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RESEARCH PAPER

Mathematical model for dynamic analysis of internal combustion engines

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Abstract

The existence of friendly programming environments, which allow the transposition of models developed for different mechanical systems into numerical procedures, easy to access, make it necessary to develop models of mechanical systems used in industry. In this work, we propose to do this for an internal combustion engine. The offered model allows the unitary solution of problems of this type, which involves the calculation of the forces appearing in the engine elements. It offers the possibility to analyze different constructive types of engines. The model is a complex model that finally provides the forces existing in different elements of the engine as well as the developed engine torque.

Keywords: IC engines; crankshaft; camshaft; vector model; numerical procedures

1. Introduction

A basic work in the field is represented by [1], which tries to systematize the main problems that may appear when modeling an Internal Combustion (IC) engine. Different aspects of the analysis are followed, such as design, optimization, stresses that appear in the engine, forces that develop, defects that may appear, etc. IC engines represent an important element in the present world in the transport of goods and people and will remain so for a long time. For this reason, the development of calculation models that allow obtaining results that can be easily used by designers is a major objective for engineers. If you take into account the huge value of this industry, the importance of such studies is easily justified. There is a rich literature that deals with the study of the forces that appear in the components of an IC engine, Researchers study a wide range of problems related to the forces and moments that occur in internal combustion engines and the mechanical effects they can cause. In the paper [2], a software is presented that has the role of simulating the performance of the engine for turbocharged IC engines. Graphical programming software, LabVIEW, was also used. A compact, multi-channel, real-time data acquisition system was created for the checks. Data is collected both in the case of static and dynamic tests. In this way, the test of an engine is made more efficient and precise. IC engines have energy losses due to the friction that occurs in the slider-crank mechanism. This mechanism is distinguished by its constructive and functional simplicity, which is why it equips all IC engines at the moment. A hypocycloid gear mechanism as an alternative to the classic slider-crank mechanism is proposed in [3]. A kinematic and dynamic study of such a mechanism is presented in the lecture. The effect of attaching a flywheel to this type of engine is also being studied. The advantages of the proposed solution are the higher efficiency than the classic mechanism and the possibility of perfectly balancing the

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engine. The disadvantage is a more complicated technical solution, which involves increased costs. A study of the forces occurring in the cam mechanism of an engine is made in [4]. This is how a camshaft for a 6-cylinder IC engine is conceived and designed. CATIA V5R20 design software was used. Then the ANSYS 2019R2 software analyzes the behavior of different materials and in different operating conditions, determining the field of stresses and deformations. A model that takes into account clearance between piston skirt and cylinder liner is presented in [5]. The general motion of the mechanism and the secondary motion of the piston caused by the changes in direction of forces are taken into account. The kinematic analysis of a single-cylinder engine equipped with a crank-rocker mechanism is performed in [6]. The significant kinetic parameters were determined for such an engine. Two variants were studied in which the cylinder volume is 402 cc and 1140 cc). The forces that appear in the bearing were thus determined, forces that are considered high and must be reduced. The crank-rocker mechanism has some advantages over the usual mechanism with slider-crank. It is obvious that the advantages implied by the use of such a mechanism must also be studied. In the papers [7-10] fundamental aspects of the calculation of an IC engine are presented, followed by researchers in the proposed mathematical models.

The vibrations that appear in IC engines represent an important aspect dealing with the dynamics of the motor mechanism. A series of results regarding engine vibrations or theoretical aspects for the study of vibrations are presented in [11-17]. Aspects related to the materials used in IC engines are presented in [18]. As in other engineering fields, the Finite Element Method (FEM) represents a useful and validated tool to obtain results in the modeling of such a system . Studies of certain aspects related to dynamics of IC engines carried out applying this method are presented in [19-22]. An analysis of the vibrations occurring in an IC engine fueled with biodiesel, using FEM, with the verification of the results by experimental methods is carried out in [23]. The specificity of this type of analysis is given by the low calorific value of biodiesel. As a result, the duration of combustion increases and the level of vibrations will increase if it is compared to a classic power engine. The dynamic response of such an engine change obviously, in a negative way and for this reason the analysis from this point of view is required. Similar studies are developed in [24, 25]. A mathematical model of an IC engine with the help of which the forces appearing in the system are determined and the piston vibrations are studied is presented in [26]. The Matlab/Simulink programming environment is used to implement the model. Stability is demonstrated in the case of engine operation at a constant speed. A consistent and highly elaborated model for the study of IC engines is presented in [27]. In the case of this model, the main functional parameters of the engine are followed. Numerical methods used for the study of such mechanical systems are presented in [28, 29]. Other results that are useful in the study of materials from these systems are presented in [30-32].

In the work, a model is developed that uses vector methods to analyze the forces that appear in the single cylinder of an IC engine. The modeling is broad enough to analyze different constructive types of IC engines. The calculation of the main forces that appear in this assembly and that can be used, later, to determine the field of stresses and strains in the elements of an IC engine is followed.

2. Model and Methods

First, the kinematics and dynamics of a single cylinder will be studied. For an engine with several cylinders, regardless of its type, the equations of motion can be obtained starting from the calculation of a single cylinder and adapting to the actual situation encountered. In Fig. 1 is presented such a monocylinder.

2.1. Monocylinder

We attach the following coordinate systems to the slider crank mechanism:

- A fixed one, with one of the axes parallel to the axis of the cylinder, with the origin in A_o and the unit vectors of the base $(\overline{u}_1, \overline{n}_1)$;
- Two mobile systems, one with the origin in A_o and having the unit vectors base $(e_1, \bar{t}_1)$ moving with and attached to the crank and the other with the origin in A_1 and having the unit vectors $(e_1, \bar{t}_1)$ moving with and attached to the connecting rod.

