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RESEARCH PAPER

Plane waves in micropolar fibre-reinforced solid and liquid interface for non-insulated boundary under magneto-thermo-elasticity

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Abstract

This work is centered on propagation, reflection and transmission of waves in a micropolar fibre reinforced thermo-elastic solid and inviscid liquid interface in the presence of magnetic fields. Green and Lindsay thermo-elastic theory is utilized for non-insulated boundary of the solid media. P-wave incident at joint surface of the micropolar fibre reinforced thermo-elastic solid-liquid media in the presence of magnetic field produces four coupled reflected waves; quasi-longitudinal displacement (qLD), quasi-transverse displacement (qTD), quasi-transverse microrotational (qTM) and quasi-thermal (qT) wave, and two waves transmitted through the inviscid liquid medium; quasi-Longitudinal transmitted (qLT) and quasi-thermal transmitted (qTT) waves. Harmonic solution method is employed in conjunction with Snell's laws cum Maxwell's equation governing electromagnetic fields in the formulations and determination of solution to the micropolar fibre-reinforced solid/liquid modeled problem. Reflection and Transmission coefficients which correspond to reflected waves are presented analytically and graphically via numerical computations for a particular chosen material using Mathematica Software. Magnetic and thermal relaxation times field parameters have varied degree of effects to the propagation, reflection and transmission of waves in the media as observed. The study would be helpful in understanding the behavior of propagation, reflection and transmission of waves in micropolar fibre-reinforecd magneto- thermo-elastic-acoustic machination fields in solid/liquid interface and future works on the behavior of seismic waves, resulting in fluid interaction especially in geotechnical, physics, amongst others.

Keywords: Micropolar fibre-reinforced, liquid, amplitude ratios, thermal effects, P-wave, magnetic fields;

1. Introduction

Compressional wave in nature which could as well be described as P-wave (primary waves), travel faster than any other type of waves and especially it is twice the speed of secondary waves (SV-waves) via any type of material. Propagation of waves through materials for instance; composite materials e.g., fibre-reinforced composites

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and its deformation and analysis of stress is an interesting area of research in continuum mechanics let alone when there are fluid interactions. The strength and weightlessness occasioned by fibre reinforced materials made it possible for due recommendation to the fields of engineering, Science and Technology, composite materials in aeronautics, astronautics, earthquake sciences and so on, since in principle there exist stresses in solid and fluids to which analysis need to be ascertained and especially when they interfaced. Moreover, some materials possess pores Cowin and Nunziato [1, 2], Puri and Cowin [3] which constitutes characteristics that influences the chemical reactivity of solids and in turn its mechanical reactivity. This made it possible for researchers to sought ways to improve on certain materials by proposing new models such as the composite material models, to get rid of the anomaly encountered. And subsequently upon this, first grade micro-continuum which is a material property also due to deformation in terms of translational and rotational forms, is a major potential concept that characterizes the behavior of materials with complex structures in wave propagation, reflection and transmission. It consists of micropolar, microstretch, and micromorphic theories Eringen [4]. Thus, it's of the view that most large bodies such as moon, planets and the earth possess angular velocities as proposed by Schoenberg and Censor [5] however, different from micro-continuum properties.

Nevertheless, magnetic fields are vectors which give description of magnetic influences of both magnetized and current electricity conducting material. These are encountered in daily activities as in the case of permanent magnets. On a different note, thermal wave starts by assuming linear proportionality of heat flux and temperature gradient which was deduced from Fourier's law of heat conduction. Hence, to examine and explain the mechanical composition of materials with all these physical phenomena, continuum models are capitalized on. Consequently, Lord and Shulman [6] postulated a generalized thermo-elasticity theory by inclusion of thermal relaxation times and heat flux terms in Fourier's law of heat conduction. Lindsay and Green [7], furthered the work by incorporating two time relaxation constants to the model while Green and Naghdi [8-10] gave three forms of relations that provides wider treatment to heat flux models.

