results are good harmony with the results of Abadi and Daneshmehr [45].

Table 1. A comparison study: Critical buckling load of the laminated microscaled beam with pinned-rolled supports for *0/90/0*.

	Critical buckling load	
Length scale parameter (l_n) μm	Abadi and Daneshmehr	Present
0	0.109672	0.10981
1	0.112363	0.1158
7	0.118283	0.1206
10	0.127432	0.1294

In figure 2 and 3, Critical buckling loads of laminated cantilever micro beam in both MCST and CT ($l_n = 0$) without elastic medium (k_w=0) are plotted for stacking sequences of *0/30/0* and *30/0/30*, respectively. As can be seen from figures 2 and 3, the difference between MCST and CT significantly increase with decrease in *L/h*. In higher ratio of *L/h*, the critical buckling loads of two theories are coincide with each other. It is observed from figures 2 and 3, the difference between MCST and CT in the θ/0/θ are bigger than those of 0/θ/0 stacking sequences. It shows that the stacking sequence effects on the behaviour size effect.

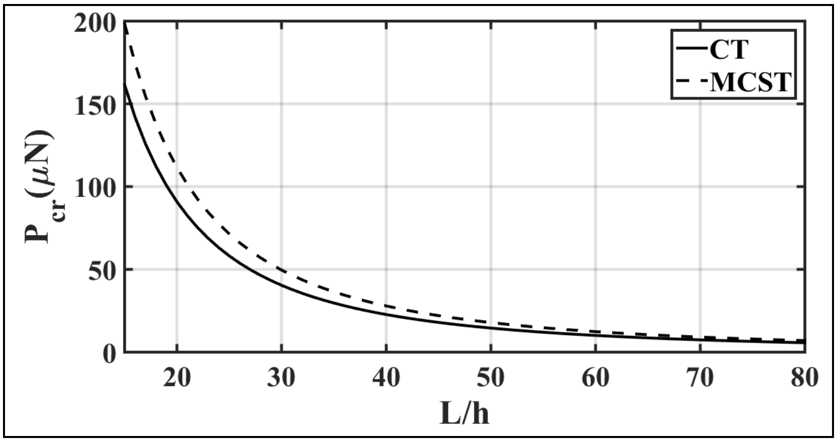

Fig 2: Critical buckling loads versus *L/h* by MCST and CT for *0/30/0* stacking sequence and k_w=0.

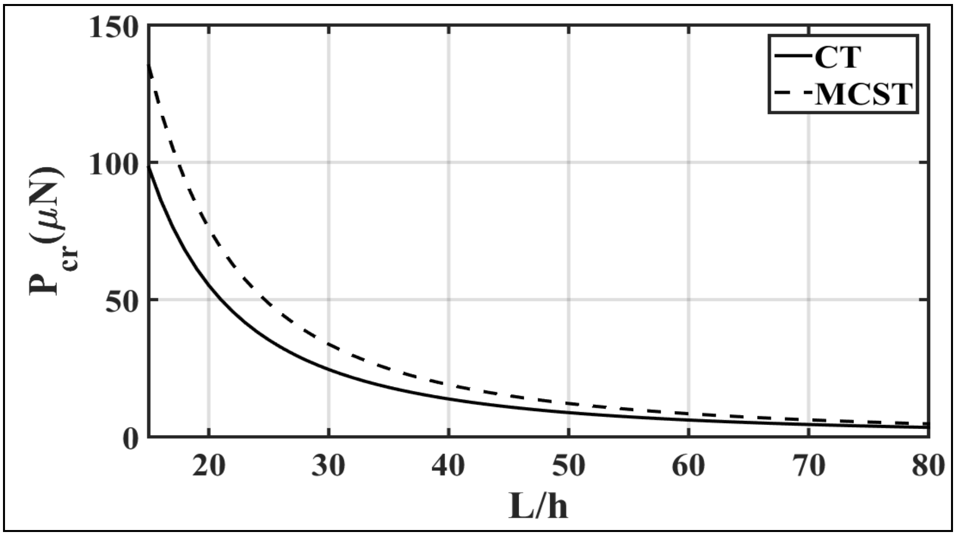

Fig 3: Critical buckling loads versus *L/h* by MCST and CT for 30/0/30 stacking sequence and k_w=0.