The length of the crank is $r = A_o A_1$, the length of the connecting rod $l = A_1 A_2$ and the eccentricity is $e = A_2 A_3$. It is noted: $\lambda = \frac{r}{l}$, $\nu = \frac{e}{l}$. With these notations it is possible to write: $\overline{A_o A_1} = r\bar{e}_o$;

$$\overline{A_1A_2} = l\bar{e}_1 \; ; \quad \overline{A_2A_3} = \bar{n}_1 \, . \tag{1}$$

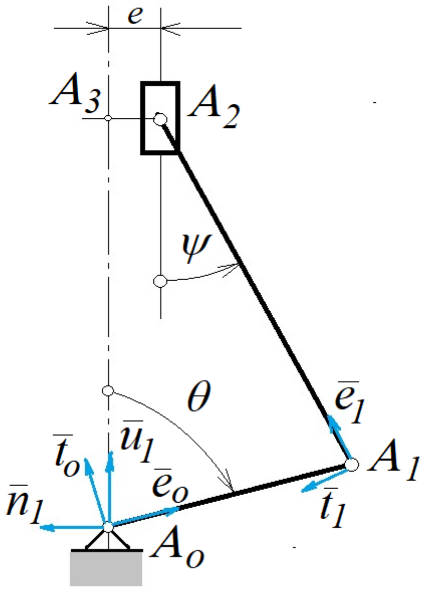

Fig.1: Slider crank for one cylinder

Taking $\overline{A_o}$ as the origin for measuring the displacement of the piston, the distance $D = A_oA_3$ will indicate the law of motion of the piston and $\overline{A_oA_3} = D\bar{u}_1$. The contour equation of the slider-crank mechanism is:

$$\overline{A_oA_1} + \overline{A_1A_2} + \overline{A_2A_3} + \overline{A_3A_o} = 0 \, , \tag{2}$$

or:

$$r\bar{e}_o + l\bar{e}_1 + e\bar{n}_1 - D\bar{u}_1 = 0 \, . \tag{3}$$

If this relation is multiplied in a dot product by $\bar{n}_1$, we get:

$$r(\bar{e}_o\bar{n}_1) + l(\bar{e}_1\bar{n}_1) + e = 0 \, . \tag{4}$$

If it is denoted by θ is the angle of rotation of the crankshaft, ψ is the angle made by the connecting rod $\bar{e}_1$ with $\bar{u}_1$, Eq. (4) becomes:

$$-r\sin\theta + l\sin\psi + e = 0 \, . \tag{5}$$

The components of $\bar{e}_o$ and $\bar{t}_o$ are, respectively:

$$\bar{e}_0 = (\cos\theta, -\sin\theta) \;\; ; \;\; \bar{t}_0 = (\sin\theta, \cos\theta) \, . \tag{6}$$

When performing dot products, it is taken into account that we have:

$$\bar{u}_1 = (1,0) \;\; ; \;\; \bar{n}_1 = (0,1) \, . \tag{7}$$

It results

$$\sin\psi = \frac{r\sin\theta - e}{l} = \lambda\sin\theta - \nu \, . \tag{8}$$

By the geometry of the slider-crank mechanism $\cos\psi$ is always positive, so:

$$\cos\psi = \sqrt{1 - (\lambda\sin\theta - \nu)^2} \, , \tag{9}$$

and the components of the vectors $\bar{e}_1$ and $\bar{t}_1$ compared to the system defined by unit vectors $(\bar{u}_1, \bar{n}_1)$ has:

$$\bar{e}_1 = (\cos\psi, \sin\psi), \tag{10}$$

$$\bar{t}_1 = (\cos\left(\psi + \frac{\pi}{2}\right), \sin\left(\psi + \frac{\pi}{2}\right)) = (-\sin\psi, \cos\psi). \tag{11}$$

Multiplying in a dot product rel.(xxx) by $\bar{u}_1$ it obtains:

$$r(\bar{e}_o \bar{u}_1) + l(\bar{e}_1 \bar{u}_1) - D = 0, \tag{12}$$

relationship that gives us distance *D*:

$$D = r(\bar{e}_o \bar{u}_1) + l(\bar{e}_1 \bar{u}_1) = r\cos\theta + l\cos\psi\ . \tag{13}$$

By deriving the relation (3) we obtain:

$$\bar{\omega}_0 \times \overline{A_0 A_1} + \bar{\omega}_1 \times \overline{A_1 A_2} - \bar{v}_1 = 0, \tag{14}$$

where $\bar{\omega}_0$ is the angular velocity of the crankshaft:

$$\omega_o = \frac{\pi n}{30} \quad \text{rad/s}, \tag{15}$$

if the engine speed *n* is expressed in *rpm*. It was noted with ω_1 the angular velocity of the connecting rod and with v_1 the velocity of the piston relative to the cylinder. Eq. (14) can be written:

$$-\omega_o r(\bar{K} \times \bar{e}_o) + \omega_1 l(\bar{K} \times \bar{e}_1) - v_1 \bar{u}_1 = 0, \tag{16}$$

where $\bar{K}$ is an unit vector perpendicular to $\bar{u}_1$ and $\bar{n}_1$ so that the system defined by the unit vectors $(\bar{u}_1, \bar{n}_1, \bar{K})$ is a straight reference system. Projecting Eq.(16) according to the directions $\bar{u}_1$ and $\bar{n}_1$ obtains:

$$-\omega_o r\bar{u}_1(\bar{K} \times \bar{e}_o) + \omega_1 l\bar{u}_1(\bar{K} \times \bar{e}_1) - v_1 = 0, \tag{17}$$

$$-\omega_o r\bar{n}_1(\bar{K} \times \bar{e}_o) + \omega_1 l\bar{n}_1(\bar{K} \times \bar{e}_1) - v_1(\bar{n}_1 \bar{u}_1) = 0. \tag{18}$$

From Eq. (18) it results:

$$-\omega_o r\bar{n}_1(\bar{K} \times \bar{e}_o) + \omega_1 l\bar{n}_1(\bar{K} \times \bar{e}_1) = 0, \tag{19}$$

wherefrom it results:

$$\omega_1 = \frac{\omega_o r\bar{n}_1(\bar{K} \times \bar{e}_o)}{l\bar{n}_1(\bar{K} \times \bar{e}_1)} = \frac{\omega_o r(\bar{n}_1 \bar{t}_0)}{l(\bar{n}_1 \bar{t}_1)} = \frac{\omega_o r(\bar{u}_1 \bar{e}_0)}{l(\bar{u}_1 \bar{e}_1)} = \frac{\omega_o r\cos\theta}{l\cos\psi} = \omega_o \lambda \frac{\cos\theta}{\cos\psi}, \tag{20}$$

and:

$$v_1 = -\omega_o r(\bar{u}_1 \bar{t}_0) + \omega_1 l(\bar{u}_1 \bar{t}_1) = -\omega_o r\sin\theta - \omega_1 l\sin\psi = -\omega_o r\sin\theta - \omega_o r\frac{\cos\theta}{\cos\psi}\sin\psi =$$

$$= -\omega_o r\frac{\sin(\theta + \psi)}{\cos\psi}\ . \tag{21}$$

By deriving Eq. (14) we obtain:

$$\bar{\varepsilon}_0 \times \overline{A_0 A_1} - \omega_0^2 \overline{A_0 A_1} + \bar{\varepsilon}_1 \times \overline{A_1 A_2} - \omega_1^2 \overline{A_1 A_2} - \bar{a}_1 = 0, \tag{22}$$

where: $\bar{\varepsilon}_1$ is the angular velocity.

