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RESEARCH PAPER

A thermo-elastic model with a single relaxation time of an unbounded medium heated by a heat supply and moving vertically

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Abstract

The current paper presents a thermoelastic model with a single relaxation time to examine the thermoelastic interaction in an isotropic infinite medium. The unbounded medium is exposed to a thermal shock with varying temperatures due to a vertically moving heat source in a planar region. The basic partial differential equations were solved using the Laplace transform method. Physical fields are studied and compared in terms of how the speed of the heat source, the relaxation time parameter, and the time parameters affect their behavior. Graphical presentations are used to analyze physical field variables like temperature changes, thermal stress and deformation.

Keywords: Thermoelastic theory; Lord and Shulman; heat supply; unbounded medium.

1. Introduction

The motion equations, the compatibility equations, and the foundation law are the basic laws of thermal elasticity, identical to those of the theory of elasticity. The idea of linear thermoelasticity is based on the linear addition of thermal stresses to mechanical strains as the primary catalytic force. Conventional thermoelastic theory predicts that the effect of thermal shock will be immediately felt at all application sites when the shock is delivered to a homogeneous and isotropic elastic body, and an external change in temperature causes the shock. This indicates that thermal shock affects the pattern and amount of thermal stress and temperature in solving related thermoelastic problems. This suggests that the paradoxical perturbation of thermoelasticity can be transmitted at any speed in experimental particle physics. Therefore, the assumption based on the conventional Fourier law of heat transport must be replaced by a better mathematical formula because this response is not physically valid and interacts with heat transfer processes.

Many serious efforts and different theories have been put forward to solve this physical inconsistency of conventional thermal elasticity, which says that heat wave disturbances can spread at an infinite rate. These theories explain why heat signals can only move at a certain speed by introducing the additional phrase "heat flow rate" to Fourier's law or making the amount of temperature one of the variables. In this context, Lord and Shulman [1] proposed one of the most famous improvements to the theory of thermoelasticity, which depends on the heat flux rate by including a single period of relaxation. Within the framework of the thermoplastic process, Green and Lindsay [2] proposed another revised hypothesis of thermoelasticity which allows for two phases of thermal relaxation. Furthermore, Green and Naghdi [3-5] introduced three models to aid the development of extended thermoelastic

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frameworks: GN-I, II, and III. Tzou [6-8] further proposed a new model (the DPL model) in which a two-stage delay separates the heat flow from the temperature gradient. In recent years, Abouelregal [9, 10] developed a thermal conductivity model with a high-order time derivative and further developed Green and Naghdi's extended theorems without energy dissipation by Mohammadi, et al. [11, 12]. In addition, he built a unique model of thermoelastic conduction with two temperature and high-order time derivatives and biphasic delays by Abouelregal [13, 14].

The phenomenon of thermal conductivity of materials involving mobile heat sources has been investigated in various fields in recent decades Hu and Liu [15]. Among the most important areas that include such sources are welding and cutting materials, drilling, hardening/laser forming, plasma spraying, and heat treatment of metals of all kinds. It is also used in manufacturing electronic components, even shooting the barrel of a gun, burning solid fuel, and others [16-18].

The field of temperature change within materials is most commonly described using the heat transfer equation with time-dependent local source terms for the transfer of heat sources. This is the most important physical variable for such real-world applications Sun, et al. [19]. Several additional thermo-physical properties of the material, such as metallic microstructures, heat stress, residual stress, and partial deformation, can be calculated after obtaining the temperature range Mirkoohi, et al. [20, 21]. It is important to infer precisely how the temperature range around moving heat sources will change over time during these engineering processes. Since a moving heat source can be placed on a surface or within a material, the resulting mathematical formula will include a source term in the boundary case or a controlled heat transfer equation, depending on which one applies. Depending on the situation, a moving heat source is usually represented by a point, a line, or a flat surface, each of which has a different shape see in Akbari, et al. [22, 23].

Regardless of the moving heat source, its power is almost always focused in a finite area that depends on time. From studies and experiments, it became clear that in the confined area around the moving heat source, the temperature of the material will change dramatically. As a result, it is clear that the adaptive grid technique, which dynamically focuses on several grid points in local areas with rapid temperature change, may yield significant efficiency gains over the static grid method when the problem is solved with the same precision Huang and Russell [24]. Many different techniques, whether using an analytical or numerical approach, have been used to analyze the temperature change and thermal properties associated with the issue of moving heat sources in much literature [25-29].