In spite of this, several authors McCarthy and Eringen [11], Kumar, Gogna and Debnath [12], Biswas, Sengupta, and Debnath [13], Kumar and Singh [14], have contributed to the study of micropolar elasticity theory in the literatures especially with effects of stretch, viscoelasticity, amplitude ratios of reflections of waves as the case may be. Singh [15] opined the reflection and transmission between micro-polar viscoelastic solid and liquid interfaces having stretch. Baljeet [16] accounted for the reflection of elastic plane surface waves with impedance boundary. Sengupta and Nath [17], developed a study on surface waves in fibre-reinforced media. Chattopadhyay, Venkateswarlu, Saha [18], investigated reflection of quasi-SV waves and quasi-P using both free and rigid boundary conditions for a fibre-reinforced medium. Chaudhary, Kaushik and Tomar [19], gave account on reflection and transmission of plane waves for two self-reinforced media. Khan, Anya and Hajra [20] studied the effects of surface waves under the influence of gravity for non-homogeneous fibre-reforced media possessing voids. Tauchert [21], examined the thermal stresses in micropolar elastic solids. Chattopadhyay and Choudhury [22],studied and deduced results on reflection and transmission of waves under the magnetic effects for a self-reinforced medium. Kumar, Sharma and Garg [23], worked on reflection of plane waves in transversely isotropic micropolar visco-thermo-elastic media. Also, Abd-Alla, Abo-Dahab, Aftab [24], made contributions on the behavior of magneto-thermoelastic surface waves in a rotating fiber reinforced viscoelastic media of higher order. Parveen Lata [25], studied reflection and refraction of plane waves in a layered nonlocal elastic and anisotropic thermoelastic medium. Eigenvalue approach was utilized by Sinha and Bera [26], to solve the problem regarding generalized infinite rotating medium of thermo-elasticity with one relaxation parameter by introducing heat source. Sunita, Suresh and Kapil [27], recently examined reflection at free surface of fibre-reinforced thermoelastic rotating medium with two- temperature and phase-lag. Roy and Acharya [28], examined the Propagation and reflection of plane waves in a rotating magneto-elastic fibre-reinforced semi space with surface stress. Singh and Sindhu [29], examined the Propagation of waves at interface between a liquid half-space and an orthotropic micropolar solid half-space. Also, Gupta [30], investigated on waves in micropolar transversely isotropic halfspace and inviscid liquid interface. Anya and Khan [20, 31-33] investigated on plane waves at free surfaces of a rotating micropolar fibre-reinforced medium with voids and also on magneto-thermo-elastic waves under GL theory: a case of reflection and propagation. The authors Asemi et al [34-37], carried out investigations on nanomaterials and nonlinear vibration analysis of piezoelectric nanoelectromechanical resonators and also in their research involved incorporated nonlocal effects. Baghani et al [38], Farajpour et al [39-46] developed models to study the dynamic and stability analysis of the rotating nanobeam in a nonuniform magnetic field considering the surface energy, buckling analysis of variable thickness of materials, large amplitude vibration of magneto-electro-elastic nanoplates, and higher-order nonlocal strain gradient plate model for buckling of orthotropic nanoplates with thermal effects, piezoelectric nanofilm-based electromechanical sensors, as the case maybe. Other authors like Ghayour et al [47], dealt with propagation of wave and its formulation in a fluid filled submerged visco-elastic finite cylindrical shells. Goodarzi et al [48] made investigation of the effect of pre-stressed on vibration frequency of rectangular nanoplate based on a visco-Pasternak foundation. Also in a similar vein, some others authors Mohammadi et al [34-68], modeled a problem in thermo-mechanical vibration analysis of annular and circular graphene sheet embedded in an elastic medium, Levy type solution for nonlocal thermo-mechanical vibration of orthotropic mono-layer material, nonlinear vibration analysis of the viscoelastic composite nanoplate with three directionally imperfect porous FG core, and Hygro-mechanical vibration analysis of a rotating viscoelastic nanobeam embedded in a visco-Pasternak elastic medium and in a nonlinear thermal environment. Moosavi et al [66] and Safarabadi et al [54] respectivel studied Vibration analysis of nanorings using nonlocal continuum mechanics and shear deformable ring theory and Effect of surface energy on the vibration analysis of rotating nanobeam while Danesh et al [64] gave account of axial vibration analysis of a tapered nanorod based on nonlocal elasticity theory and differential quadrature method.

Be that as it may, the current examination is aimed to account for the propagation, reflection and transmission of magneto-thermo elastic plane waves at joint surfaces of micropolar fibre-reinforced solid and inviscid liquid interfaces under G-L theory when the boundary is not insulated. The formulations were made along the x_1x_2 -plane and the equations of motions analytically derived. By using appropriate boundary conditions at the interface the amplitude ratios were achieved. Four reflected waves exists for incident P-wave at the interface of the solid material; quasi-longitudinal displacement (qLD), quasi-transverse displacement (qTD) or quasi-transverse microrotational (qTM), and quasi-thermal waves while two waves exists in the liquid medium owing to no insulation of the solid medium; quasi-longitudinal transmitted (qLT) and qausi-thermal transmitted (qTT) waves. In addition, Mathematica Software aided in our numerical computations of results. These results i.e., the amplitude ratios are equally shown graphically by considering variations in the physical parameters to ascertain their effects to the modelled system. Some particular results could also be deduced in the absence of thermal effects, and magnetic fields, yielding the results of micropolar fibre-reinforced medium and liquid interface.