Eq. (22), if the vector expressions are taken into account $\overline{A_0 A_1}$ and $\overline{A_1 A_2}$, can be written:

$$r\varepsilon_0 \bar{K} \times \bar{e}_0 - \omega_0^2 r \bar{e}_0 + l\varepsilon_1 \bar{K} \times \bar{e}_1 - \omega_1^2 l \bar{e}_1 - a_1 \bar{u}_1 = 0 . \tag{23}$$

Projecting this equation on $\bar{e}_1$ respective $\bar{n}_1$ unit vectors, the equations are obtained:

$$\begin{cases} r\varepsilon_0 \bar{e}_1 \left(\bar{K} \times \bar{e}_0\right) - \omega_0^2 r\left(\bar{e}_1 \bar{e}_0\right) + l\varepsilon_1 \bar{e}_1 \left(\bar{K} \times \bar{e}_1\right) - \omega_1^2 l - a_1 \left(\bar{e}_1 \bar{u}_1\right) = 0 \\ r\varepsilon_0 \bar{n}_1 \left(\bar{K} \times \bar{e}_0\right) - \omega_0^2 r\left(\bar{n}_1 \bar{e}_0\right) + l\varepsilon_1 \bar{n}_1 \left(\bar{K} \times \bar{e}_1\right) - \omega_1^2 l\left(\bar{n}_1 \bar{e}_1\right) = 0 \end{cases} . \tag{24}$$

If it is taken into account that $\bar{K} \times \bar{e}_0 = \bar{t}_0$, $\bar{K} \times \bar{e}_1 = \bar{t}_1$, the previous relations can be written:

$$\begin{cases} r\varepsilon_0 \left(\bar{e}_1 \bar{t}_0\right) - \omega_0^2 r\left(\bar{e}_1 \bar{e}_0\right) + l\varepsilon_1 \left(\bar{e}_1 \bar{t}_1\right) - \omega_1^2 l - a_1 \left(\bar{e}_1 \bar{u}_1\right) = 0 \\ r\varepsilon_0 \left(\bar{n}_1 \bar{t}_0\right) - \omega_0^2 r\left(\bar{n}_1 \bar{e}_0\right) + l\varepsilon_1 \left(\bar{n}_1 \bar{t}_1\right) - \omega_1^2 l\left(\bar{n}_1 \bar{e}_1\right) = 0 \end{cases} , \tag{25}$$

wherefrom:

$$a_1 = \frac{1}{\left(\bar{e}_1 \bar{u}_1\right)}\left[r\varepsilon_0\left(\bar{e}_1 \bar{t}_0\right) - \omega_0^2 r\left(\bar{e}_1 \bar{e}_0\right) - \omega_1^2 l\right] =$$

$$= \frac{1}{\cos\psi}\left[r\varepsilon_0 \sin(\theta+\psi) - \omega_0^2 r\cos(\theta+\psi) - \omega_1^2 l\right], \tag{26}$$

$$\varepsilon_1 = \frac{1}{\left(\bar{n}_1 \bar{t}_1\right)}\left[-\frac{r}{l}\varepsilon_0\left(\bar{n}_1 \bar{t}_0\right) + \omega_0^2 \frac{r}{l}\left(\bar{n}_1 \bar{e}_0\right) + \omega_1^2\left(\bar{n}_1 \bar{e}_1\right)\right] =$$

$$= \frac{1}{\cos\psi}\left[-\lambda\varepsilon_0 \cos\theta - \omega_0^2 \lambda \sin\theta + \omega_1^2 \sin\psi\right] . \tag{27}$$

We have used the relation $\left(\bar{e}_1 \bar{t}_1\right)$=0.

Substituting ω_1 it obtains:

$$a_1 = \frac{1}{\cos\psi}\left[r\varepsilon_0 \sin(\theta+\psi) - \omega_0^2 r\left[\cos(\theta+\psi) - \frac{\cos^2\theta}{\cos^2\psi}\right]\right], \tag{28}$$

$$\varepsilon_1 = \frac{\lambda}{\cos\psi}\left[-\varepsilon_0 \cos\theta + \omega_0^2\left(-\sin\theta + \lambda\frac{\cos^2\theta}{\cos^2\psi}\sin\psi\right)\right]. \tag{29}$$

2.2. V Engine

The results can be applied easily to a V Engine (Fig.2). To study the kinematic cycles of the two cylinders, we take into account that $\theta = \theta_1$ where γ is the angle made by the axes of the two cylinders. So the kinematic elements of the second slider crank mechanism will be calculated with the Eqs. (20),(21),(28) and (29) in which θ is replaced by θ_1. It is observed that if $\gamma = \pi$, the engine with opposite cylinders is obtained (Fig.3).

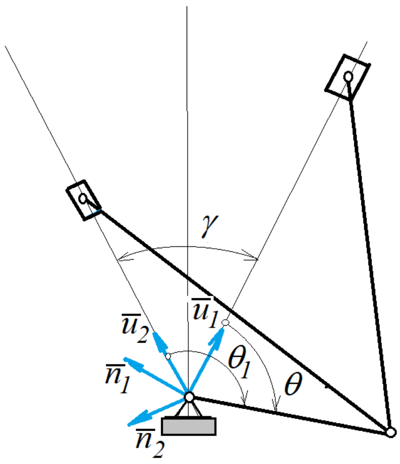

Fig.2: Model for a V engine

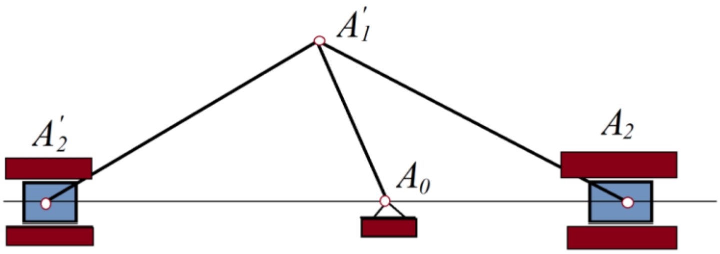

Fig.3: Model of an engine with opposite cylinders

2.3. *Star cylinder engine*

If N is the number of cylinders in a plane of the engine, then the angle between two adjacent cylinders is $\frac{2\pi}{N}$ (Fig. 4).