In recent years, nanosensors are a typical type of nano-electromechanical system (NEMS) that is used for detection at the nanoscale and is also one of the most useful nano-devices. In many different fields, such as environmental nanotechnology, nanomedicine, nanomechanics, etc., accurate and sensitive detection of nanoparticles, including proteins and viruses, is critical [30-34]. Continuum modeling of nanostructured materials has garnered a lot of attention from the scientific community in recent years due to the impracticality of conducting experiments of this scale and the high processing cost of conducting simulations using molecular dynamics. Nanorods, nanoparticles, graphene sheets, and nanorings are just a few examples of nanostructures that have been studied using the nonlocal elasticity theory to analyze the influence of small scale on bending, resonance, and buckling [11, 35-37].

The originality of the present research lies in its attempt to characterize thermoelastic interactions in an unbounded elastic medium within the framework of the hyperbolic thermoelasticity model with a single-phase lag. It was taken into account that the outer surface of the half space is subjected to thermal shock as well as a time-dependent heat source moving perpendicular to the outer surface of the medium. Since the laser beam is in motion with respect to the part during cutting and scribing, heat transfer model from a moving source of heat is an area of study. If the scanning speed is held constant, then the erosion front and subsequent temperature distribution will remain unchanged with respect to a reference frame cantered on the laser beam.

To obtain the analytical solution, the coupled differential equations are decoupled using Laplace transform procedures, and the resulting system is solved analytically. The problem was solved numerically using the Honig and Hirdes approach, which is a numerical implementation of the inverse Laplace transform. Graphs of the numerical findings for various models are also provided.

After the introduction, the remainder of the research can be organized as follows: The basic equations of the theoretical thermoelasticity model are established and discussed in Section 2. The problem of one-dimensional heat conduction in the presence of heat sources is described in Section 3. The fourth section solves the problem analytically after applying the Laplace transform. An application of the proposed problem is presented in Section 5, where it is assumed that the free surface is adherent and due to thermal shock. Due to the difficulty of obtaining the analytical formulas of the inverse Laplace transforms, an accurate and proven numerical algorithm was applied in the fifth section. In the sixth section, the numerical results were discussed. In the last section of this article, the most important conclusions obtained are presented.

2. Governing equations of generalized thermoelasticity

The experimental Fourier law is the oldest model presented to study how heat travels through matter. This law takes into account the existence of a linear relationship between the heat flow $\vec{Q}$ and the temperature gradient $\nabla\theta$ as follows:

$$\vec{Q}(\vec{r},\tau) = -K\nabla\Theta(\vec{r},\tau). \tag{1}$$

The simple equation for energy conservation is as follows:

$$\rho C_S \frac{\partial\Theta}{\partial t} + \Omega T_0 \frac{\partial}{\partial\tau}\left(\nabla\cdot\vec{U}\right) = -\nabla\cdot\vec{Q} + H \tag{2}$$

When Fourier's law (1) and the conservation equations (٢) are put together, the classical equivalent Fourier thermal conductivity equation can be found. Over the past decades, Fourier's law has been used in a wide range of mechanical and engineering fields for ease of understanding and application. But on the other hand, Fourier's law gives an equation of a parabolic type heat transfer with an infinite speed of conductive heat transfer, which is contrary to what can be seen in the real world. Biot [38] devised the theory related to thermal elasticity to solve the paradox problem, which eliminates the problem that the deformation does not affect the temperature change. For this theory to be in line with the experimental data, the velocity of the thermoelastic signal is not expected to be limited by its field equations, which are parabolic and hyperbolic combined equations.

To address the conflict and inconsistencies of the classical models, the concepts of thermoelasticity have been expanded, and many suggestions have been made. Cattaneo [39] provided a clear mathematical correction to the Fourier law of heat transfer propagation error. Cattaneo's idea is an important improvement that limits the speed of heat wave propagation. In this proposal, Cattaneo introduced the thermal relaxation time t_0 with the rate of heat flux as:

$$\left(1 + t_0\frac{\partial}{\partial t}\right)\vec{Q}(\vec{r},\tau) = -K\nabla\Theta(\vec{r},\tau) \tag{3}$$

Applying the Cattaneo relaxation-time model, the associated theory can no longer claim that heat waves can travel at infinite speeds. Lord and Shulman [1] proposed the initial generalized theory of thermoelasticity with one relaxation period for homogeneous and isotropic bodies based on this previous suggestion. Instead of using Cattaneo's concept of thermal conductivity, this theory uses a new law of thermal conductivity that explains heat flow and its time derivative.

