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RESEARCH PAPER

Influence of Booster's Geometry and Circuit's Resistor on Performance of the Auxetic Energy Harvester - Experimentally Validated Analysis

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Abstract

Modal and frequency response analysis of the piezoelectric energy harvester utilizing the auxetic booster has been performed in this paper. This harvester has composed of a cantilever, auxetic substrate, and piezoelectric layer. The influence of the piezoelectric's electrical circuit and the harvester's geometrical properties on the fundamental natural frequency, output voltage, and harvested power of the energy harvester have been investigated. The electrical circuit of this electromechanical system consists of a resistor that influences the energy harvester's output voltage and harvested power. A comprehensive parametric study has been performed to find the optimum resistor of the energy harvester. All the analysis has been performed using the finite element method. Mesh size sensitivity analysis of the models is presented, and the finite element model is verified by previous experimental studies. Furthermore, the effect of this energy harvester's damping ratio on the system's outputs has been investigated. The results show that the system's output alters considerably in different damping ratios, and it is necessary to determine the system's damping ratio of the system. The damping ratio of the auxetic energy harvester has been measured through the experimental investigation. The present study illustrates that harvested power of a trapezoidal auxetic energy harvester in resonant frequency could improve by 260 percent by utilizing the optimum resistor. Also, increasing the auxetic booster's thickness could improve the output voltage and harvested power by 48 percent and 22 percent.

Keywords: Finite element method, Auxetic structures, Piezoelectric energy harvester, Electrical circuit;

1. Introduction

Wireless sensors for structural health monitoring and wearable electronic devices could be powered by harvesting from sources such as heat, wind, ambient vibrations, magnetic fields, and many other sources. The attention to converting vibrations into operating electrical energy has improved rapidly over recent years as the world places a great emphasis on renewable energy to develop our environment and health. Sensors such as

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accelerometers and velocity-meters face wiring and battery replacement challenges. These challenges have been overcome by self-powered energy harvester sensors [1-4]. Due to its advantages, including simple structure, scalability, and high power out-put, piezoelectric energy harvesting has attracted more interest than other energy harvesting approaches, such as electromagnetic and electrostatic [5]. In the present framework, piezo-electric transducers are examined to convert the mechanical energy of vibration or applied force to electrical energy. Electronic portable devices that are not dependent on traditional methods of energy supply, such as lithium batteries, could be powered by piezoelectric self-power sensors [6]. Many researchers have studied the application of piezoelectric ceramics on vibration energy harvesting. Sodano et al. explained that discharged nickel-metal hydride batteries could be recharged utilizing piezoelectric vibration sensors [7]. In addition, they have studied three types of piezoelectric devices to specify their capabilities to convert ambient vibration into electrical energy and their capacity to recharge a discharged battery. The model of a cantilever beam with a piezoelectric layer has been offered by some researchers [8-10]. Their theoretical model had good agreement with the experimental results. They have examined base excited beams with several forms of dynamic loading. Harvesting energy from non-linear vibration and broadband excitation has been studied by Adhikari et al [11]. They presented a new formation consisting of a cantilever beam with a tip mass mounted vertically and excited in the lateral direction.

Aim to understand the most influential parameters influenced by shape changes, the differences in power output between a rectangular and triangular bimorph have been studied experimentally by Siddiqui *et al* [12]. A trapezoidal cantilever beam has been used for low-frequency piezoelectric energy harvesting by Zhang *et al* [13]. Also, they presented a theoretical model, optimization method, and comparison with the experiment. Chen and Bedekar proposed a new high-power energy harvester device design through a two-piece trapezoidal geometry approach [14]. They simulated the performance of the composite two-piece trapezoidal piezoelectric PZT-PZN polycrystalline ceramic material using COMSOL Multiphysics. Kim et al. defined harvesting power efficiency as the ratio of device output power to mechanical input power [15]. They presented a closed-form solution for harvesting efficiency that allowed device comparison and compared efficiency-optimized versus power-optimized electrical loading conditions. Harvester for full-scale bridge structure and subsequent monitoring undergoing forced dynamic testing and train passages has been presented by Cahill *et al* [16]. Also, they presented associated analysis techniques for system and event identification. Utilizing energy harvesting from operational vibration and based on Radio Frequency (RF) pulse transmission, a new autonomous method for the health monitoring of rotating devices has been presented by Khazaee *el al* [17]. Using a Voltage Multiplier (VM) circuit and storing the energy in a capacitor, they designed an energy harvesting assembly to generate and rectify the harvested energy from the machine vibration. An intelligent wireless network for the Signal Health Monitoring (SHM) of bridge structures has been developed by Maruccio *et al*. [18] which a novel piezoelectric energy harvester supplies the required electric power. Matching the fundamental resonance frequency of the piezoelectric energy harvester with the input frequency of the host structure is another challenge in energy harvester design. Many researchers have indicated that even 5% mismatch may result in 100-time smaller power generation than the maximum value obtained around resonance. The mainstream of frequency-tuning techniques has been focused on decreasing the resonance frequencies of the devices. One of the frequently utilized techniques is adding a proof mass, which can be attached to the free end of a piezoelectric cantilever [19]. Another method of tuning the energy harvester's resonant frequency is utilizing engineered structures such as metamaterials.

Poisson's ratio of a material is the negative ratio between the lateral contractile strain and the longitudinal tensile strain for a material undergoing tension in the longitudinal direction; it illustrates how much a material becomes thinner when it is stretched. Therefore, most of the materials have a positive Poisson ratio. In the case of the counterintuitive behavior of the auxetic materials, this material undergoes lateral expansion when strained longitudinally and becomes thinner when compressed. Auxetic materials offer some unique properties in comparison with common materials. Classical elasticity theory predicts that the auxeticity of materials should lead to enhancements in specific mechanical properties, such as increased plane strain fracture resistance and increased shear modulus, indentation resistance, fracture toughness, and acoustic response compared to conventional materials [20]. In particular, it has been indicated auxetic behavior could be achieved in various highly porous materials, including foams with re-entrant and chiral microstructure, microporous polymeric materials, networks of rigid units, and skeletal structures [21-25]. Moreover, negative Poisson's ratio has also been illustrated in non-porous systems, such as laminates, sheet assemblies of carbon nanotubes, composites, and polycrystalline thin films [26]. A piezoelectric bimorph with auxetic behaviors for increased power output in vibration energy harvesting has been presented by Li *et al* [5]. Their piezoelectric harvester comprises a 2D auxetic substrate sandwiched between two piezoelectric layers. The FE analysis results illustrated the auxetic substrate could increase the lateral stress of a bimorph by 16.7 times, and the average power generated by the auxetic bimorph is 2.76 times that generated by a conventional bimorph.