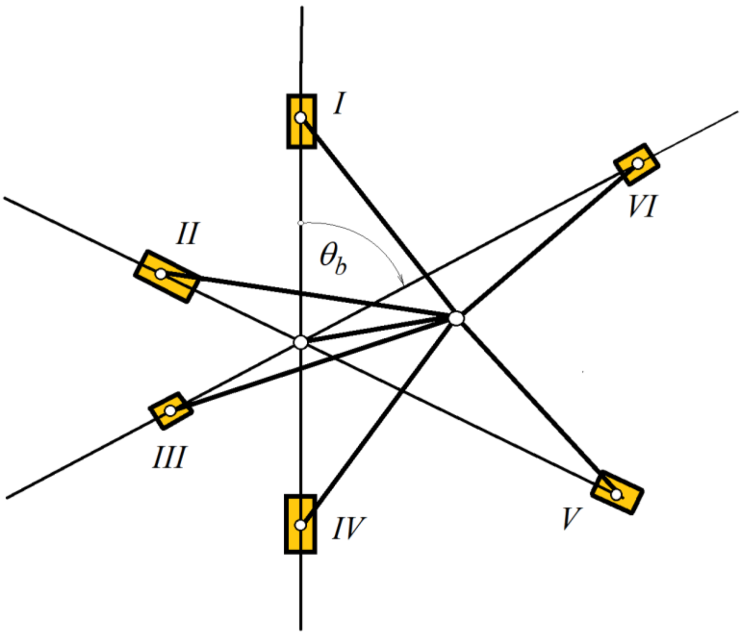

Fig.4: Star cylinder engine

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The reference cycle of a reference cylinder being determined by the angle θ_o, the speeds and accelerations of the cylinders are obtained with relation (xxx) in which θ successively takes the values:

$$\theta_o, \theta_o + \frac{2\pi}{N}, \ldots \theta_o + \frac{2\pi}{N} i, \ldots \theta_o + \frac{2\pi}{N}(N-1), \tag{30}$$

the cylinders are numbered starting from the reference cylinder in a direct trigonometric sense.

2.4. *Engine with master and slave connecting-rod*

For the first piston and connecting rod, the problem is presented in section 2.1. For the connecting rod, the crank arm is changed to $\overline{A_o A_5}$ and the angle θ to θ_1 (Fig. 5).

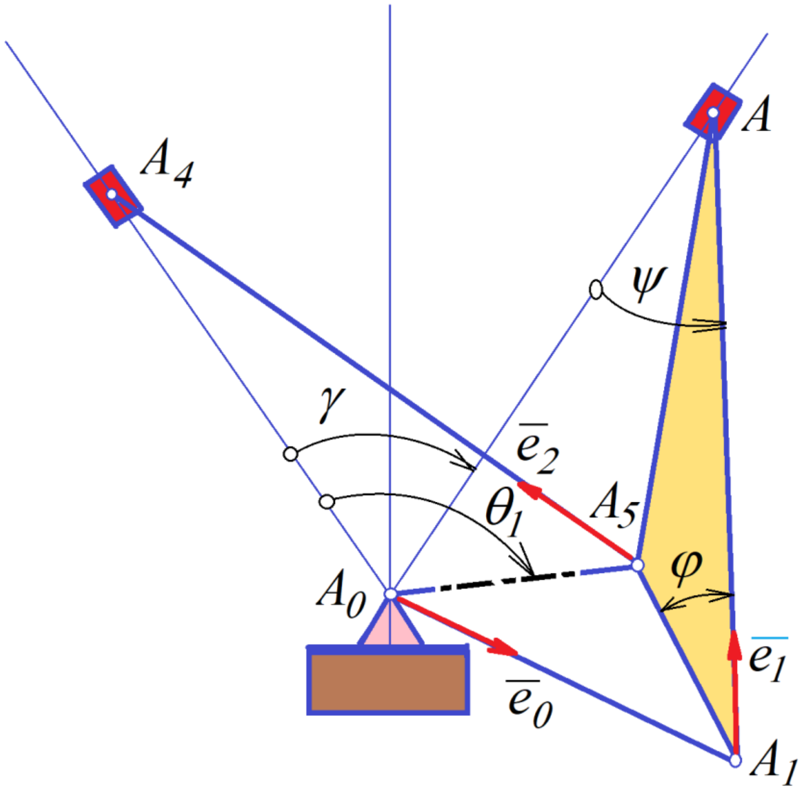


Fig.5: Model of a V engine

Let be φ in this case the angle made by $\overline{A_1 A_5}$ with the connecting rod unit vector $\bar{e}_1$, s be the length of the segment $\overline{A_1 A_5}$, so φ and s also define the point A_5. The length of the slave connecting rod is l_2:

$$l_2 = \overline{A_5 A_4} . \tag{31}$$

If the angle θ_1 was determined, the problem was reduced to that of the single cylinder. It is noted with $\bar{e}_b$ the unity vector of the vector $\overline{A_1 A_5}$. Regarding the system $(e_1, \bar{t}_1)$, $\bar{e}_o$ has the components (cos φ, sin φ), so regarding the system $(\bar{u}_2, \bar{n}_2)$ it will have the components:

$$(\cos(\varphi + \psi - \gamma); \sin(\varphi + \psi - \gamma)) . \tag{32}$$

The relationship can be written:

$$\overline{A_o A_5} = \overline{A_o A_1} + \overline{A_1 A_5} = r\bar{e}_o + s\bar{e}_b . \tag{33}$$

Regarding the coordinate system defined by $(\overline{u}_2, \overline{n}_2)$, $\overline{e}_b$ has the components $(\cos(\theta-\gamma); \sin(\theta-\gamma))$. So:

$$\overline{A_o A_5} = (r\cos(\theta+\gamma) + s\cos(\varphi+\psi-\gamma)\ ;\ -rc\sin(\theta+\gamma) + s\sin(\varphi+\psi-\gamma))\ . \tag{34}$$

It is noted:

$$r_1 = r\cos(\theta+\gamma) + s\cos(\varphi+\psi-\gamma)\quad ;\quad r_2 = -rc\sin(\theta+\gamma) + s\sin(\varphi+\psi-\gamma). \tag{35}$$

The angle θ results from:

$$\tan\theta_1 = \frac{r_2}{r_1} = \frac{-rc\sin(\theta+\gamma) + s\sin(\varphi+\psi-\gamma)}{r\cos(\theta+\gamma) + s\cos(\varphi+\psi-\gamma)}\ , \tag{36}$$

and the modulus of the segment $\overline{A_o A_5}$ is:

$$A_o A_5 = \sqrt{r_1^2 + r_2^2} = \sqrt{r^2 + s^2 + 2rs\cos(\theta+\varphi+\psi)}\ . \tag{37}$$

Having determined these quantities, using Eqs. (20,21) and (28,29) in which θ is replaced by θ1, *r* by $A_o A_5$ and *l* by l_2.