Lord and Shulman's model suggests that the generalized heat transfer equation can be written as:

$$K\nabla^2\Theta = \left(1 + t_0\frac{\partial}{\partial\tau}\right)\left(\rho C_e\dot{\Theta} + \Omega T_0\dot{e} - H\right). \tag{4}$$

The stress tensor S_{kl} can be defined by the relation

$$S_{kl} = 2\mu e_{kl} + (\lambda e_{mm} - \Omega\Theta)\delta_{kl}. \tag{5}$$

where e_{kl} stands for the strain tensor, which can be found by following the formula:

$$e_{kl} = 0.5\left(U_{k,l} + U_{l,k}\right), \tag{6}$$

one of the formulas for the equations of motion is as follows:

$$S_{kl,l} + F_k = \rho \ddot{U}_k. \tag{7}$$

Introducing Eqs. (5) and (6) into Eq. and (7), we get

$$\mu U_{k,ll} + (\lambda + \mu) U_{l,kl} - \Omega \Theta_{,k} + F_k = \rho \ddot{U}_k. \tag{8}$$

3. Problem formulation

Applying the generalized elasticity theory and for the purposes of the study and discussion, it will be taken into account that the region $X \geq 0$, which represents half of the infinite space, has a constant temperature of T_0 (see Figure 1). The boundary surface of the medium $X = 0$ will be thought to be fixed and exposed to thermal shock and a moving heat source $Q(X, \tau)$. As $Q(X, \tau)$ propagates across the surface of half the area, it causes heat to be emitted perpendicular to the x-axis. In the Cartesian coordinate system (X, Y, Z), we will also assume that the X coordinate is parallel to the central axis.

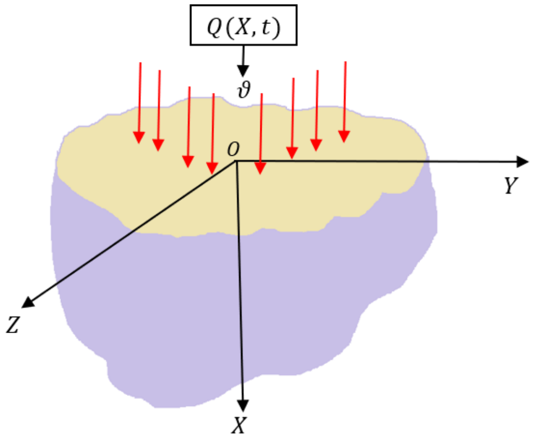

Figure 1: An infinite space due to a moving heat supply

The proposed half-space problem can be simplified to a one-dimensional problem, in which case the only component of the displacement is $U_X = U(X, \tau)$. As a direct consequence of this, the thermal stress, denoted by the symbol S_{xx} in equation (7) can be expressed as:

$$S_{xx} = (\lambda + 2\mu) \frac{\partial U}{\partial X} - \Omega \Theta \, . \tag{9}$$

By substituting equation (9) into equation (8) and using the relation (6), we get

$$(\lambda + 2\mu) \frac{\partial^2 U}{\partial X^2} - \Omega \frac{\partial \Theta}{\partial X} = \rho \frac{\partial^2 U}{\partial \tau^2} \, . \tag{10}$$

In addition to this, equation (4) describes the modified LS heat transport theory and can be expressed as

$$\left(1 + t_0 \frac{\partial}{\partial \Omega}\right) \left(\frac{\rho C_s}{K} \frac{\partial \theta}{\partial \Omega} + \frac{\Omega T_0}{K} \frac{\partial^2 U}{\partial \tau \partial X} - \frac{1}{K} H\right) = \frac{\partial^2 \Theta}{\partial X^2}. \tag{11}$$

The following dimensionless variables will be taken into account

$$\{x,u\} = \eta c_0\{X,U\},\ \ \{t,\tau_0\} = \eta c_0^2\{\tau,t_0\}, \theta = \frac{1}{T_0}\Theta,$$
$$\sigma_x = \frac{1}{\mu}S_{xx},\ h = \frac{H}{KT_0c_0^2\eta^2}, c_0 = \frac{\lambda+2\mu}{\rho}, \eta K = \rho C_s. \tag{12}$$

Using the preceding data, the non-dimensional form of equations (9) to (11) can be derived as follows:

$$\sigma_x = a\frac{\partial u}{\partial x} - p\theta, \tag{13}$$

$$a\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = p\frac{\partial \theta}{\partial x}, \tag{14}$$

$$\frac{\partial}{\partial t}\left(1 + t_0\frac{\partial}{\partial t}\right)\theta + q\left(1 + t_0\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial t\partial x} = \left(1 + t_0\frac{\partial}{\partial t}\right)h + \frac{\partial^2\theta}{\partial x^2}, \tag{15}$$

where

$$a = \frac{\lambda+2\mu}{\mu}, \quad p = \frac{\Omega T_0}{\mu},\ \ q = \frac{\Omega}{\rho C_E}.$$