2. Formulation of the problem

The constitutive relations for a micro-polar heat conducting fibre-reinforced linearly elastic anisotropic medium with reinforcement direction '$\vec{a}$' is given by:

$$\sigma_{ij} = B_{ijmn}E_{mn} + P_{ijmn}\vartheta_{mn} - \beta_{ij}\left(1+\nu_o\frac{\partial}{\partial t}\right)(T-T_o) \tag{1}$$

$$m_{ij} = B_{jimn}\hat{E}_{mn} + P_{mnji}\vartheta_{mn} \tag{2}$$

The deformations and wryness tensors are taken as:

$$E_{ij} = u_{\mathrm{j,i}} + \varepsilon_{jim}\phi^*_m, \quad \vartheta_{mn} = \phi^*_{m,n}, \tag{3}$$

and the balance laws in the presence of external applied magnetic field F_j under G-L theory are given below:

$$\sigma_{ij,i} + F_j = \rho\ddot{u}_j \tag{4}$$

$$m_{ij,i} + \varepsilon_{jmn}\sigma_{mn} = \rho J\ddot{\phi}^*_j \tag{5}$$

$$\frac{\partial}{\partial x_i}(\kappa_{ij}\frac{\partial T}{\partial x_j}) = \rho c_v(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2})T + T_o\beta_{ij}(\frac{\partial}{\partial t})E_{ij} \tag{6}$$

The thermal constants ν_o and τ_o, as stated in the above equations are termed thermal relaxation times equations and they satisfy the inequalities $\nu_o \geq \tau_0 \geq 0$. if $\tau_o > 0$, consequently $\nu_o > 0$, the Eq. (6) predicts a finite speed of propagation of thermal signals and if $\nu_o = \tau_0 = 0$, the Eqs. (1) and (6) reduce to the coupled theory. The presumption that $\tau_o = 0$ and $\nu_o > 0$ is also tenable; in this case the equation of motion continues to be affected by the temperature rate, while Eq. (6) predicts an infinite speed for the propagation of heat. In Eq. (1), we have made use of the condition $|T - T_o| << T_0$ to replace $T - T_o$ by T in the last term of Eq. (1). K_{ij} represents conductivity tensor, c_v is the specific heat at constant deformation, β_{ij} denotes the thermal moduli, ϕ^*_j, u_j σ_{ij}, m_{ij}, are the microrotation vector, displacement vector, stress tensor, and couple stress tensor respectively; ρ is the bulk mass density, J is the microinertia; B_{ijmn}, B_{jimn} are characteristics constants of the material and also non symmetric properties of B_{ijmn}, B_{jimn} and Q_{ijmn} holds. For simplicity we chose $\vec{a}=(a_1,a_2,a_3)$ such that $a_i=\delta_{i3}$ entails fibre direction such that δ_{ij} connotes Kronecker-Delta function, ε_{jim} is the Levi-Civita tensor. Index after comma represents partial derivative with respect to coordinate and superscript dot specifies partial derivative with respect to time. Consider the deformation in x_1x_2-Plane and $\phi^*=(0,0,\phi_3^*)$. $F_i=\mu_0\varepsilon_{ijk}J_jH_k$. Linearized Maxwell equations in tensor form governing the electromagnetic field for a perfectly conducting medium as: $\varepsilon_{ijk}H_{k,j}=\varepsilon_0\varepsilon_{ijk}J_j\dot{E}_k$, $\varepsilon_{ijk}E_{k,j}=-\mu_0\dot{H}_i$, $H_{i,i}=0$, $E_{i,i}=0$, $E_i=\mu_0\varepsilon_{ijk}\dot{u}_jH_k$, are considered. $H_i=H_0\delta_{i3}+h_i$, h_i is induced magnetic field and ε_o is electric permeability and the material lies in x_1x_2-plane. Thus the magnetic force F_i is given as $F_i=\mu_0H_0^2\left(e_{,i}-\varepsilon_0\mu_0\ddot{u}_i\right)$, $F_3=0$ and $h_i(x_1,x_2,x_3)=-u_{k,k}\delta_{i3}$, and $e=u_{1.1}+u_{2,2}$. In these equations, F_i represents magnetic force, J_i is current density, H_i is magnetic vector field and μ_o is magnetic permeability. In view of the fact that the tensors are not symmetric in micropolar, Eqs.(4)- (6) in component form take the forms:

$$B_1u_{1,11}+(B_2+B_3)u_{2,12}+B_4u_{1,22}+B_1^*\phi_{3,2}^*-\beta_1(1+\nu_o\frac{\partial}{\partial t})T_{,1}=(\varepsilon_0\mu_0^2H_0^2\ddot{u}_1+\rho\ddot{u}_1) \tag{7}$$

$$B_5u_{2,11}+B_2u_{1,12}+B_6u_{2,22}-B_5\phi_{3,1}^*-\beta_1(1+\nu_o\frac{\partial}{\partial t})T_{,2}=(\varepsilon_0\mu_0^2H_0^2\ddot{u}_2+\rho\ddot{u}_2), \tag{8}$$