De Bellis and Bacigalupo [27] investigated the periodic anti-tetrachiral auxetic lattice structures and their mechanical and piezoelectric response. They studied the acoustic behavior of the periodic piezoelectric material with auxetic topology and detected the possible band gaps. Umino [28] has suggested a vibration energy harvester with high power generation efficiency in a low-frequency wide band. The suggested device is a bimorph type made of a middle elastic layer and two piezoelectric layers with a flexible mechanical metamaterial structure. The strain of the piezoelectric layer and the power generation level has improved by controlling the flexibility of the elastic layer by the microstructure. The harvested power of the suggested devise is 1.6 times greater than that of the conventional flat plate and provides the minimum electric power required as a sensor node for WANs. A two-dimensional auxetic lattice structure from a PZT piezoceramic has been fabricated by Fey [29]. (2015). They investigated the in-plane strain response upon applying a uniaxial compression load and an electric field perpendicular to the lattice plane by a 2D image data detection analysis. They concluded strain amplification and anisotropic deformation might be particularly interesting in designing structures for sensors offering enhanced sensitivity. The potential of integrating lightweight honeycomb structures with existing piezoelectric device configurations (bimorph) to achieve higher specific power has been investigated by Chandrasekharan and Thompson [30]. They have shown that replacing the solid continuous substrate of conventional bimorph with honeycomb structures of the same material at low excitation frequency ranges results in a significant rise in the power-to-weight ratio of the piezoelectric harvester. At higher driving frequency ranges, they have shown that, unlike the traditional piezoelectric bimorph with the solid continuous substrate, the honeycomb substrate bimorph can preserve optimum global design parameters by manipulating honeycomb unit cell parameters.

An auxetic piezoelectric energy harvester has been fabricated by Ferguson [31] to increase the output power of a strain energy harvester that is excited by limited strain vibrations. Their harvester is composed of a piezoelectric element bonded to an auxetic substrate. Their experimental results indicated the auxetic energy harvesters could harvest electric power up to 191.1 microwatt, 14.4 times that of the peak power harvested by the plain harvesters. An auxetic booster to improve the efficiency of vibration energy harvesting has been developed by Eghbali *et al* [32]. Their model had auxetic structures and applied extra stretching strain in two perpendicular directions. Compared with the case in which the PZT is attached to the cantilever without the booster, they have indicated adding such intermediate boosters at a low-frequency range can increase the harvested power by factors of 3.9 and 7.0 for the two proposed geometries. An auxetic latticed resonator backed by a rectangular acoustic tube has been developed by Eghbali *et al.* [33] to increase the efficiency of acoustic energy harvesting. They have illustrated that utilizing the auxetic structures can considerably enhance the performance of the acoustic energy harvesting system.

A novel homogenized model based on the Equivalent Single Layer (ESL) formulation has been suggested by Tornabene *et al* [34]. This model has been used to investigate the dynamic response of doubly-curved sandwich panels with a honeycomb core. Also, a homogenized approach for the modal analysis of doubly-curved shells with a lattice layer has been presented by Tornabene *et al* [35]. They have investigated complex geometries of panels utilizing differential geometry outcomes. The free transverse vibration and supersonic flutter analyses of cantilevered trapezoidal plates composed of an orthotropic honeycomb core and two homogeneous isotropic face sheets have been presented by Torabi *et al* [36]. They have modelled the plate based on the first-order shear deformation theory, and the aerodynamic pressure of external flow with desired flow angle has been estimated via the piston theory. An analytical relation for calculating all the effective elastic constants of a periodic hexagonal honeycomb has been presented by Sorohan *et al* [37]. All nine effective elastic properties have been obtained analytically utilizing the beam and membrane plate theory. Two and three-dimensional finite element models to analyze the equivalent orthotropic mechanical properties of honeycombs have been presented by Stefan *et al* [38]. Their models are developed on a representative volume element with appropriate periodic boundary conditions. They obtained the in-plane and out-of-plane elastic properties by investigating six load cases for three-dimensional models. The elastic performance of periodic hexagonal honeycombs over a wide range of relative densities and cell geometries has been investigated by Malek and Gibson [39]. They have utilized both analytical and numerical approaches. They also compared the new analytical equations with previous analytical models, with a numerical analysis based on a computational homogenization technique, and with data for rubber honeycombs over a wide range of relative densities and cell geometries.

This paper is structured as follows. The auxetic piezoelectric energy harvester is introduced in section 1. Mesh convergency and verification study, which is required for finite element study, are investigated in section 2. The trapezoidal auxetic energy harvester is presented in section 3. The parametric analysis on the electrical circuits resistor and thickness of booster and piezoelectric layer is performed in section 4. The damping measurement methodology is illustrated in section 5. The effect of using electrical circuits resistor and thickness of the auxetic booster is investigated in section 6. Also, the experimental setup and examination are explained in this section. Finally, the conclusion is presented in section 7.

To the best of the authors' knowledge, only the effect of rectangular auxetic substrates on piezoelectric energy harvesters has been investigated in previous studies, and the impact of the booster’s thickness and the electrical circuit's resistor has not been explored yet. This study has developed an investigation to increase energy harvesters' output power. A parametric study will be conducted to investigate the effect of the geometry of the auxetic booster and the electrical circuit's resistor on the performance of the energy harvester. Utilizing the rectangular and trapezoidal booster present study investigates the levels of voltage RMS and harvested power frequency response in different mechanical and electrical conditions. Also, modal analysis of the auxetic piezoelectric energy harvester, which has not been presented in previous studies, has been investigated in the present study.