It is suggested that the initial and continuing aspects of the problem are as follows:

$$\theta = 0 = \frac{\partial\theta}{\partial t} = 0, \quad u = 0 = \frac{\partial u}{\partial t} \quad \text{at} \quad t = 0, \tag{16}$$

In the current problem, it will be taken into account that the heat source h, which moves at a constant speed ϑ in the direction of the surface of the medium and constantly releases energy in the positive direction of the x-axis, affects the stress and heat of the half region $x \geq 0$ affected by this heat source. So, the non-dimensional source of heat h of uniform magnitude h_0 can be thought of as having the following shape:

$$h = h_0\delta(x - \vartheta t)\,, \tag{17}$$

where $\delta(\cdot)$ denotes the delta function.

4. The solution in the Laplace transform domain

The following relationship forms the definition of the well-known Laplace transform:

$$\bar{g}(x,t) = \int_0^\infty e^{-st}g(x,t)dt, \quad s > 0. \tag{18}$$

When applying the Laplace transform to the initial conditions (16), the expressions (13)-(15) in the field of the Laplace transform become:

$$\bar{\sigma}_{xx} = a\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} - p\bar{\theta}, \tag{19}$$

$$a\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}x^2} - s^2\bar{u} = p\frac{\mathrm{d}\bar{\theta}}{\mathrm{d}x}, \tag{20}$$

$$\psi_1\bar{\theta} + \psi_2\frac{\mathrm{d}\bar{u}}{\mathrm{d}x} - \psi_3 e^{\left(-\frac{sx}{\vartheta}\right)} = \frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2}, \tag{21}$$

where

$$\psi_1 = s(1+t_0 \mathrm{s}),\ \psi_2 = sq(1+t_0 \mathrm{s}), \psi_3 = \frac{h_0 s(1+\tau_0 \mathrm{s})}{\vartheta}.$$

The equations (20) and (21) may be rewritten as follows:

$$\left(\beta_1 \frac{\mathrm{d}^2}{\mathrm{d}x^2} - s^2\right)\bar{e} = b\frac{\mathrm{d}^2\bar{\theta}}{\mathrm{d}x^2}\ , \tag{22}$$

$$\psi_2 \bar{e} = \left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \psi_1\right)\bar{\theta} + \psi_3 e^{\left(-\frac{sx}{\vartheta}\right)}. \tag{23}$$

Eliminating $\bar{\theta}$ from Eqs. (22) and (23), one gets

$$\left(\beta_1 \frac{\mathrm{d}^4}{\mathrm{d}x^4} - \left(\psi_1\beta_1 + s^2(1+\psi_2 b)\right)\frac{\mathrm{d}^2}{\mathrm{d}x^2} + s^2\psi_1\right)\bar{e} = -\frac{\psi_3 b s^2}{\vartheta} e^{\left(-\frac{sx}{\vartheta}\right)}\ . \tag{24}$$

Which can be expressed as:

$$(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\bar{\phi}\ = -\alpha_1 e^{\left(-\frac{sx}{\vartheta}\right)}\ , \tag{25}$$

where $\alpha_1 = \frac{\psi_3 b s^2}{\vartheta \beta_1}$ and the coefficients k_1^2 and k_2^2 satisfy the following equation

$$k^4 - \frac{(\psi_1\beta_1 + s^2 + s^2\psi_2 b)}{\beta_1}k^2 + \frac{s^2\psi_1}{\beta_1} = 0\ . \tag{26}$$

The comprehensive solutions of equation (25) under regularity constraints (16) can be expressed as:

$$\bar{e}(x) = A_1 \mathrm{e}^{-k_1 x} + A_2 \mathrm{e}^{-k_2 x} - A_3 e^{\left(-\frac{sx}{\vartheta}\right)}, \tag{27}$$

where $A_3 = \frac{\alpha_1}{\left(\frac{s}{\vartheta}\right)^4 - \frac{(\psi_1\beta_1+s^2+s^2\psi_2 b)}{\beta_1}\left(\frac{s}{\vartheta}\right)^2 + \frac{s^2\psi_1}{\beta_1}}$.