$$B_5^*\phi_{3,11}^*+B_4^*\phi_{3,22}^*-2B_4\phi_3^*+B_4(u_{2,1}-u_{1,2})=\rho J\ddot{\phi}_3^*, \tag{9}$$

$$K_1T_{,ii}=\rho c_v\left(\dot{T}+\tau_o\ddot{T}\right)+T_o\beta_1\dot{u}_{i,i}, \tag{10}$$

where;

$$\left.\begin{aligned}&B_1=(\lambda+\beta+2\alpha-2\mu_T+4\mu_L+\mu_0H_0{}^2),\quad B_2=(\lambda+\alpha+\mu_0H_0{}^2),\quad B_3=2(\mu_L-\mu_T),\\&B_4=2\mu_T, B_5=2\mu_L,\ B_6=(\lambda+2\mu_T+\mu_0H_0{}^2),\ B_1^*=B_4-B_3,\end{aligned}\right\}, \tag{11}$$

$\mu_l-\mu_T$ α,β are fiber reinforced parameters. We employ the following dimensionless constants for convenience: $(x_1',x_2',u_1',u_2')=c_0\eta_0(x_1,x_2,u_1,u_2)$, $(t',\tau_0',\nu_0')=c_0^2\eta_0(\mathrm{t},\tau_0,\nu_0)$, $\sigma_{ij}'=\sigma_{ij}/\rho c_0^2$, $\phi_3^{*\prime}=B_5\phi_3^*/\rho c_0^2$, $m_{ij}'=m_{ij}/\rho c_0^2$, $T'=\beta_1\mathrm{T}/\rho c_0^2$, where $\eta_0=\rho C_v/K_1$, $c_0^2=B_1/\rho$, are for the solid medium and $\sigma_{ij}'=\sigma_{ij}/\rho c_0^2$, $(x_1',x_2',u_1',u_2')=c_{01}\eta_{01}(x_1,x_2,u_1,u_2)$, $(t',\tau_0',\nu_0')=c_{01}^2\eta_{01}(\mathrm{t},\tau_0,\nu_0)$, where $\eta_{01}=\rho_5C_v/K_1$, $c_{01}^2=\lambda_5/\rho_5$, are for the inviscid liquid medium. The dimensionless constants are introduced into Eqs. (7-10) and the upper sign " $'$ " dropped, gives;

$$u_{1,11}+B_{11}u_{2,12}+B_{13}u_{1,22}+B_{11}^*\phi_{3,2}^*-(1+\nu_o\frac{\partial}{\partial t})T_{,1}=(\eta\ddot{u}_1+\ddot{u}_1), \tag{12}$$

$$B_{14}u_{2,11}+B_{12}u_{1,12}+B_{15}u_{2,22}-\phi_{3,1}^*-(1+\nu_o\frac{\partial}{\partial t})T_{,2}=(\eta\ddot{u}_1+\ddot{u}_1), \tag{13}$$

$$B_{12}^*\phi_{3,11}^*+B_{13}^*\phi_{3,22}^*-B_7\phi_3^*+B_8(u_{2,1}-u_{1,2})=J\ddot{\phi}_3^*, \tag{14}$$

$$T_{,ii}=\left(\dot{T}+\tau_o\ddot{T}\right)+B_9\dot{u}_{i,i}\quad . \tag{15}$$

Here, $(B_{11},B_{12},B_{13},B_{14},B_{15},B_{11}^*,B_{12}^*,B_{13}^*)=((B_2+B_3),B_2,B_4,B_5,B_6,B_1^*,B_5^*,B_4^*)/B_1$, $B_7=2(B_4\mathrm{K}_1^2)/\rho B_1^3C_v^2$, $B_8=(B_4B_5\mathrm{K}_1^2)/\rho B_1^3C_v^2$, $B_9=(T_o\beta_1^2)/B_1\rho C_v$, and $\eta=(\varepsilon_0\mu_0^2H_0^2)/\rho$.

3. Normal mode analysis and solution of the problem

In this section, consider joint media of a homogeneous micropalar fibre-reinforced solid in the half-space $x_2 < 0$ and liquid medium occupying the half-space $x_2 > 0$. This is such that they are in contact at $x_2 = 0$. Let the normal mode analysis or harmonic solution approach be applicable such that the incident waves have the displacement chosen as;

$$(u_i, \phi_3^*, (T - T_o) = \theta) = (R, P, \theta_o) e^{i\{k(x_j p_j) - \omega t\}}, \quad i = j = 1, 2. \tag{16}$$