2. Auxetic piezoelectric energy harvesters

The constitutive equations for piezoelectric materials could be derived by considering an electric enthalpy function H defined, for the linear static case without body charge or forces, as [2]

$$H(\varepsilon, E) = \iiint_V \left(\frac{1}{2}\varepsilon_{ij} C_{ijmn} \varepsilon_{mn} - \frac{1}{2} E_i k^{\varepsilon}_{in} E_n + E_n e_{nij} \varepsilon_{ij} \right) dv \tag{1}$$

In this equation, the independent variables are elastic strain $\boldsymbol{\varepsilon_{mn}}$ and the electric field $\mathbf{E_n}$. On the right side of the equation $\mathbf{C_{ijmn}}$ are the elastic constitutive constants measured at a constant electric field, $\mathbf{e_{nij}}$ the piezoelectric constants (measured at a constant strain or electric field), and $\mathbf{k^{\varepsilon}_{in}}$ are the dielectric constants measured at constant strain. Constitutive equations of piezoelectricity can be presented by differentiating this equation with respect to the independent variables, as

$$\sigma_{ij} = \frac{\partial H(\varepsilon, E)}{\partial \varepsilon_{ij}} = C_{ijmn}\varepsilon_{mn} + e_{nij} E_n \tag{2}$$

$$D_i = \frac{\partial H(\varepsilon, E)}{\partial E_i} = e_{imn}\varepsilon_{mn} - k_{in} E_n \tag{3}$$

where $\boldsymbol{\sigma_{ij}}$ is the stress and $\mathbf{D_i}$ is the electric displacement. Eq. (6) shows the relation between the electrical field and voltage, and the electrical field is related to the electrical displacement through Eq. (5)

$$\varepsilon_{mn} = \frac{1}{2}\left(u_{m,n} + u_{n,m}\right) \tag{4}$$

$$D_m = k_{mn} E_n \tag{5}$$

$$E_m = -\phi_{,m} \tag{6}$$

From Eq. (2), in the linear case, the stress applied to a piezoelectric material will be converted to elastic deformation and an electrical field proportional to the piezoelectric constitutive matrix. This electrical field will also result in a gradient through Eq. (6), generating the voltage from the energy harvester. Also, this electrical field provokes energy loss since Eqs. (3)- (5) indicates that the electrical field will create a self-induced electrical field with the opposite sign, proportional to the permittivity constants. Hence the voltage output from the energy harvester is maximized by decreasing the permittivity constants and increasing the piezoelectric constants.

Base vibration of the cantilever beam would produce longitudinal stress and deformation. Utilizing auxetic structures in the cantilever provides the transverse deformation of the beam, and bonding a uniform layer on the auxetic beam makes the transverse stress appear on the bonded layer and auxetic beam. Fig. 1 illustrates the auxetic piezoelectric energy harvester model. The bonded layer and auxetic beam's stiffness ratio influence the amplitude and distribution of transverse stress. Also, the auxetic cells' design is the effective parameter on the stress distribution of the bonded layer. The piezoelectric layer has been utilized as a bonded layer on the auxetic beam to assemble the auxetic energy harvester. Energy harvesters are usually used to provide sensors and other electronic devices power. Harvested power, which depends on the cantilever base acceleration and stress distribution on the piezoelectric layer, is the most significant parameter in energy harvesters. Increasing the harvested power of the piezoelectric energy harvester will be investigated by utilizing auxetic structures and changing the shape of the substrate in this research.

The present study has studied utilizing an auxetic structure to increase the harvested energy in vibration energy harvesters. This investigation has been accomplished by using the finite element method. Auxetic energy harvesters employed by previous experimental research have been utilized to verify the present finite element model. Fig. 1 indicates the model's geometry of previous studies, and their model's physical and mechanical properties have been explained [31, 32]. This model is composed of a steel cantilever, a steel substrate, and a piezoelectric layer fabricated by PZT8. Decreasing the size of elements leads to a considerable effect on unconverged finite element models. The finite element model results should be independent of reducing the element size.

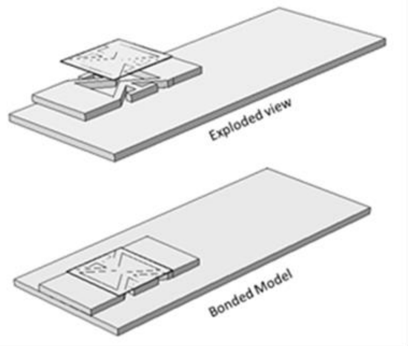

Fig. 1. Auxetic piezoelectric energy harvester.

Because of the importance of strength analysis and vibration characteristics of the harvester in the vibrational surrounding, the model's convergence would be investigated by free vibration and static response. Boundary distributed force is applied to the free end of the cantilever for static analysis. Fig. 2 and Fig. 3 show maximum deflection and the first natural frequency of the energy harvester's model converge by increasing the number of elements. Fig.2 indicates that ten thousand elements are adequate for converged response in free vibration analysis. The first natural frequency grew up less than 0.1 percent after increasing the number of elements by more than ten thousand. Also, Fig. 3 illustrates that this number of elements is enough to converge the first natural frequency of the energy harvester, which has not changed by increasing elements by more than ten thousand. It should be noted that a hexahedron element (20 nodes) has been used in the present study. The model has been designed and analyzed in Comsol Multiphysic® version 5.6.
In order to validate the present finite element model, the results of the experimental investigation [31] are simulated by this model. The geometry of this model is shown in Fig. 1. A harmonic strain at an amplitude of $250\mu\varepsilon$ peak-to-peak has been applied to this sample at a frequency of *10Hz*. In the FE model, a thin elastic layer modelled the bonding between the piezoelectric layer and the auxetic substrate. The adhesion strength of this layer has been defined by the spring constant per unit area (KA).