From Eqs. (22) and (27) we have:

$$\bar{\theta}(x) = \left(\frac{\beta_1 k_1^2 - s^2}{b s^2 k_1^2}\right)A_1 \mathrm{e}^{-k_1 x} + \left(\frac{\beta_1 k_2^2 - s^2}{b s^2 k_2^2}\right)A_2 \mathrm{e}^{-k_2 x} - \left(\frac{\beta_1\left(\frac{s}{\vartheta}\right)^2 - s^2}{b s^2\left(\frac{s}{\vartheta}\right)^2}\right)A_3 e^{\left(-\frac{sx}{\vartheta}\right)}\ . \tag{28}$$

Integrating Eq. (27) with respect to x, we obtain:

$$\bar{u}(x) = -\frac{1}{k_1}A_1 \mathrm{e}^{-k_1 x} - \frac{1}{k_2}A_2 \mathrm{e}^{-k_2 x} + \frac{\vartheta}{s}A_3 e^{\left(-\frac{sx}{\vartheta}\right)}\ . \tag{29}$$

We can extract the transformed thermal stress $\bar{\sigma}_{xx}$ by substituting equations (28) and (29) into (19) as:

$$\bar{\sigma}_{xx}(x) = \beta_1 A_1 \mathrm{e}^{-k_1 x} + \beta_1 A_2 \mathrm{e}^{-k_2 x} - \beta_1 A_3 e^{\left(-\frac{sx}{\vartheta}\right)} - \left(\frac{\beta_1 k_1^2 - s^2}{s^2 k_1^2}\right)A_1 \mathrm{e}^{-k_1 x} - \left(\frac{\beta_1 k_2^2 - s^2}{s^2 k_2^2}\right)A_2 \mathrm{e}^{-k_2 x} + \left(\frac{\beta_1\left(\frac{s}{\vartheta}\right)^2 - s^2}{s^2\left(\frac{s}{\vartheta}\right)^2}\right)A_3 e^{\left(-\frac{sx}{\vartheta}\right)}. \tag{30}$$

5. Applications

To calculate the integration constants A_1 and A_2 it will be taken into account that the free surface of the infinite media ($x = 0$) is constrained (fixed) and subjected to a thermal shock. The boundary conditions can be expressed mathematically as:

$$\theta(0,t) = \theta_0 H(t), \quad u(0,t) = 0, \tag{31}$$

where $H(t)$ denotes the unit step function.

As shown below, in the Laplace transform field, the boundary conditions can be transformed as follows:

$$\bar{\theta}(x,s) = \frac{\theta_0}{s}, \qquad at \quad x = 0, \tag{32}$$

As shown below, to obtain a system of linear equations with the constants A_1 and A_2, the boundary conditions (32) must be applied

$$\left(\frac{\beta_1 k_1^2 - s^2}{bs^2 k_1^2}\right) A_1 + \left(\frac{\beta_1 k_2^2 - s^2}{bs^2 k_2^2}\right) A_2 = \left(\frac{\beta_1 \left(\frac{s}{\vartheta}\right)^2 - s^2}{bs^2 \left(\frac{s}{\vartheta}\right)^2}\right) A_3 + \frac{\theta_0}{s}, \tag{33}$$

$$\frac{1}{k_1} A_1 + \frac{1}{k_2} A_2 = \frac{\vartheta}{s} A_3 \tag{34}$$

It is possible to find the values of the integration constants A_1 and A_2 by solving the previous system of algebraic equations.

6. Laplace transform inversion

This section will calculate the numerical values of physical fields in the time domain using the Riemann sum approximation approach because it is difficult to obtain direct inverse transformations of these fields. There is no fool proof strategy that can be relied upon in every situation. In this article, inverse Laplace transforms will be calculated using one of the most accurate approximation methods among others in the field of thermoelasticity. In this approach, the inverse of any function in the Laplace domain, denoted by $\bar{R}(x,s)$, can be numerically computed in the time domain, denoted by $R(x,t)$, by using the tried-and-true approximation algorithm [40]:

$$R(x,t) = \frac{e^{ct}}{t}\left(\frac{1}{2}\bar{R}(x,c) + Re \sum_{n=1}^{w} \bar{R}\left(x, c + \frac{in\pi}{t}\right)(-1)^n\right) \tag{35}$$

where w is a finite number that cannot exceed infinity. For numerical computations, this extension is straightforward and easy to program. Several computational investigations showed that the c value satisfies the relation $ct \cong 4.7$ [8], which is necessary for faster convergence.