Where R, P, ϕ_0^*, and θ_0 are amplitudes of u_1, u_2, ϕ_3^* and θ respectively. ω, is the angular velocity or frequency of the wave, $c = \dfrac{\omega}{k}$ is the phase velocity of the wave, and k is the wave number. Making use of Eq. (16) into Eqs. (12-15) respectively, yields the non-dimensional equations below:

$$(k^2 D_1 - \{c^2(\eta + 1)\})R + \{k^2 B_{11} p_1 p_2\}P - i\{kiB_{11}^* p_2\}\phi_0^* + \{ik(1 - ickv_o)p_1\}\theta_o = 0, \tag{17}$$

$$\{k^2 B_{12} p_1 p_2\}R + \{k^2 D_2 - (c^2(\eta + 1))\}P + i\{ikp_1\}\phi_0^* + \{ik(1 - ickv_o)p_2\}\theta_o = 0, \tag{18}$$

$$\{ikB_8 p_2\}R - \{ikB_8 p_1\}P + \{k^2 D_3 + B_7 - Jk^2 c^2\}\phi_0^* = 0, \tag{19}$$

$$-(kB_9 ck\ p_1)R - (kB_9 ck\ p_2)P + \left\{(ick + \tau_o k^2 c^2) - k^2\right\}\theta_o = 0. \tag{20}$$

Here $D_1 = p_1^2 + B_{13} p_2^2$, $D_2 = B_{14} p_1^2 + B_{15} p_2^2$, $D_3 = B_{12}^* p_1^2 + B_{13}^* p_2^2$.

For non-trivial solution, Eqs.(3.6)-(3.8), becomes the quartic equation as follows:

$$d^4 + C_1 d^3 + C_2 d^2 + C_3 d + C_4 = 0. \tag{21}$$

Where $d = k^2$. This shows that the characteristic Eq. (21) with complex coefficients C_1, C_2, C_3, and C_4 (See appendix) yields four complex roots; detailing that four waves propagates, with complex phase velocities: c_1, c_2, c_3 and c_4 corresponding to the wave number $k_1, k_2, k_3,$ and k_4, in the solid medium respectively. This also entails that the two dimensional model of magneto-thermo-elastic micropolar fibre-reinforced solid half space under G-L theory for a non-insulated boundary have four waves; quasi-P wave, quasi-SV wave, quasi-transverse microrotational wave and thermal wave travelling in the solid medium. Following [29] for the liquid medium, consider $B_1 = B_2 = B_6 = \lambda_5$, $\rho = \rho_5$, and $B_4 = B_5 = B_1^* = B_4^* = B_5^* = 0$ into Eqs. (7-10) to obtain the non-dimensional equation for a non-trivial solution as:

$$E_{11} d_1^2 + E_{12} d_1 + E_{13} = 0 \tag{22}$$

Where $d_1 = k^{*2}$, and E_{11}, E_{12}, *and* E_{13} given in appendix, are complex coefficients of the characteristic equation in the inviscid liquid medium such that Eq. (3.10) possesses two complex phase velocities: c_5, and c_6 corresponding to the wave number k_5, and k_6. Thus, in the liquid medium quasi-longitudinal and quasi-thermal wave can propagate. Any one of four waves can be chosen as incident wave.

In Fig. 1, when quasi-P wave (A_0) is incident at the boundary $x_2 = 0$ of a rotating magneto-thermo-elastic micropolar fibre-reinforced non-insulated anisotropic solid at the boundary and liquid interface under G-L theory, there exist reflected waves as quasi-P (A_1) or qLD, quasi-SV (A_2) or qTD, quasi-TM (A_3) and quasi-thermal (A_4). Also the transmitted waves exists as; transmitted qLD (A_5) and transmitted thermal wave (A_6), See Fig.1

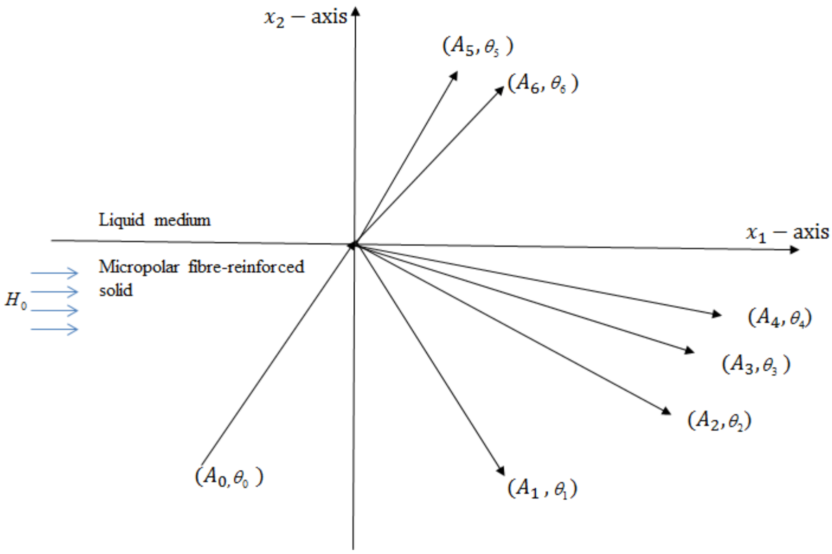

Fig 1: Schematic of the problem showing incidence, reflected and transmitted waves in the solid/liquid interface under magnetic effects.