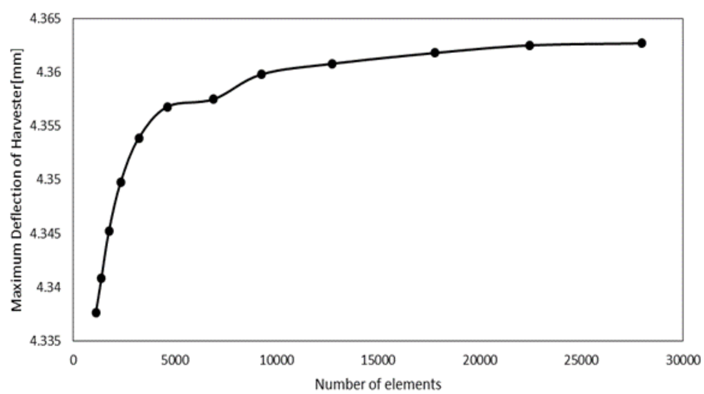

Fig. 2. Mesh convergency investigation of static response.

Fig. 4 indicates the comparison of harvested power of the experimental sample and the FE model's results. This comparison has been performed for the plain harvester and auxetic harvester. These harvesters' spring constant per unit area is $173GN/m^3$ and $159GN/m^3$, respectively. This figure shows good agreement between the experimental and simulation results for plain and auxetic models.

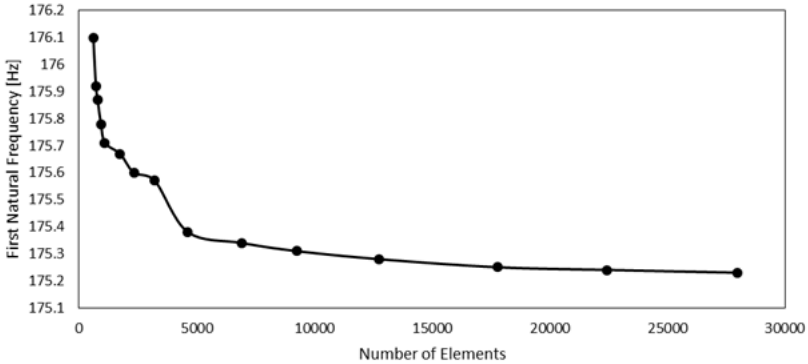

Fig. 3. Mesh convergency investigation of first natural frequency.

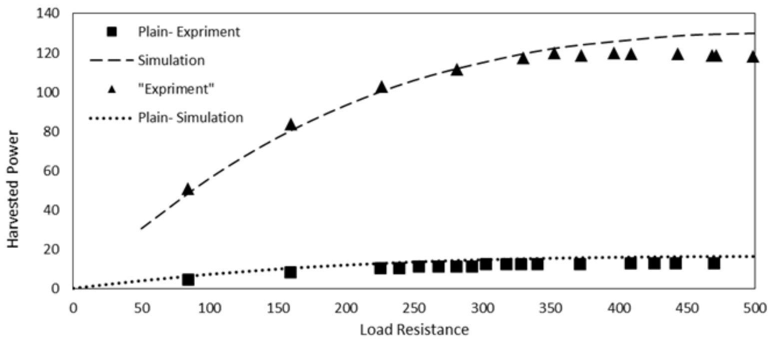

Fig. 4. Comparison of the present FE model and experimental sample.

3. Trapezoidal auxetic energy harvester

To the best knowledge of the writers, auxetic cells were only utilized in rectangular energy harvesters in previous research. The auxetic energy harvester in the present study consists of a steel cantilever, steel booster, and piezoelectric patch. The physical and geometrical properties of the energy harvester have been illustrated in Table 1. The parameters of this table will not be changed in the present research's analysis unless it is explained in parametric studies. The parameters of this table will not be changed in the present research's analysis unless it is explained in parametric studies.

Table 1. The physical and geometrical properties of the auxetic energy harvester.

	Material	Geometry parameters	
Cantilever	Steel St 37	Length [*mm*]	120
		Width [*mm*]	25
		Thickness [*mm*]	1
Substrate (booster)	Steel St 37	Length [*mm*]	40
		Width [*mm*]	20
		Thickness [*mm*]	2
Piezoelectric layer	Piezoceramic (PZT 4)	Length [*mm*]	24
		Width [*mm*]	24
		Thickness [*mm*]	0.46

Using auxetic cells with the trapezoidal pattern on the trapezoidal beam to increase the harvested power is investigated in this section. As shown in Fig. 5, the size of one edge of the harvester remains constant, and the size of the other side has decreased. Also, the size of one side of the auxetic cell has been reduced. The ratio between the edge size of the auxetic cell is the same as that of the harvester base edge size.

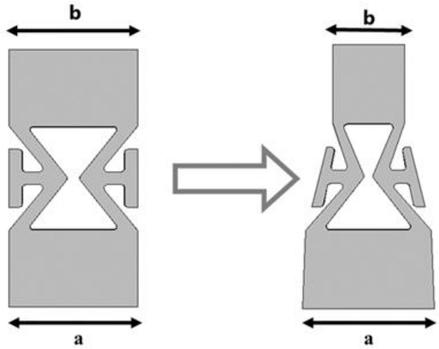

Fig. 5. Changing the geometry of the rectangular auxetic energy harvester to a trapezoidal harvester.

In other words, '*a*' remains constant in Fig. 5 while '*b*' decreases. The present investigation shows changes in the shape of the harvester base lead to changes in the natural frequency of the harvester. The natural frequency of the harvester is a significant factor in harvesting energy from vibrating surroundings. Changing the geometry of the auxetic cell and harvester base from rectangular to trapezoidal leads to two consequences: first, decreasing the harvester's natural frequency, and second, increasing the harvested energy in comparison to the rectangular harvester. It should be noted perfect bonding assumption has been simulated for a thin elastic layer in this investigation.