7. Numerical results

Some analysis of the numerical results of the various physical fields estimated in the preceding section will be provided to confirm and explain the correctness of the analytical conclusions suggested in this study. For this goal, numerical simulations and discussions of two potential scenarios were carried out. Magnesium crystal is the material to be used for numerical evaluations and calculations. For this material, the various physical parameters will be as follows [41]:

$$K = 1.7 \times 10^2 \text{W m}^{-1}\text{K}^{-1}, \qquad \alpha_t = 1.78 \times 10^{-5}\ \text{K}^{-1}, \qquad T_0 = 298\text{K}$$
$$\rho = 1740\text{Kgm}^{-3}, C_E = 1040\text{Jkg}^{-1}\text{K}^{-1},\ \gamma = 2.68 \times 10^6 \text{Nm}^{-2}\text{K}^{-1},$$
$$\mu = 3.278 \times 10^{10}\text{Nm}^{-1},\ \lambda = 2.17 \times 10^{10}\text{Nm}^{-1}.$$

7.1 The influence of the speed of the source of heat

Several types of engineering, such as laser processing of materials, spot welding, and milling, can learn a lot from studying thermoelasticity problems that involve the transfer of heat sources. Because the heat source is moving, temperature, deformation, and stress all change in very big ways. The construction of the current study makes use of both the time-dependent basic solution of the heat transfer problem as well as the theoretical solutions to the thermoelastic and thermodynamic problems, respectively. In this part, we'll look at how heat sources with a moving point affect their surroundings.

We will investigate what happens to the non-dimensional displacement and the change in dynamic temperature and thermal pressure when the velocities ϑ of the moving heat source differ (see Figures 2-4). Figures 2-4 show that when the velocity parameters ϑ of a heat source change, all field variables change in a completely different way in magnitude with similar behavior.

The following is what we see:

- There is a substantial relationship between the velocity of the heat source ϑ and the distributions of the studied fields.
- In comparison to the velocity of the heat source, the rate of stress σ_{xx} increase is quite sluggish.
- The magnitude of the displacement u increases as a function of the increasing axial distance x until it reaches its maximum value. It gradually starts decreasing until it disappears in the middle.
- With a rising value of heat source speed and a fixed value of distance x, the displacement u decreases, and these fluctuations are pretty visible.
- When the medium is stable, the displacement values at the surface $x = 0$ remain constant, matching the given boundary conditions. It is clear from Figure 3 that the displacement component follows the same pattern for all different magnitudes of velocity.
- From Figure 2, it can be seen that the temperature decreases when the speed of heat source transfer increases, which is the opposite of what can be expected. The amount of thermal energy the heat source can output in a certain time period is fixed. The strength of the emitted energy per unit length, however, diminishes with increasing source velocity.
- The thermodynamic temperature has an oscillating pattern concerning distance, but the amplitude of the oscillation decreases with increasing distance from the heat source, although it has the same pattern consistent with [25].
- When a moving heat source is used, the medium undergoes thermal expansion displacement (see Figure 3). With more time having passed, a larger heat disturbed zone has developed, resulting in thermal expansion deformation evolving transversely through the medium.
- Figure 4 further shows that the nondimensional thermal stress σ_{xx} amplitudes drop with increasing heat source velocity due to a decrease in thermal energy strength per unit length.
- It is also observed that the oscillations are identical in behavior when the moving heat source speed increases; the values of peak temperature and displacement and the absolute values of pressures decreased in some periods within the medium. These values converge farther from the surface of the infinite mean. When there is a greater rise in the velocity of the heat source, there is a corresponding increase in the size of the effect field, which is also consistent with [42].

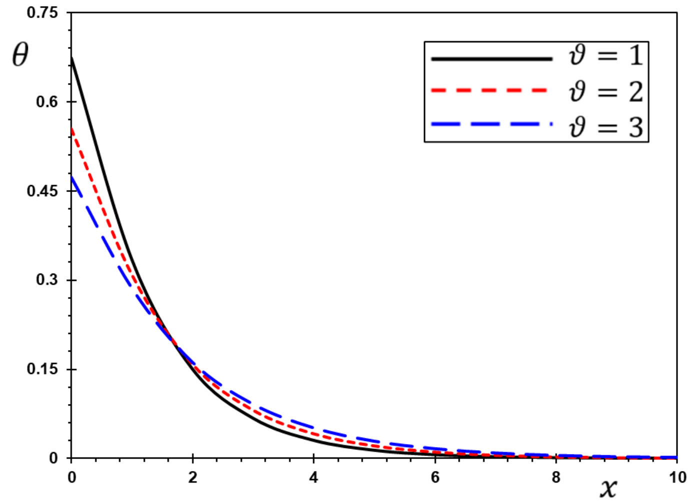

Figure 2: The change of temperature θ depending on the various velocities of the moving heat sources ϑ