3.1 Boundary conditions

$\sigma_{22}^{\alpha}+\bar{\sigma}_{22}^{\alpha}=\sigma_{22}^{l\,\prime}+\bar{\sigma}_{22}^{l\,\prime},\ \sigma_{21}^{\alpha}=0,\ \sigma_{12}^{\alpha}=0,\ \ m_{23}^{\alpha}=0, T_{,2}^{\alpha}=T_{,2}^{l}$ for non-insulated boundary and $u_2^{\alpha}=u_2^{l\prime}$ at $x_2=0$, α =0, 1 ,2, 3, 4 and l =5, 6, and the sign "$'$" indicates liquid medium. Where, Maxwell's stresses Abd- Alla et al [24] are as follows: $\bar{\sigma}_{ij}=\mu_0 H_0\left[H_i h_j+H_j h_i-H_k h_k\delta_{ij}\right], \Rightarrow \bar{\sigma}_{ij}=\mu_0 H_0\begin{bmatrix}-h_3 & 0 & h_1\\ 0 & -h_3 & h_2\\ h_1 & h_2 & h_3\end{bmatrix}$.

Here, $h_3=-H_0(u_{1,1}+u_{2,2})$, $h_1=h_2=0$, and $\sigma_{22}+\bar{\sigma}_{22}=0\Rightarrow \sigma_{22}+\mu_0 H_0^2(u_{1,1}+u_{2,2})=0$, $m_{23}=0\Rightarrow\phi_{3,2}^{*}=0$.

That is at $x_2=0$, normal force stresses, normal components of displacement vectors, and temperature gradients are continuous, while tangential stresses, tangential couple stresses vanishes. We choose the displacement components, micro-rotation vectors and temperature as:

$$\left.\begin{aligned}
&u_1^{\alpha}=A_{\alpha}d_1^{\alpha}e^{i\mu_{\alpha}}, u_1^{l}=A_l d_1^{l}e^{i\mu_l}, u_2^{l}=I^{l}A_l d_1^{l}e^{i\mu_l},\\
&u_2^{\alpha}=W^{\alpha}A_{\alpha}d_1^{\alpha}e^{i\mu_{\alpha}}, T^{l}-T_0^{l}=I^{*l}A_l d_1^{l}e^{i\mu_l},\\
&\phi_3^{*}=ik_{\alpha}G^{\alpha}A_{\alpha}d_1^{\alpha}e^{i\mu_{\alpha}}, T^{\alpha}-T_0^{\alpha}=X^{\alpha}A_{\alpha}d_1^{\alpha}e^{i\mu_{\alpha}},
\end{aligned}\right\}\tag{23}$$

Here, $\mu_{\alpha}=k_{\alpha}(x_1p_1^{\alpha}+x_2p_2^{\alpha}-c_{\alpha}t)$, $\alpha=0$ correspond to incident wave, $\alpha=1,2,3,4$ corresponds to reflected waves in the solid medium and $\mu_l=k_l(x_1p_1^{l}+x_2p_2^{l}-c_lt)$, where $l=5$ and 6 corresponds to transmitted waves in the inviscid liquid medium. Also, the coupled relations for the solid medium are obtained from Eqs. (17-20) i.e.

$$\begin{aligned}
W_1^{\alpha}&=\{p_1^{\alpha}(k_{\alpha}^2(D_1^{\alpha}-(c_{\alpha}^2(\eta+1)))+B_{11}^{*}p_2^{\alpha}(k_{\alpha}^2B_{12}p_1^{\alpha}p_2^{\alpha})\}\{ic_{\alpha}k_{\alpha}+k_{\alpha}^2(\tau_0c_{\alpha}^2-1)\}+\\
&\quad ik_{\alpha}(1-ic_{\alpha}k_{\alpha}v_0)(B_{11}^{*}p_2^{(\alpha)2}+p_1^{(\alpha)2})(B_9k_{\alpha}\omega p_1^{\alpha}),\\
W_2^{\alpha}&=\{B_{11}^{*}p_2^{\alpha}(k_{\alpha}^2D_2^{\alpha}-(c_{\alpha}^2(\eta+1)))+p_1^{\alpha}(k_{\alpha}^2B_{12}p_1^{\alpha}p_2^{\alpha})\}\{ic_{\alpha}k_{\alpha}+k_{\alpha}^2(\tau_0c_{\alpha}^2-1)\}+\\
&\quad ik_{\alpha}(1-ic_{\alpha}k_{\alpha}v_0)(B_{11}^{*}p_2^{(\alpha)2}+p_1^{(\alpha)2})(B_9k_{\alpha}c_{\alpha}k_{\alpha}p_2^{\alpha}),\\
W^{\alpha}&=-F_1^{\alpha}/F_2^{\alpha}, G^{\alpha}=(ik_{\alpha}B_8(p_1^{\alpha}F^{\alpha}-p_2^{\alpha}))/k_{\alpha}^2D_3^{\alpha}+B_7-Jk_{\alpha}^2c_{\alpha}^2,\\
X^{\alpha}&=(k_{\alpha}B_9(p_2^{\alpha}W^{\alpha}+p_1^{\alpha}))/ic_{\alpha}k_{\alpha}+k_{\alpha}^2(\tau_0c_{\alpha}^2-1).
\end{aligned}$$