4. Parametric study

It is advantageous to describe the electrical domain of the coupled system by the simple circuit shown in Fig. 6. Characterizing the piezoelectric element as a current source in parallel with its internal capacitance is well-known in the circuitry-based energy harvesting literature. The components of the circuit are the internal capacitance of the piezoceramic layer C_p, the resistive load R_l and the dependent current source *i(t)*. The voltage across the resistive load is represented by *v(t)*. Applying Kirchhoff's laws to the electrical circuit illustrated in Fig. 6 leads to the following equation [40].

$$C_{\tilde{p}} \frac{dv(t)}{dt} + \frac{v(t)}{R_l} - i_{\tilde{p}}(t) = 0 \tag{7}$$

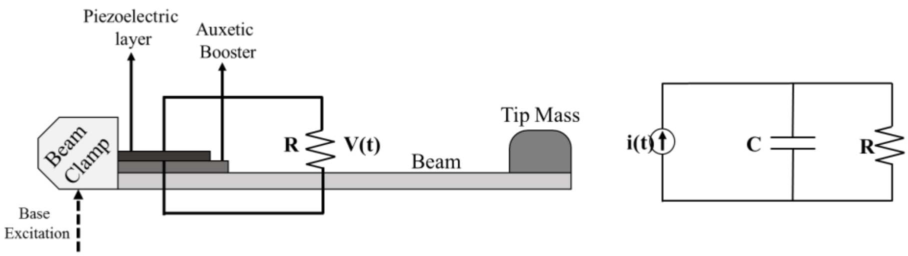

Fig. 6. Cantilever with the resistive load and the corresponding electrical circuit.

The coupled electromechanical behavior of the piezoelectric influences the energy harvester's mechanical characteristics. Fig. 7 indicates the variation of the first natural frequency of harvesters with edge sizes 5 mm, 10 mm, and 20 mm in the resistor range *0-1000 kΩ*. As expected, reducing the edge size of the energy harvester leads to decreasing the first natural frequencies of these harvesters. Also, increasing the resistor of the electrical circuit increases the first natural frequency lower than one percent. This investigation shows that the resistor of the electrical circuit could not take an essential role in tuning the fundamental frequency of the energy harvester and tuning the energy harvester with the host structure's frequency, utilizing the geometry. Physical parameters of the energy harvester would be suggested.

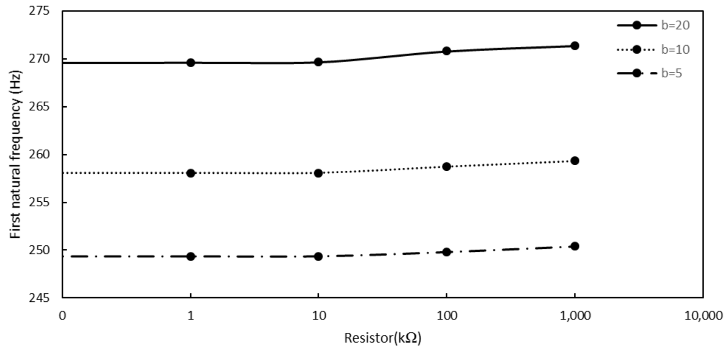

Fig. 7. First natural frequency for harvesters with edge sizes 5 mm, 10 mm, and 20 mm.

In order to investigate the effect of external resistors on harvested power, a parametric study has been performed in three damping ratios (*η=0.1, 0.2, 0.3*). This investigation has been conducted by applying the *1g (9.8067 m/s^2)* base excitation to the energy harvester. The frequency of base excitation is *260 Hz* which is near the first resonant frequency of the harvester. The results of this examination –which has been illustrated in Fig. 8- indicate the maximum harvested power for the harvester with the damping ratio 0.10 is *932μW* which is *267* percent more than the minimum level of harvested power in the range *0-1000 kΩ* of the electrical circuit's resistor. Also, the present investigation shows maximum harvested power is *427μW* and *243μW* for the harvester with damping ratios of *0.15* and *0.20*, respectively. These investigations illustrate the *274* percent difference between the maximum and minimum level of harvested power for these damping ratios.

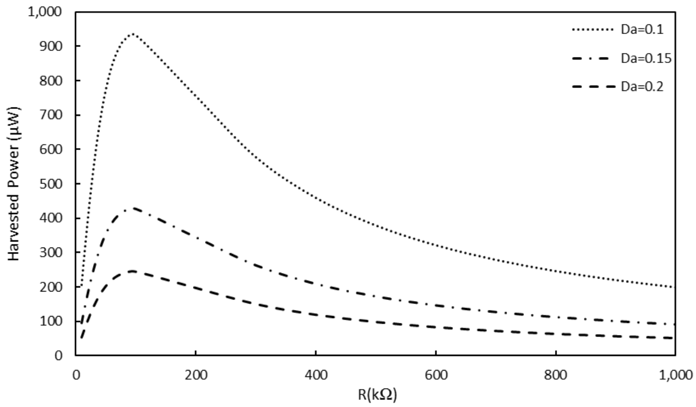

Fig. 8. Harvested power versus load resistance for three damping ratios.

Also, the output voltage RMS of the energy harvester has been illustrated in Fig. 9. This parametric study shows the output voltage RMS growth with increasing the resistor of the electrical circuit. This value has been converged for approximately *220 kΩ* resistor. The converged value for the harvester with the damping ratio *0.1* is *14.1v*, and it is decreased to *9.5v* and *7.1v* for the harvesters with damping ratios *0.15* and *0.2*, respectively.

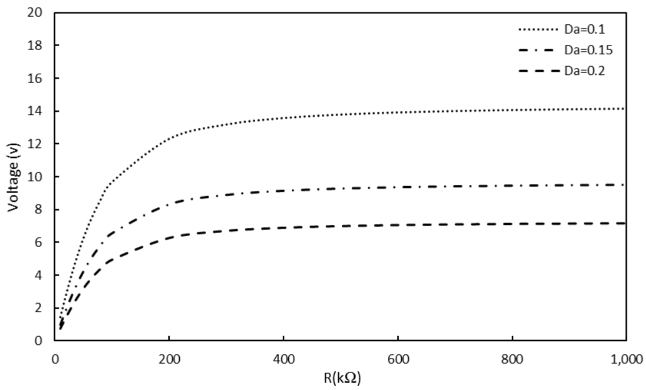

Fig. 9. RMS value of the output voltage variation.