2.5. *Dynamics of a monocylinder*

The calculation of the forces requiring the elements of the slider crank system (Fig. 6) can be done by two methods:

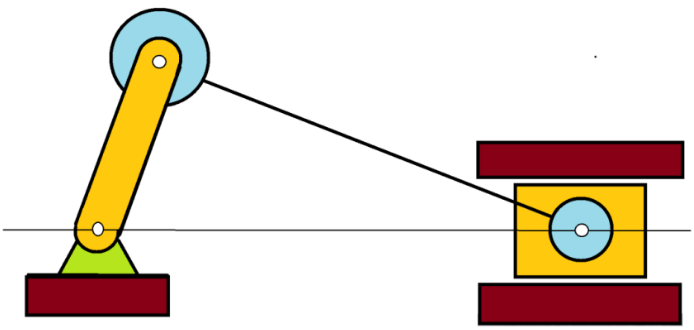
I. The problem is simplified by reducing the assembled connecting rod to two masses:

- one of translation m_1, concentrated in A_2 which gathers at the masses that execute the movement of translation m_p (piston, bolt, segments,....)
- the second mass, m_2, concentrated in A_1. It will rotate with angular speed ω_1 around the axis of the shaft, at a distance *r* from it.

Masses m_1 and m_2 are determined by the law of static moment (Fig. 6):

$$m_1 = m_b \frac{l_2}{l}\quad ;\quad m_2 = m_b \frac{l_1}{l}, \tag{38}$$

where m_b is the mass of the connecting rod.



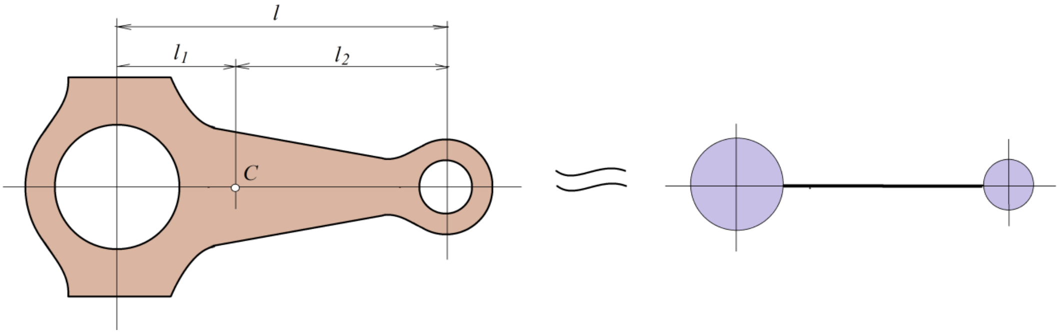
a) The two parts distributed in the mechanism

b) For car engines m_1 and m_2 it is approximately calculated with the formulas:

c) $$m_1 = 0.275 m_b\quad ;\quad m_2 = 0.725 m_b\,. \tag{39}$$

d) We cut the bar that joins the masses m_t=m_1+m_p (total translational mass of the connecting rod) and m_2, the effort *S* is introduced in the connecting rod (Fig. 7).

e) $$N\overline{n}_1 - F_g\overline{u}_1 - G_1\overline{u}_o + F_1^i\overline{u}_1 - S\overline{e}_1 = 0\,, \tag{40}$$



f) The real rod and the equivalent rod

Fig.6: The rod decomposed in two parts

where:

N is the force that appears at the contact between the piston and the cylinder;

F_g is the force given by gas pressure;

$F_1^i = -m_t a_1$ is the inertial force of the translational mass,

$G_1 = m_t g = (m_P + m_1) g.$

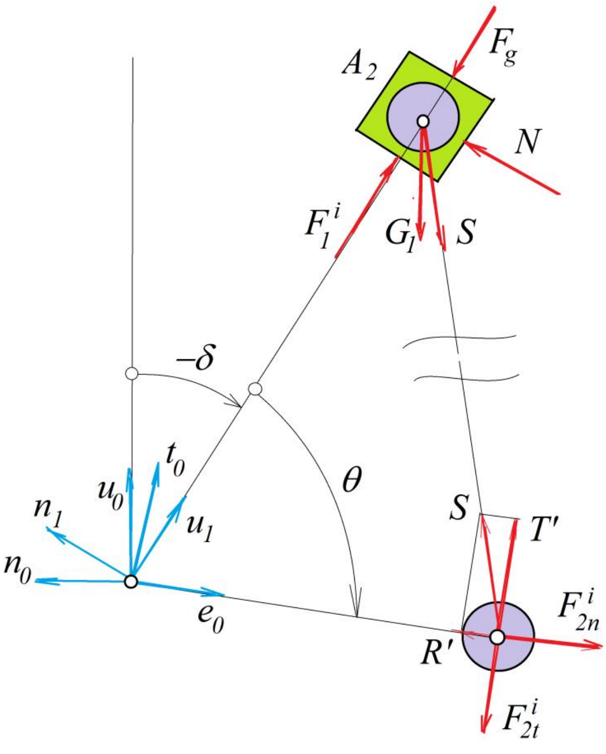


Fig.7: Forces acting on the two parts of the rod

Projecting the equations (40) on the directions defined by the unit vectors $\bar{t}_1$ and $\bar{u}_1$ respectively, a system of two equations with two unknown is obtained:

$$\begin{cases} N(\bar{n}_1\bar{t}_1) - F_g(\bar{u}_1\bar{t}_1) - G_1(\bar{u}_o\bar{t}_1) + F_1^i(\bar{u}_1\bar{t}_1) = 0 \\ -F_g\bar{u}_1^2 - G_1(\bar{u}_o\bar{u}_1) + F_1^i\bar{u}_1^2 - S(\bar{e}_1\bar{u}_1) = 0 \end{cases}. \tag{41}$$

It results:

$$N=\frac{\left(F_g-F_1^i\right)\left(\bar{u}_1\bar{t}_1\right)+G_1\left(\bar{u}_o\bar{t}_1\right)}{\left(\bar{n}_1\bar{t}_1\right)}=$$
$$=\frac{-\left(F_g-F_1^i\right)\sin\psi-G_1\sin\left(\psi+\delta\right)}{\cos\psi}. \tag{42}$$

We take into account that we have:

$$\bar{t}_1=\left(\cos(\psi+\frac{\pi}{2}+\delta)\,;\sin(\psi+\frac{\pi}{2}+\delta)\right), \tag{43}$$

in the fixed coordinate system defined by $\left(\bar{u}_o,\bar{n}_o\right)$.

$$S=\frac{-F_g\bar{u}_1^2-G_1\left(\bar{u}_o\bar{u}_1\right)+F_1^i\bar{u}_1^2}{\left(\bar{e}_1\bar{u}_1\right)}=\frac{-F_g+F_1^i-G_1\cos\delta}{\cos\psi}. \tag{44}$$

We take into account that we have:

$$\bar{u}_1=\left(\cos\delta;\sin\delta\right), \tag{45}$$

in the fixed coordinate system defined by $\left(\bar{u}_o,\bar{n}_o\right)$.