In order to investigate the effect of fiber orientation angle (θ) on the critical buckling loads of the laminated cantilever micro beam, θ- P_{cr} relation is presented in figures 4 and 5 for L/h=20 without elastic medium (k_w=0) in both MCST and CT for 0/θ/0 and θ /0/ θ stacking sequences, respectively. Also, In order to better see the effects of fiber orientation angle with together L/h, L/h and θ versus P_{cr} presented in both MCST and CT for 0/θ/0 and θ /0/ θ in figures 6 and 7 stacking sequences, respectively. Figures 4-7 shows that the critical buckling loads dramatically decrease with increase in θ because of decreasing in the rigidity of the laminated micro beam. This decreasing comes into focus on the In θ /0/θ stacking sequences.

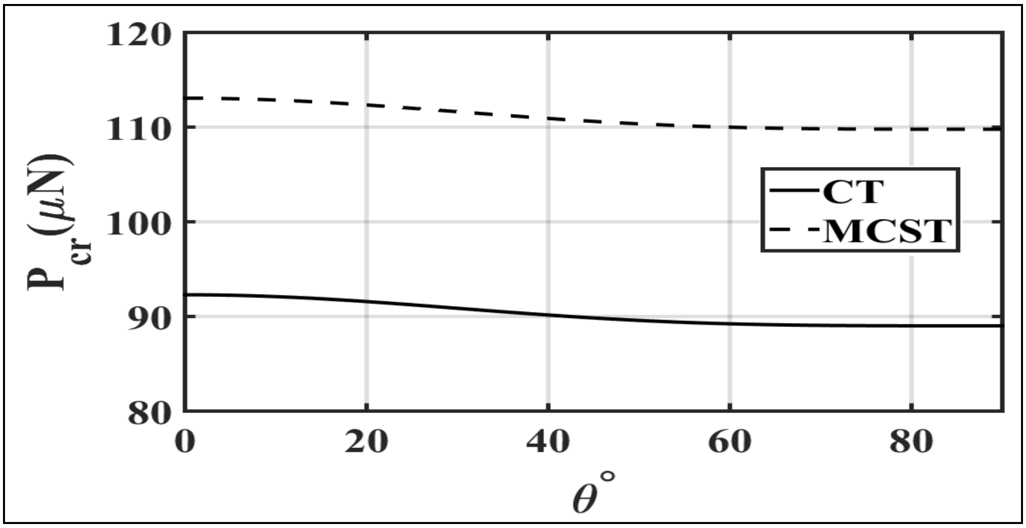

Fig 4: Critical buckling loads versus θ by MCST and CT for L/h=20 for 0/ θ/0 stacking sequence and k_w=0.

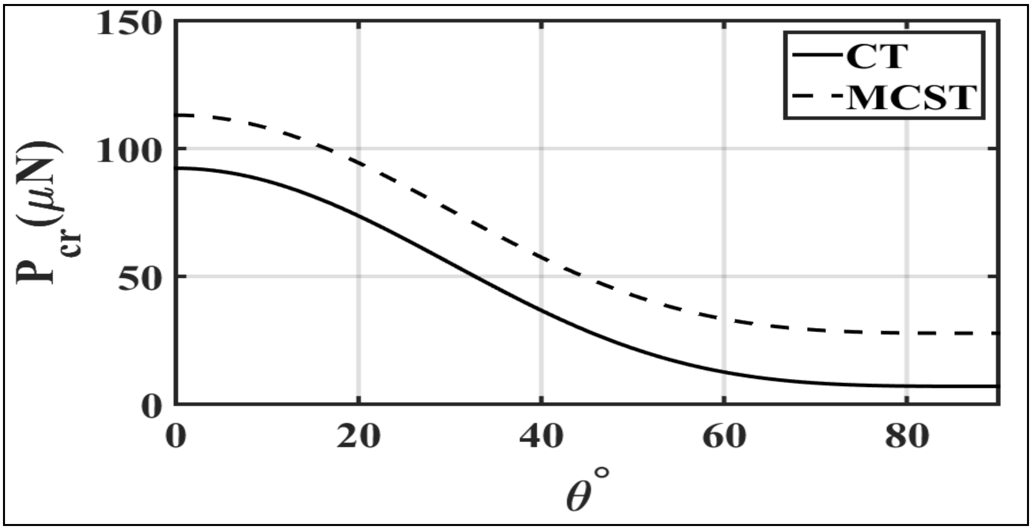

Fig 5: Critical buckling loads versus θ by MCST and CT for L/h=20 for θ /0/ θ stacking sequence and k_w=0.