Here $D_1^{\alpha} = p_1^{(\alpha)2} + B_{13} p_2^{(\alpha)2}$, $D_2^{\alpha} = B_{14} p_1^{(\alpha)2} + B_{15} p_2^{(\alpha)2}$, and $D_3^{\alpha} = B_{12}^* p_1^{(\alpha)2} + B_{13}^* p_2^{(\alpha)2}$, and similarly, the coupled equations for the inviscid liquid medium take the forms:

$$\mathrm{I}^l = \{p_2^l(k_\alpha^2 D_1^l - (c_l^2(\eta+1))) - p_1^l(k_\alpha^2 p_1^l p_2^l)\} / \{p_1^l(k_\alpha^2 D_2^l - (c_l^2(\eta+1))) - p_2^l(k_\alpha^2 p_1^l p_2^l)\}$$
$$I^{*l} = (k_l B_{10}(p_2^l I^l + p_1^l)) / i c_l k_l + k_l^2(\tau_0 c_l^2 - 1) \text{, where } D_1^l = p_1^{(l)2} \text{and } D_2^l = p_2^{(l)2}.$$

Using Eq. (23) into dimensionless boundary conditions, we obtain the system: $a_{ij} Z_j = b_i$. Where;

$$a_{1j} = (\{k_j\{(A_2' + (\mu_0 H_0^2 / B_1)) p_1^j + (A_6' + (\mu_0 H_0^2 / B_1)) W^j p_2^j\} + (i + c_j \nu_0 k_j) X^j\} d_1^j) /$$
$$\{k_0\{(A_2' + (\mu_0 H_0^2 / B_1)) p_1^{(0)} + (A_6' + (\mu_0 H_0^2 / B_1) W^{(0)} p_2^{(0)}\} + (i + c_0 \nu_0 k_0) H^{(0)}\} d_1^{(0)},$$
$$a_{1l} = -(\{k_l\{(1 + (\mu_0 H_0^2 / \lambda_5)) p_1^l + (1 + (\mu_0 H_0^2 / \lambda_5)) I^l p_2^j\} + (i + c_l \nu_0 k_l) I^{*l}\} d_1^l) /$$
$$\{k_0\{(A_2' + (\mu_0 H_0^2 / B_1)) p_1^{(0)} + (A_6' + (\mu_0 H_0^2 / B_1)) W^{(0)} p_2^{(0)}\} + (i + c_0 \nu_0 k_0) X^{(0)}\} d_1^{(0)},$$
$$a_{2j} = ((k_j\{\mathrm{W}^j A_3' p_1^j + B_{13} p_2^j + B_{11}^* G^j\} d_1^j)) / \{k_0\{\mathrm{W}^{(0)} A_3' p_1^{(0)} + B_{13} p_2^{(0)} + B_{11}^* G^{(0)}\} d_1^{(0)}, \quad a_{25} = 0,\ a_{26} = 0,$$
$$a_{3j} = (\{k_l\{\mathrm{W}^j B_{14} p_1^j - G^j\} d_1^j) / \{k_0\{\mathrm{W}^{(0)} B_{14} p_1^0 - G^{(0)}\} d_1^0, \quad a_{35} = 0,\ a_{36} = 0,$$
$$a_{4j} = (k_j^2 p_2^j G^j d_1^j) / k_0^2 p_2^0 G^0 d_1^0, \quad a_{45} = 0,\ a_{46} = 0, a_{5l} = -\{I^l d_1^l / \mathrm{W}^{(0)} d_1^{(0)}\}(B_1 \rho / \lambda_5 \rho_5)^{1/2}, a_{5j} = W^j d_1^j / W^{(0)} d_1^{(0)},$$
$$a_{6j} = k_j p_2^j X^j d_1^j / k_0 p_2^{(0)} X^{(0)} d_1^{(0)}, \quad a_{6l} = -k_l p_2^l I^{*l} d_1^l / k_0 p_2^{(0)} X^{(0)} d_1^{(0)} (\lambda_5^3 \rho_5 / B_1^3 \rho)^{1/2}, j = 1,2,3,4, l = 5,6,$$