The present parametric study indicates damping ratio is one of the significant parameters in the harvested power and Voltage RMS of the energy harvester. The maximum harvested power for the harvester with an assumed damping ratio *Da=0.1* is *932µW*, and it decreases to *243µW* for the assumed damping ratio *Da=0.2*, which means almost *283%* reduction in harvested power of the optimum resistor. This investigation indicates the damping ratio should be measured by experimental study. The following section explains the method of damping measurement to increase the simulation's accuracy.

In order to evaluate the effect of the electrical circuit resistor on the modal behavior of the energy harvester, the fundamental frequency of the energy harvester has been investigated through the resistor range of *1-1000 kΩ*. This examination has been performed for the rectangular harvester (*b=20*) and trapezoidal harvesters (*b=15, 10, 5*).

Another investigation should be performed to analyze the effect of the booster thickness on the natural frequencies and harvested power of the energy harvester. The results of this investigation are shown in Fig. 10. This figure illustrates that increasing the booster's thickness from 0.5 mm to 5 mm increases the first natural frequency of the energy harvester. The increase for harvesters with edge sizes 20, 10, and 5 are 92 percent, 87 percent, and 85 percent, respectively. Also, the effect of the piezoelectric layer thickness on the energy harvester's natural frequency has been investigated in three harvester edge sizes. The results of this study are illustrated in Fig. 11. Increasing the thickness of the energy harvester from 0.2 mm to 0.8 mm leads to increasing the natural frequency by 9.0 percent, 8.5 percent, and 7.9 percent for the harvester with edge sizes of 20 mm, 10 mm, and 5 mm, respectively. Mode shapes of the first six modes of the energy harvester with an edge size of 10 mm have been illustrated in Fig. 12. The first mode shape of the energy harvesters is the most significant in utilizing the harvesters in surrounding vibration.

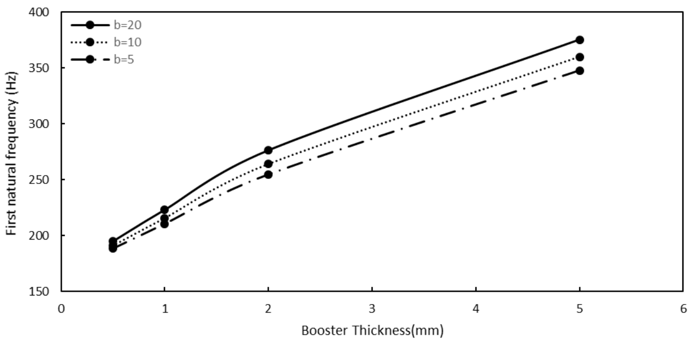

Fig. 10. Effect of booster thickness on the first natural frequency of the energy harvester.

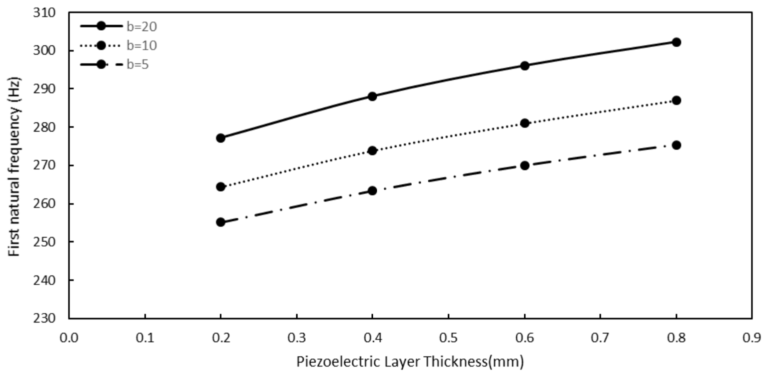

Fig. 11. Effect of piezoelectric layer thickness on the first natural frequency of the energy harvester.

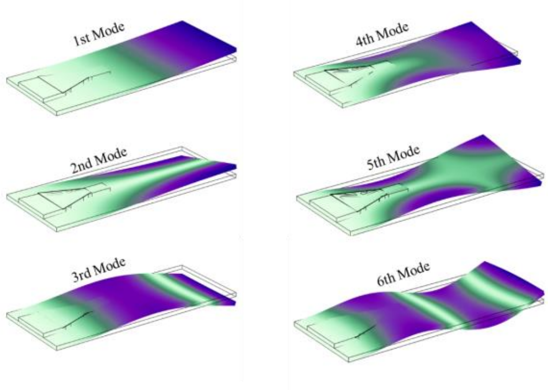

Fig. 12. Mode shape of first six modes of the trapezoidal energy harvester.

5. Damping measurement

The damping ratio is an effective parameter in vibration and dynamic analysis. It should be noted that several mechanisms contribute to the total damping in structures. Different mechanisms may involve different stress levels, temperatures, or frequencies. Thus damping depends on both frequency and mode number. It is essential to specify all the working conditions to identify the effect of various variables on the total damping of structures.

The free-decay method is a suitable way of evaluating the damping in the structure. The structure is set into free vibration by a shock load such as a small explosive; the fundamental mode governs the response since all the higher modes are damped out quickly. It is not usually possible to excite any mode other than the fundamental one using this method. The logarithmic decrement δ could be determined as the following equations [41] by measuring and recording the decay in the oscillation, as illustrated in Fig. 13,

$$\delta = \ln\left(\frac{X_2}{X_1}\right) = \ln\left(\frac{X_{n-1}}{X_n}\right) = \frac{1}{n}\ln\left(\frac{X_1}{X_n}\right) \tag{8}$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \tag{9}$$

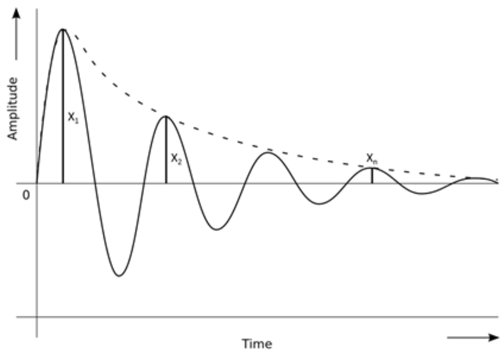

Fig. 13. Vibration decay of system with viscous damping.