It results from here:

$$T'=S\left(\bar{e}_1\bar{t}_0\right)=S\sin(\theta+\psi)=\frac{\left(-F_g+F_1^i-G_1\cos\delta\right)\sin(\theta+\psi)}{\cos\psi}, \tag{46}$$

$$R'=S\left(\bar{e}_1\bar{e}_0\right)=S\cos(\theta+\psi)=\frac{\left(-F_g+F_1^i-G_1\cos\delta\right)\cos(\theta+\psi)}{\cos\psi}. \tag{47}$$

The forces T and R that act on crank are obtained adding the inertia forces to the determined forces T' and R'.

$$T=T'+F_{2n}^i=\frac{\left(-F_g+F_1^i-G_1\cos\delta\right)\sin\left(\theta+\psi\right)}{\cos\psi}+F_{2n}^i, \tag{48}$$

where:

$$F_{2t}^i=-m_2r\varepsilon_1, \tag{49}$$

and:

$$R=R'+F_{2t}^i=\frac{\left(-F_g+F_1^i-G_1\cos\delta\right)\cos(\theta+\psi)}{\cos\psi}+F_{2t}^i, \tag{50}$$

where:

$$F_{2n}^i=-m_2\omega_1^2r. \tag{51}$$

If G_1 is neglected, simpler expressions are obtained for N,S,T',R',T,R:

$$N=-\left(F_g-F_1^i\right)\tan\psi\ ; \tag{52}$$

$$S = \frac{-\left(F_g - F_1^i\right)}{\cos\psi} \; ; \tag{53}$$

$$T' = -\left(F_g - F_1^i\right)\frac{\sin(\theta+\psi)}{\cos\psi} \; ; \tag{54}$$

$$R' = -\left(F_g - F_1^i\right)\frac{\cos(\theta+\psi)}{\cos\psi} \; ; \tag{55}$$

$$T = -\left(F_g - F_1^i\right)\frac{\sin(\theta+\psi)}{\cos\psi} - G_2\frac{r\varepsilon}{g} \; ; \tag{56}$$

$$R = -\left(F_g - F_1^i\right)\frac{\cos(\theta+\psi)}{\cos\psi} + G_2\frac{\omega_1^2 r}{g} . \tag{57}$$

II. Connecting rod is considered a rigid body (Fig. 8). The dynamic equilibrium equations are written for the connecting rod considered as a rigid body. If it is noted with F_b^i the inertial force due to the mass of the assembled connecting rod, with M_b^i the moment of inertia of the connecting rod, with F_p^i the force of inertia of the assembled piston m_p, the moments balance equation is written with respect to point A_1:

$$\overline{A_1A_2} \times \left[\left(-F_g + F_p^i\right)\overline{u}_1 - G_p\overline{u}_0 + N\overline{n}_1\right] + \overline{A_1C} \times \left(-G_b\overline{u}_0 + F_b^i\right) + \overline{M_b^i} = 0 \tag{58}$$

and the equilibrium equation:

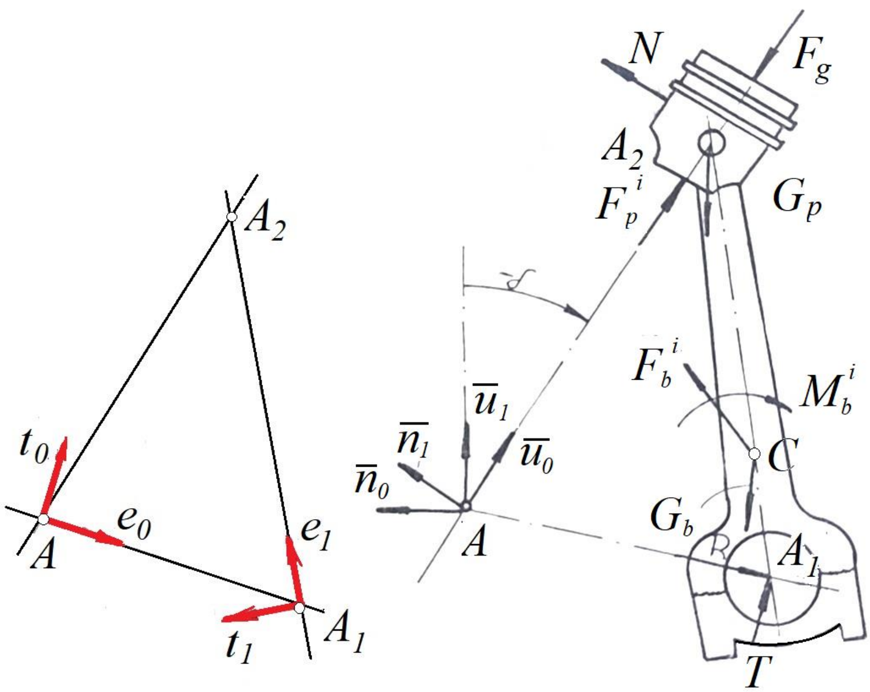


Fig.8: Free body diagram for the rod as a solid

where:

$$G_b = m_b g\,; \tag{59}$$

$$\overline{F}_b^i = -m_b \bar{a}_C = -m_b\left(\bar{a}_1 - \omega_1^2\,\overline{A_2C} + \bar{\varepsilon}_1 \times \overline{A_2C}\right) = -m_b\left(a_1\bar{u}_1 - \omega_1^2 l_1 \bar{e}_1 - \bar{\varepsilon}_1 l_1 \bar{t}_1\right); \tag{60}$$

$$\overline{M_b^i} = -J_b \varepsilon_1 \overline{K}\,, \tag{61}$$

J_b being the moment of inertia of the connecting rod with respect to the center of mass C; $\bar{a}_1$ is the acceleration of the piston; ω_1 is the angular velocity of the connecting rod and ε_1 the angular acceleration. Equation (xxx) becomes:

$$l\bar{e}_1 \times \left[\left(-F_g + F_p^i\right)\bar{u}_1 - G_p\bar{u}_0 + N\bar{n}_1\right] + l_2\bar{e}_1 \times \left[-G_b\bar{u}_0 - m_b\left(a_1\bar{u}_1 - \omega_1^2 l_1\bar{e}_1 - \bar{\varepsilon}_1 l_1 \bar{t}_1\right)\right] - J_b\varepsilon_1\overline{K} = 0\,, \tag{62}$$

or:

$$\left(\bar{e}_1 \times \bar{u}_1\right)\left[\left(-F_g + F_p^i\right)l - m_b l_2 a_1\right] - \left(\bar{e}_1 \times \bar{u}_0\right)\left(lG_p + l_2 G_b\right) + lN\left(\bar{e}_1 \times \bar{n}_1\right) - l_1 l_2 \varepsilon_1 m_b\left(\bar{e}_1 \times \bar{t}_1\right) - J_b\varepsilon_1\overline{K} = 0\,. \tag{63}$$