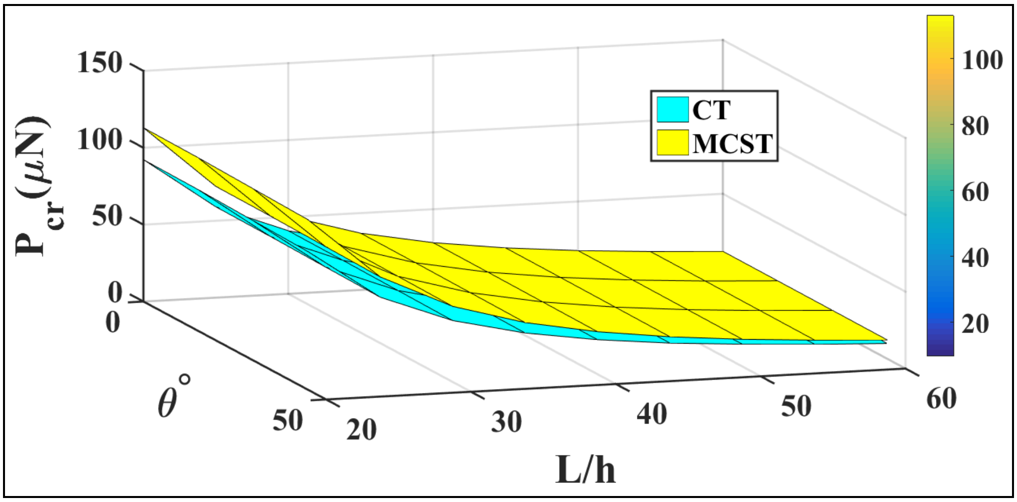

Fig 6: Critical buckling loads versus θ and L/h by MCST and CT for 0/ θ/0 stacking sequence and k_w=0.

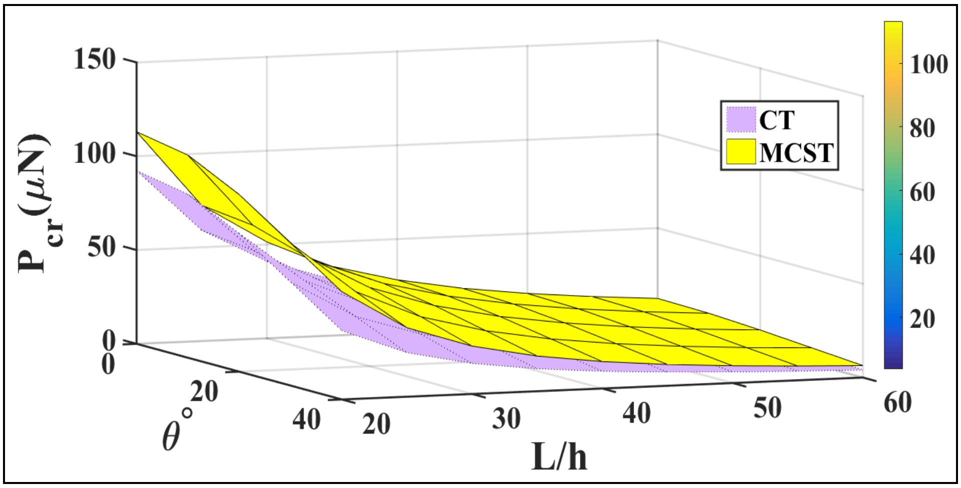

Fig 7: Critical buckling loads versus θ and L/h by MCST and CT for θ/0/ θ stacking sequence and k_w=0.