The amplitude of oscillations, logarithmic decrement, damping ratio, and damping loss factor have been shown correspondingly X_n, δ, ζ and η. The damping loss factor could be calculated for low damping by Eq. (10).

$$\eta = 2\times\zeta = 2\times\frac{\delta}{2\pi} = \frac{\delta}{\pi} \tag{10}$$

The present model with one auxetic cell has been fabricated to specify the damping loss factor. Fig. 14 illustrates this sample subassembly: a steel plate as a cantilever, a steel booster with an auxetic cell, and a PZT ceramic patch. These parts have been bonded perfectly. In order to specify the damping ratio, the present energy harvester has been excited by hammer impact. The piezoelectric patch's output voltage of the harvester with one auxetic cell shows the displacement amplitude decreases with the logarithmic decrement $\delta=0.38$, which means the damping loss factor is $\eta=0.12$. The piezoelectric patch's output voltage has been illustrated in Fig. 15.

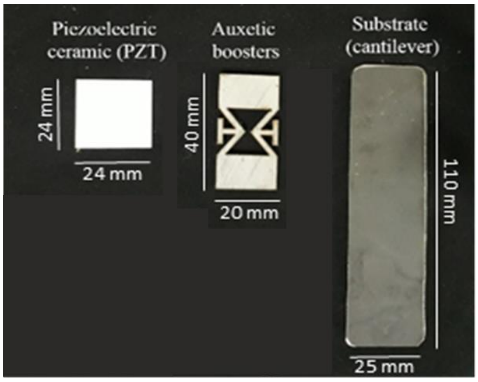

Fig. 14. Auxetic energy harvester sample.

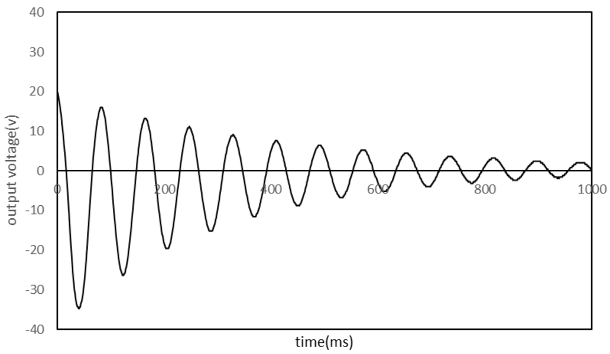

Fig. 15. Decrement of output voltage's amplitude.

6. Harvested Power FRF

The accuracy of the results of the finite element method in frequency response analysis has been improved after the measurement of the damping ratio of the energy harvester. Fig. 8 and Fig. 9 illustrate the effect of the damping ratio on the output of finite element frequency response analysis is considerable. The harvester with the rectangular booster has been fabricated to validate the finite element analysis, and the frequency response test has been performed to evaluate the harvested power experimentally. The harvester, its schematic diagram, and experimental setup are shown in Fig. 16 (a), (b), and (c), respectively.

This harvester has been composed of a steel cantilever, auxetic substrate, and piezoelectric layer. The geometry parameters of this harvester have been introduced in Table 1.

This energy harvester has been mounted on an electrodynamic shaker (TV52110-TIRA) through the clamp and connector. A power amplifier (type BBA 120 - TIRA) has been used to drive the shaker. The force transducer (B&K 8200) has been installed on the connector to measure and tune the shaker's acceleration amplitude. The harvester has been bonded to the shaker by the tight connections. The output voltage of the energy harvester and transducer has been transmitted to the data acquisition card (NI), and finally, data has been collected into the Lab View software.

In order to compare the simulation and experimental results, the power frequency response of the energy harvester with the electrical circuit's resistor *10 kΩ*, *100 kΩ*, and *1000 kΩ* has been measured. The results of this investigation have been compared with simulation results in Fig. 17. This study has been performed for the energy harvester with an edge size of *20 mm*. Results of section 5 measurements, have been utilized in the damping ratio assumption in simulations. Other geometrical and physical properties of this harvester are according to Table 1. Experimental measurements have occurred in *100 Hz*, *150 Hz*, *200 Hz*, *240 Hz*, *260 Hz*, *300 Hz*, *350 Hz*, and *400 Hz*. The difference between finite element analysis and the experimental results is in the range of 9-15 percent, which is acceptable. Results of the present investigation show 100 kΩ resistor in the electrical circuit harvest more energy than other resistors, which agrees with the Fig. 8 results. Maximum harvested power for base excitation acceleration $1 m/s^2$ and *10 kΩ*, *100 kΩ* and *1000 kΩ* resistor is *1.6 μw*, *9.5 μw*, and *3.0 μw*, respectively.

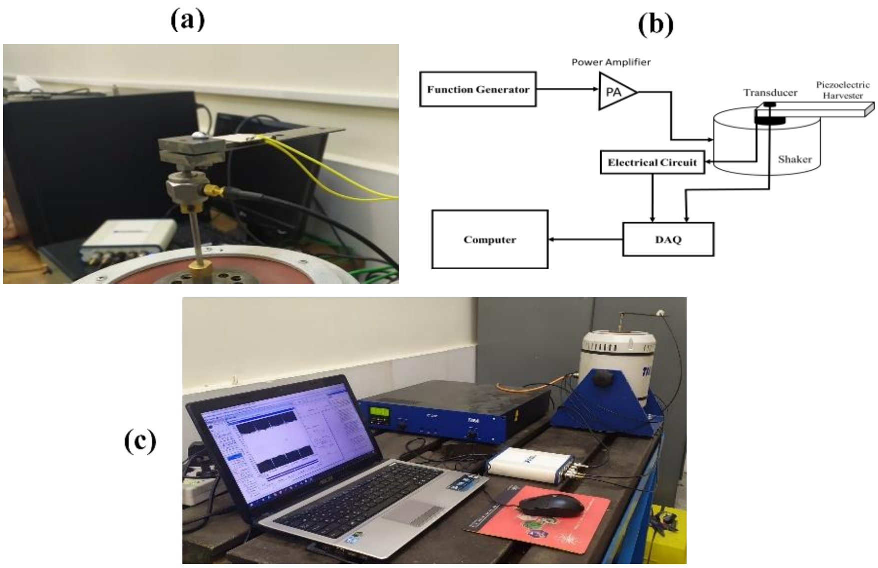

Fig. 16. (a) Fabricated energy harvester, (b) Test schematic, and (c) Test setup.