Projecting this equation on the axis $\overline{K}$ gives:

$$-\left(\bar{e}_1\bar{n}_1\right)\left[\left(-F_g + F_p^i\right)l - m_b l_2 a_1\right] + \left(\bar{e}_1\bar{n}_0\right)\left(lG_p + l_2 G_b\right) + lN\left(\bar{e}_1\bar{u}_1\right) + l_1 l_2\varepsilon_1 m_b - J_b\varepsilon_1 = 0\,, \tag{64}$$

wherefrom:

$$N = \frac{\sin\psi\left[\left(-F_g + F_p^i\right) - m_b l_2 a_1 / l\right]}{\cos\psi} + \frac{\sin(\psi+\delta)\left(G_p + l_2 G_b / l\right)}{\cos\psi} - \frac{l_1 l_2 \varepsilon_1 m_b - J_b\varepsilon_1}{l\cos\psi}\,. \tag{65}$$

Projecting the equations (xxx) on the versors $\bar{t}_0$ and e_0, the equations are obtained:

$$\begin{aligned}&\left(-F_g + F_p^i\right)\left(\bar{u}_1\bar{t}_0\right) - G_p\left(\bar{u}_0\bar{t}_0\right) + N\left(\bar{n}_1\bar{t}_0\right) - G_b\left(\bar{u}_0\bar{t}_0\right) - m_b a_1\left(\bar{u}_1\bar{t}_0\right) + \\ &\quad + m_b\varepsilon_1 l_1\left(\bar{e}_1\bar{t}_0\right) - m_b\omega_1^2 l_1\left(\bar{e}_1\bar{t}_0\right) + T = 0,\end{aligned} \tag{66}$$

and:

$$\begin{aligned}&\left(-F_g + F_p^i\right)\left(\bar{u}_1\bar{e}_0\right) - G_p\left(\bar{u}_0\bar{e}_0\right) + N\left(\bar{n}_1\bar{e}_0\right) - G_b\left(\bar{u}_0\bar{e}_0\right) - m_b a_1\left(\bar{u}_1\bar{e}_0\right) + \\ &\quad + m_b\varepsilon_1 l_1\left(\bar{t}_1\bar{e}_0\right) - m_b\omega_1^2 l_1\left(\bar{e}_1\bar{e}_0\right) + R = 0.\end{aligned} \tag{67}$$

It results:

$$\begin{aligned}T &= \left[-\left(-F_g + F_p^i\right) + m_b a_1\right]\left(\bar{u}_1\bar{t}_0\right) + \left(G_p + G_b\right)\left(\bar{u}_0\bar{t}_0\right) - N\left(\bar{n}_1\bar{t}_0\right) + m_b\omega_1^2 l_1\left(\bar{e}_1\bar{t}_0\right) - m_b\varepsilon_1 l_1\left(\bar{e}_1\bar{e}_0\right) = \\ &= \left[-\left(-F_g + F_p^i\right) + m_b a_1\right]\sin\theta + \left(G_p + G_b\right)\sin\left(\theta - \delta\right) - N\cos\theta + \\ &\quad + m_b\omega_1^2 l_1\sin\left(\theta + \psi\right) - m_b\varepsilon_1 l_1\cos\left(\theta + \psi\right),\end{aligned} \tag{68}$$

$$\begin{aligned}R &= \left[-\left(-F_g + F_p^i\right) + m_b a_1\right]\left(\bar{u}_1\bar{e}_0\right) + \left(G_p + G_b\right)\left(\bar{u}_0\bar{n}_0\right) - N\left(\bar{n}_1\bar{e}_0\right) + m_b\omega_1^2 l_1\left(\bar{e}_1\bar{e}_0\right) - m_b\varepsilon_1 l_1\left(\bar{e}_1\bar{t}_0\right) = \\ &= \left[-\left(-F_g + F_p^i\right) + m_b a_1\right]\cos\theta + \left(G_p + G_b\right)\cos\left(-\theta + \delta\right) - N\sin\theta + \\ &\quad + m_b\omega_1^2 l_1\cos\left(\theta + \psi\right) - m_b\varepsilon_1 l_1\sin\left(\theta + \psi\right).\end{aligned} \tag{69}$$

3. Torque transmitted to crankshaft

The moment equation with respect to A_o gives the motor torque provided by a cylinder (Fig. 9):

$$M_m\overline{k} = \overline{A_0A_1}\times\left(-\overline{T}\right)+\overline{A_0C_1}\times\overline{G}_m - J_0\varepsilon_0\overline{K}\,, \tag{70}$$

where $\overline{G}_m$ is the weight of the crank and the crank arm and J_o the moment of inertia of the mass G_m/g, around the axis of the shaft, determined in A_0.

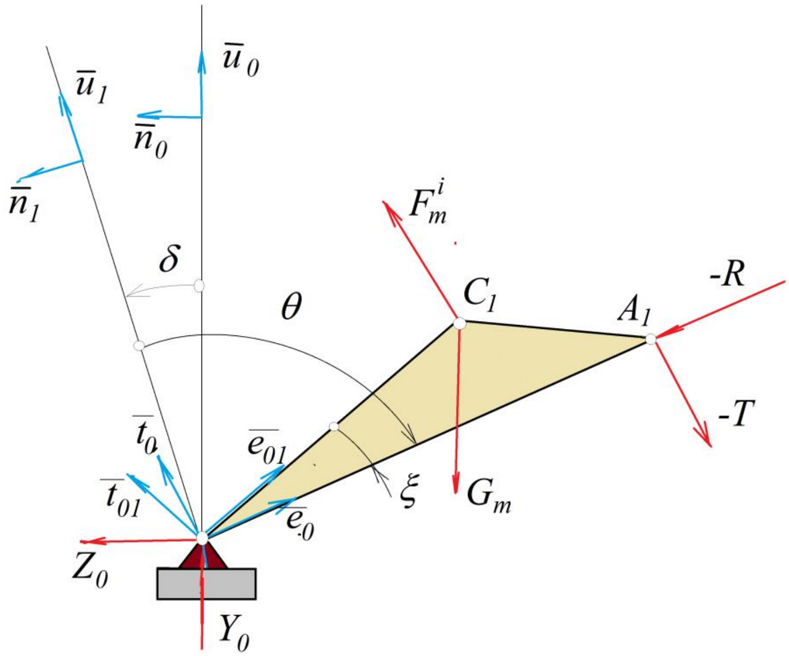


Fig.9: Free body diagram for a crank arm

It obtains:

$$M_m = \left[rT + \rho G_m \sin(\theta-\delta) + J_0\varepsilon_0\right], \tag{71}$$

with $\rho = A_oC$.