In order to investigate the effect of elastic medium on the critical buckling loads of the laminated cantilever micro beam, k_w - P_{cr} relation is presented in figures 8 and 9 for L/h=20 in both MCST and CT for stacking sequences of 0/30/0 and 30/0/30, respectively. Also, in figures 10 and 11, k_w and θ versus P_{cr} plotted in both MCST and CT for stacking sequences of 0/30/0 and 30/0/30, respectively. It is shown from figures 8-9 shows that the critical buckling loads considerably increase with increase in k_w. Effects of the k_w parameter on the critical buckling in the θ /0/θ stacking sequence is more than those of the 0/θ/0 stacking sequence. It is observed from figures 8-11 that k_w parameter, size effect has not effect on the size-dependent buckling of the laminated micro beam. The difference between MCST and CT does not change with increase in k_w.

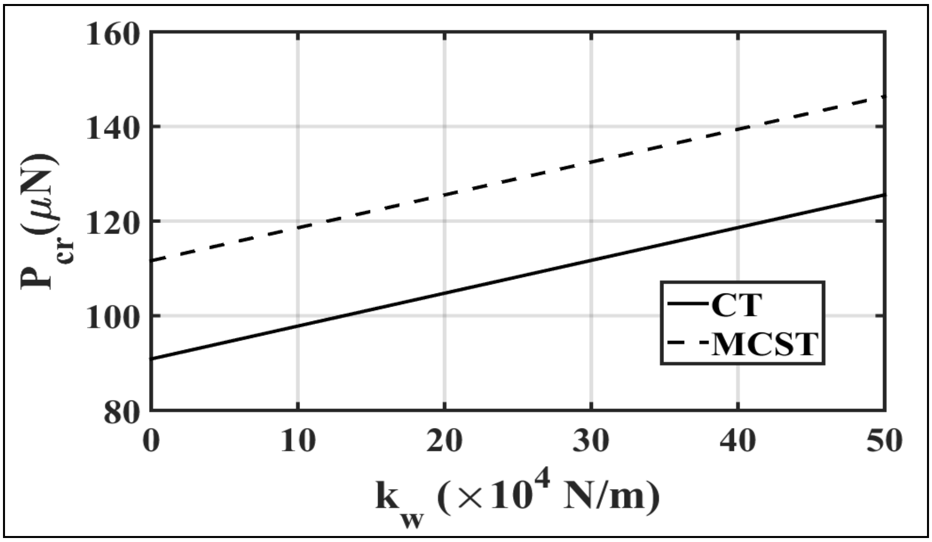

Fig 8: Critical buckling loads versus k_w by MCST and CT for 0/30/0 stacking sequence and L/h=20.

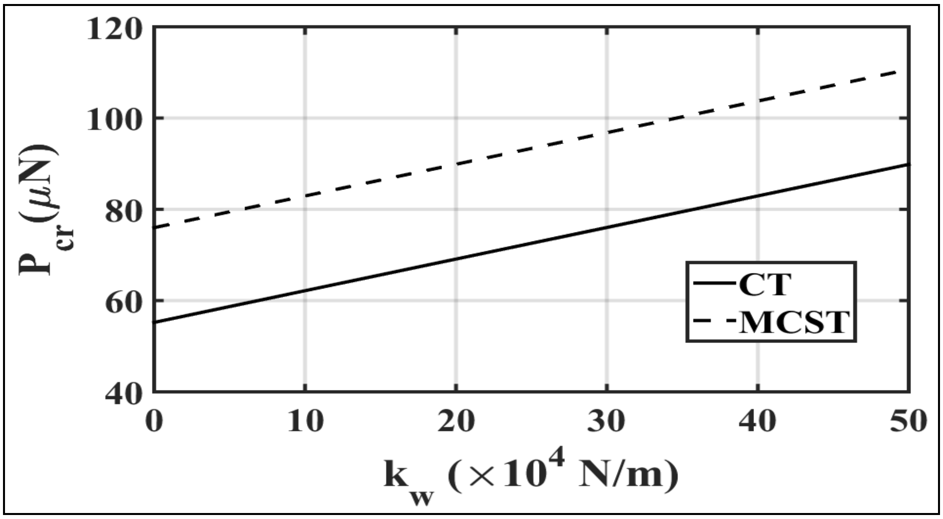

Fig 9: Critical buckling loads versus k_w by MCST and CT for 30/0/30 stacking sequence and L/h=20.