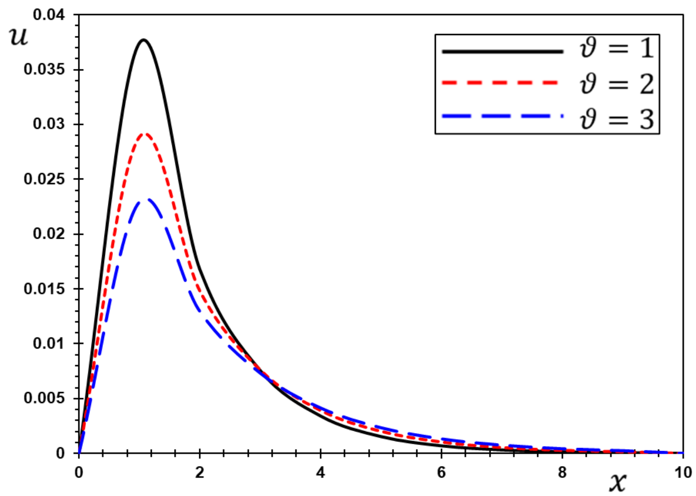

Figure 3: The change of displacement u depending on the various velocities of the moving heat sources ϑ

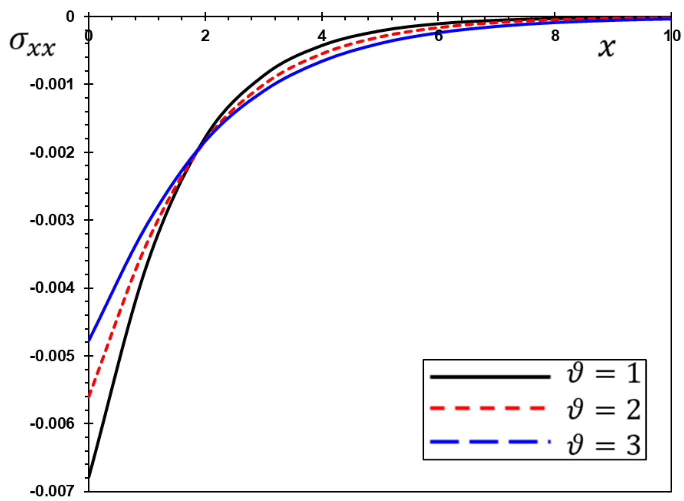

Figure 4: The change of the thermal stress σ_{xx} depending on the various velocities of the moving heat sources ϑ

7.2 The influence of the time instant

This scenario shows how the response of the studied field variables changes with a change in the instantaneous values of time t. We will assume that the moving heat source maintains a constant velocity of $\vartheta = 1.2$ plus the relaxation time $\tau_0 = 0.1$. For the purpose of data comparison, the changes in temperature, vertical displacement, and thermal pressure of the half medium are shown in Figures 5-7. It was found from the numerical results and figures that the time coefficient had a clear effect on each field of research. It is also noticed that the values of all studied areas increase along with the increase in the value of t. It is also seen that the rate at which waves travel through space slows down more rapidly as the value of t increases. The time coefficient t significantly influences each of the issues considered. As the value of t increases, all field values studied experience an increase, but the rate at which waves travel through space decreases faster.

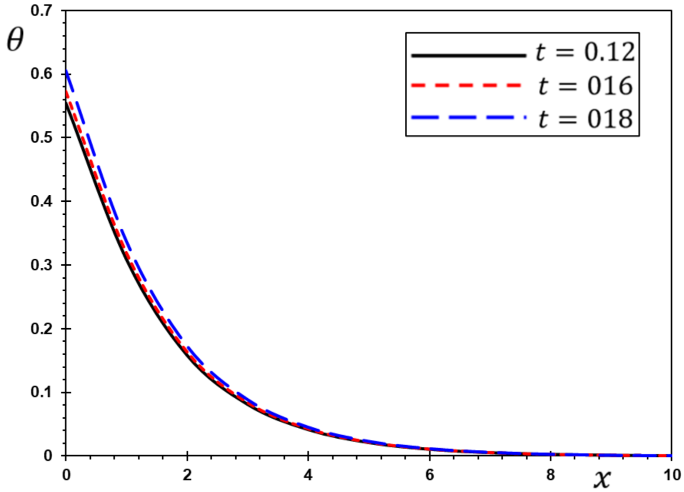

Figure 5: The temperature θ with different time instant t.