Equilibrium equation:

$$\overline{R}_o + \overline{F}_m^i - \overline{T} - \overline{R} - G_m\overline{u}_o = 0\,, \tag{72}$$

where: $\overline{R}_o(Y_o, Z_o)$ is the reaction that occurs in the Ao joint, and:

$$\overline{F}_m^i = m_m\omega_o^2\rho\,\overline{e}_{01} - m_m\rho\,\overline{e}_0\overline{l}_{01}\,, \tag{73}$$

is the inertial force due to the mass of the crank and the crank arm, by projecting on $\overline{u}_0$, respectively $\overline{n}_0$, offers:

$$Y_o + m_m\omega_0^2\rho\left(\overline{e}_{01}\overline{u}_0\right) - m_m\rho\varepsilon_o\left(\overline{t}_{01}\overline{u}_0\right) - G_m - T\left(\overline{t}_0\overline{u}_0\right) - R\left(\overline{e}_0\overline{u}_0\right) = 0\,, \tag{74}$$

and:

$$Z_o + m_m\omega_0^2\rho\left(\overline{e}_{01}\overline{n}_0\right) - m_m\rho\varepsilon_o\left(\overline{t}_{01}\overline{n}_0\right) - T\left(\overline{t}_0\overline{n}_0\right) - R\left(\overline{e}_0\overline{n}_0\right) = 0\,. \tag{75}$$

It results:

$$Y_o = -m_m\omega_0^2\rho\left(\overline{e}_{01}\overline{u}_0\right) + m_m\rho\varepsilon_o\left(\overline{t}_{01}\overline{u}_0\right) + G_m + T\left(\overline{t}_0\overline{u}_0\right) + R\left(\overline{e}_0\overline{u}_0\right) =$$

$$= -m_m \omega_0^2 \rho \cos(\theta - \delta - \xi) + m_m \rho \varepsilon_o \sin(\theta - \delta - \xi) + G_m + T \sin(\theta - \delta) + R \cos(\theta - \delta), \quad (76)$$

and:

$$Z_o = -m_m \omega_0^2 \rho \left(\bar{e}_{01} \bar{n}_0\right) + m_m \rho \varepsilon_o \left(\bar{t}_{01} \bar{n}_0\right) + T\left(\bar{t}_0 \bar{n}_0\right) + R\left(\bar{e}_0 \bar{n}_0\right) =$$

$$= -m_m \omega_0^2 \rho \sin(\theta + \delta + \xi) + m_m \rho \varepsilon_o \cos(-\theta + \delta + \xi) + T \cos(-\theta + \delta) + R \sin(-\theta + \delta). \quad (77)$$

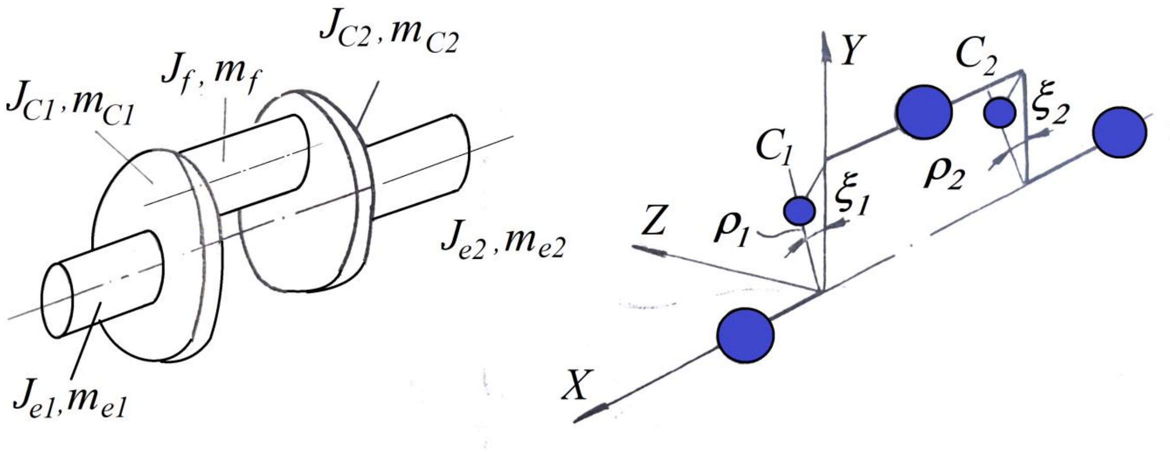


Figure 10. The crank arm parts for one cylinder

The center of mass of the crank arm is determined with the relations:

$$y_C = \frac{m_{C1} \rho_1 \cos \xi_1 + m_f r + m_{C2} \rho_2 \cos \xi_2}{m_m}; \quad (78)$$

$$z_C = \frac{m_{C1} \rho_1 \sin \xi_1 + m_{C2} \rho_2 \sin \xi_2}{m_m}, \quad (79)$$

where:

m_{C1}, m_{C2} - the mass of the crank arm 1 and 2;

m_{11}, m_{12} - the mass of the parts of the crankshaft relative to the crank arm 1 and 2 respectively;

m_f - the mass of the cylindrical part of the crank;

We have:

$$m_m = m_{C1} + m_{C2} + m_{11} + m_{12} + m_f. \quad (80)$$

It results:

$$\rho = \sqrt{\left(y_C^2 + z_C^2\right)}, \quad (81)$$

and:

$$\tan \xi = \frac{z_C}{y_C}. \quad (82)$$

For J_0 we have the calculus relation:

$$J_0 = J_{11} + J_{12} + J_f + m_f r^2 + J_{C1} + J_{C2}, \tag{83}$$

where:

J_{11}, J_{12} - The moment of inertia relative to the axis of the crankshaft of the cylindrical parts 1 and 2 belonging to the crankshaft, respectively;

J_{C1}, J_{C2} - The moment of inertia of the crank arm 1, respectively 2, relative to the axis of the crankshaft;

J_f -The moment of inertia of the cylindrical part of the crank relative to the axis of the cylinder.

4. Discussion and Conclusions

Determining the forces that require the components of IC engines and the moment that is transmitted to the crankshaft represents an important stage in the design of any type of engine. It is necessary to know how many forces are acting and based on the obtained values to make a calculation of the resistance of the materials. Also, the moment that requires torsion of a crankshaft is an important element in an engine design because it will then require the entire transmission of a car. This necessity requires the development of models that allow the calculation of the forces that appear. This is useful to do quickly and with minimal costs. The vector model presented in the paper comes to satisfy this major requirement. The forces that appear in the system are thus determined and, then, using these values, one can apply, for example, the FEM to study the stresses and strains that appear in the engine components. Knowing at any moment the engine torque and its evolution over time will also allow a calculation of the car's transmission, using the appropriate model for each part of the transmission.

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