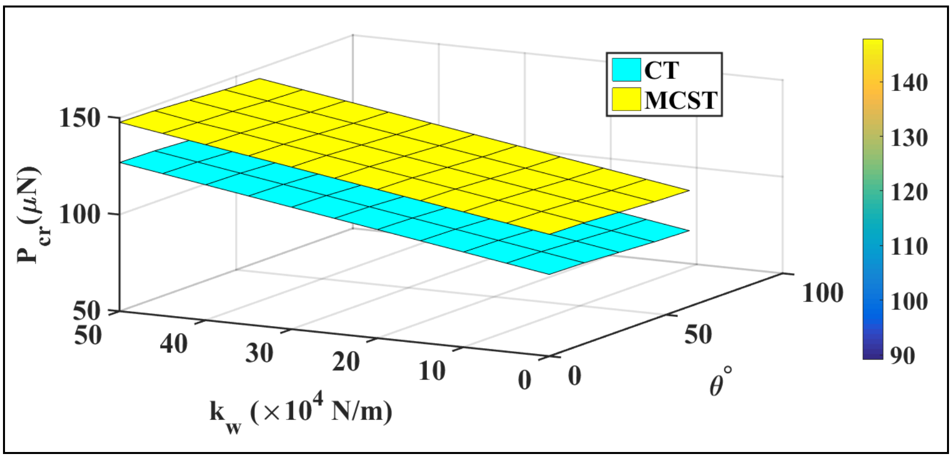

Fig 10: Critical buckling loads versus θ and k_w by MCST and CT for 0/θ/0 stacking sequence and L/h=20.

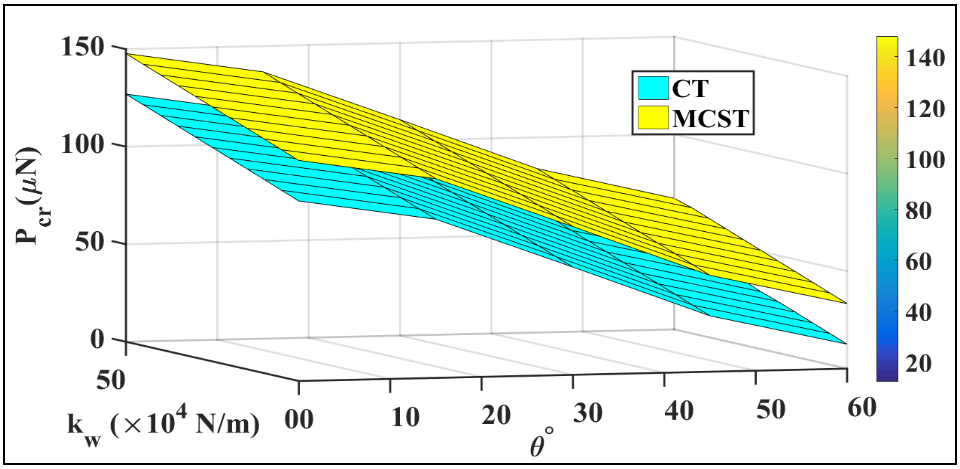

Fig 11: Critical buckling loads versus θ and k_w by MCST and CT for θ/0/θ stacking sequence and L/h=20.

4. Conclusions

In this study, the size dependent buckling of a cantilever laminated micro beam embedded in elastic medium is investigated based on MCST the Winkler foundation model by using the Ritz method with algebraic polynomials. In the numerical studies, effects of geometric parameters, length scale parameter fiber orientation angle, the parameter of the elastic medium, stacking sequence of laminas on the critical buckling loads of the laminated micro beams are obtained and discussed. The findings from this study are summarized as follow; Stacking sequence of laminated micro beam significantly effects on the behavior size dependent buckling loads. The ratio of L/h has important role on the size dependent buckling behavior of laminated micro beams. Effects of elastic medium parameter on the size dependent buckling loads can be changed according to the stacking sequence of the laminated micro beam. In higher values of kw, the critical buckling loads considerably increase.

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