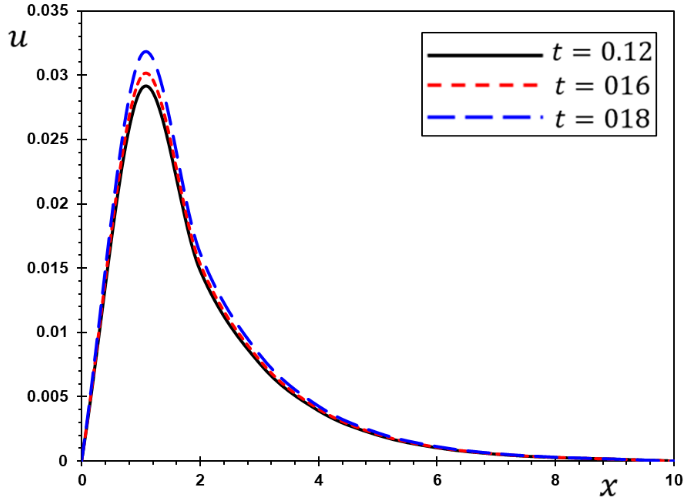

Figure 6: The displacement u with different time instant t.

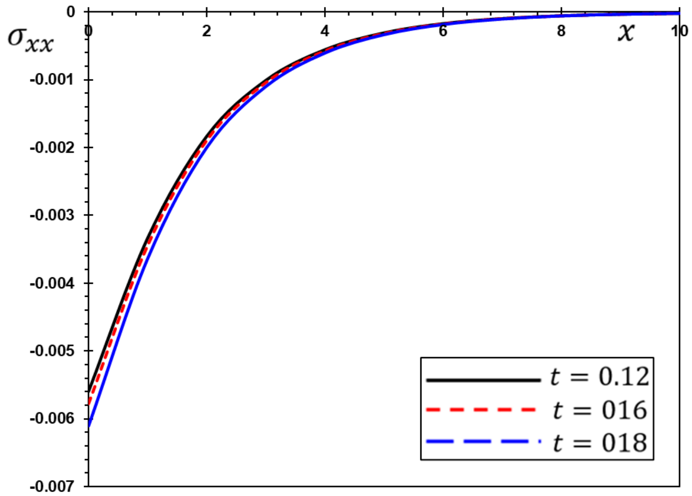

Figure 7: The stress σ_{xx} with different time instant t.

8. Conclusions

This article introduces the Lord and Shulman heat transfer model, a type of generalized thermoelasticity theory. This proposed model, which is based on the theory of generalized thermoelasticity, was applied to investigate the structural response to a variable heat source in an isotropic and homogeneous half-space. The Laplace transform method was used to obtain the solutions of different physical fields and an accurate and suitable numerical technique to produce the inverse Laplace transform. The various effects that a moving heat source can have on transient reactions and the effects that time can have on these reactions have been studied.
According to the results obtained, the following conclusions can be summarized:

- Each distribution of the studied fields is profoundly affected by the thermoelastic theory with a single relaxation factor when there are active heat sources in motion.
- The phenomenon of finite propagation velocities in all shapes and fields is shown, suggesting that the presented model is accurate and consistent with physical phenomena. This is in contrast to some traditional theories, which state that heat waves are expected to travel at infinite speeds.
- When the velocity of the moving heat source increased, the values of peak temperature and displacement decreased, and the absolute values of the thermal stress decreased.
- The results converge within the mean at a distance beyond the boundary of the infinite mean. The speed of the heat source is directly linked to how much the field of influence grows.
- The values of all studied fields rise with increasing time, although the speed with which waves propagate through space decreases more rapidly. This is due to the thermal relaxation time.

Nomenclature:

λ, μ	Lam´e's constants	K	thermal conductivity
α_e	thermal expansion coefficient	ρ	material density
C_S	specific heat	$\vec{Q}$	heat flux vector
$\Omega = (3\lambda + 2\mu)\alpha_t$	thermal coupling parameter	τ	time
T_0	environmental temperature	H	heat source
$\Theta = T - T_0$	temperature increment	S_{kl}	stress tensor
T	absolute temperature	t_0	thermal relaxation time
$\vec{U}$	displacement vector	e_{kl}	strain tensor
$e = \operatorname{div} \vec{U}$	cubical dilatation	δ_{ij}	Kronecker's delta function
F_k	external body force		

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