In order to evaluate the effect of booster thickness on the voltage RMS frequency response and harvested power frequency response, another finite element analysis has been conducted. The results of this analysis are indicated in Fig. 18 and Fig. 19.

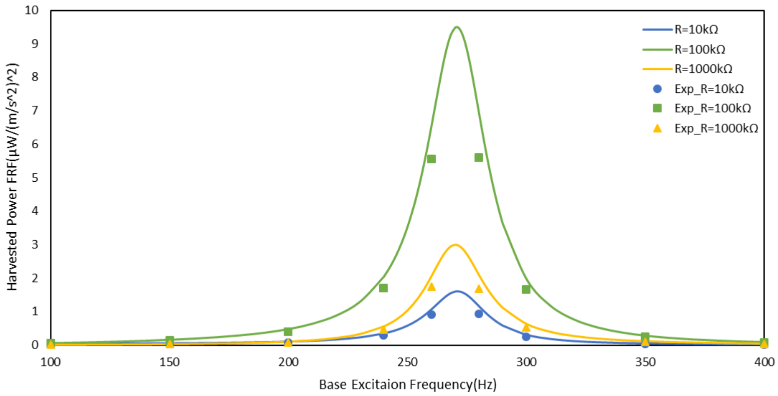

Fig. 17. Experimental and simulation results of harvested power frequency response.

Increasing the booster's thickness from 0.5 mm to 2 mm improves the maximum level of voltage RMS and harvested power. This analysis shows that increasing the booster thickness could improve the harvested power and voltage RMS levels by 48 percent and 22 percent, respectively.

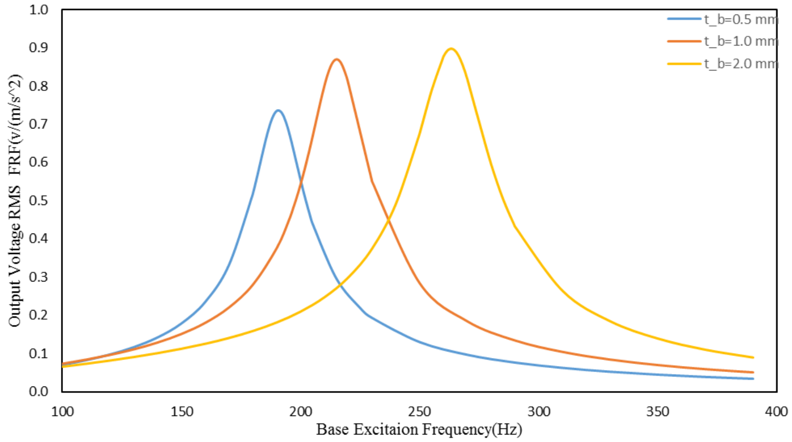

Fig. 18. Effect of booster thickness on the voltage RMS frequency response of the energy harvester.

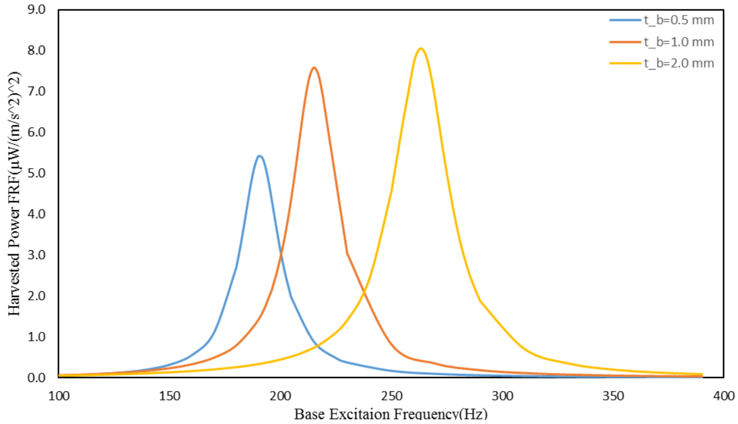

Fig. 19. Effect of booster thickness on the harvested power FRF of the energy harvester.

7. Conclusion

Modal analysis and frequency response of the auxetic piezoelectric energy harvester has been investigated in the present study. The finite element method has been used to analyze the effect of a wide range of parameters on the performance of the auxetic energy harvester. Experimental tests have been conducted to verify the output of the finite element analysis. Effect electrical circuit’s resistor has been examined on the energy harvester's fundamental natural frequency and harvested power frequency response. Also, the effect of geometrical parameters of the auxetic booster has been investigated on the harvested power frequency response and first natural frequency of the energy harvester.

The results of the investigation indicated damping ratio significantly affects the energy harvester's harvested power. In order to improve the accuracy of the finite element analysis, the damping ratio of the energy harvester has been measured experimentally. Also, the investigation output voltage indicates that increasing the electrical circuit's resistor to 250 kΩ could increase the voltage RMS of the energy harvester by 267 percent. Also, this analysis indicates increasing the resistor by more than 250 kΩ could not raise the output voltage. The effect of the auxetic booster’s thickness on the fundamental natural frequency of the energy harvester has been investigated, and results indicated increasing the thickness from 0.5 mm to 5 mm leads to increasing the fundamental frequency by 92 percent.

The present study shows increasing the auxetic booster's thickness from 0.5 mm to 5 mm could improve the output voltage and harvested power by 48 percent and 22 percent.

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