$Z_i = |A_i / A_0|$, $b_i = -1$, $i = 1,2,3,4,5,6$. $\mathrm{A}_6' = B_6^* / B_1$, $\mathrm{A}_3' = B_3 / B_1$, $\mathrm{A}_2' = B_2^* / B_1$, $B_2^* = (\lambda + \alpha)$, $B_6^* = (\lambda + 2\mu_T)$. a_{ij} as the coefficients of the system are found by by using Snell's law:

$$k_0 p_1^{(0)} = k_1 p_1^{(1)} = k_2 p_1^{(2)} = k_3 p_1^{(3)} = k_4 p_1^{(4)} = k_5 p_1^{(5)} = k_6 p_1^{(6)} = k.$$
$$k_0 c_0 = k_1 c_1 = k_2 c_2 = k_3 c_3 = k_4 c_4 = k_5 c_5 = k_6 c_6 = \omega.$$

Notice that $k_0 = k_1$ and $c_0 = c_1$. Components of propagation and unit displacement vector are as follows:

$$p_1^{(0)} = Sin\theta,\ p_2^{(0)} = Cos\theta,\ d_1^{(0)} = Sin\theta, d_2^{(0)} = Cos\theta,\ p_1^{(1)} = Sin\theta_1, p_2^{(1)} = -Cos\theta_1,\ d_1^{(1)} = Sin\theta_1, d_2^{(1)} = -Cos\theta_1,$$
$$p_1^{(2)} = Sin\theta_2, p_2^{(2)} = -Cos\theta_1,\ d_1^{(2)} = Cos\theta_2, d_2^{(2)} = Sin\theta_2, p_1^{(3)} = Sin\theta_3, p_2^{(3)} = -Cos\theta_3,\ d_1^{(3)} = Cos\theta_3, d_2^{(3)} = Sin\theta_3,$$
$$p_1^{(4)} = Sin\theta_4, p_2^{(4)} = -Cos\theta_4,\ d_1^{(2)} = Cos\theta_4, d_2^{(4)} = Sin\theta_2, p_1^{(5)} = Sin\theta_5, p_2^{(5)} = Cos\theta_5,\ d_1^{(5)} = Sin\theta_5, d_2^{(5)} = Cos\theta_5,$$
$$p_1^{(6)} = Sin\theta_6, p_2^{(6)} = Cos\theta_6,\ d_1^{(6)} = Sin\theta_6, d_2^{(6)} = Cos\theta_6, \theta = \theta_1 \text{ (due to laws of reflection)}$$

4. Computational results and discussion

The effects of magnetism and thermal parameters on the reflection and transmission coefficients of plane waves in a micropolar fibre-reinforced material under Green and Lindsay theory of thermo-elasticity are studied by utilizing the numerical micropolar fiber-reinforced physical constants [27, 29, 30] and other parameters as:

$$\mu_T = 2.46 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}};\ \mu_L = 5.66 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}};$$
$$\lambda = 5.65 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}};\ \alpha = -1.28 \times 10^9\ \mathrm{kg\,m^{-1}s^{-2}};$$
$$\beta = 220.9 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}};\ \rho = 2660 kg\,\mathrm{m^{-3}};\ \omega = 2;$$
$$H_0 = 1000;\ B_4^* = 3.71 \times 10^9;\ B_5^* = 3.9 \times 10^9;\ \mathrm{J} = 0.000196 m^2;$$
$$T_0 = 2.93 K;\ \tau_0 = 0.15 s;\ \nu_0 = 0.40 s;\ C_v = 0.787 \times 10^3\ \mathrm{J}\, kg^{-1}\,\mathrm{deg}^{-1};$$
$$\rho_5 = 1000\ \mathrm{kg\,m^{-3}};\ K_1 = 0.0963 \times 10^3\ \mathrm{J\,m^{-1}s^{-1}\,deg^{-1}};\ \lambda_5 = 11.65 \times 10^{10}\ \mathrm{kg\,m^{-1}s^{-2}}$$

Using Snell's law, the angles of reflections are deduced and the velocities computed with the help of the wave numbers. Figs. (2-4) represent the graphical representation of the amplitude ratios. This is such that we considered variations between the reflection and transmission coefficients (RC/TC or Z_i) for an incident P-wave with angle "θ" for different magneto-thermo-elastic parameters H_0, ν_0 and τ_0. Ascertaining the effects of these parameters on modulation, reflection and transmission coefficients of waves under the G-L theory of thermo-elasticity for non-insulated boundary are then discussed below.

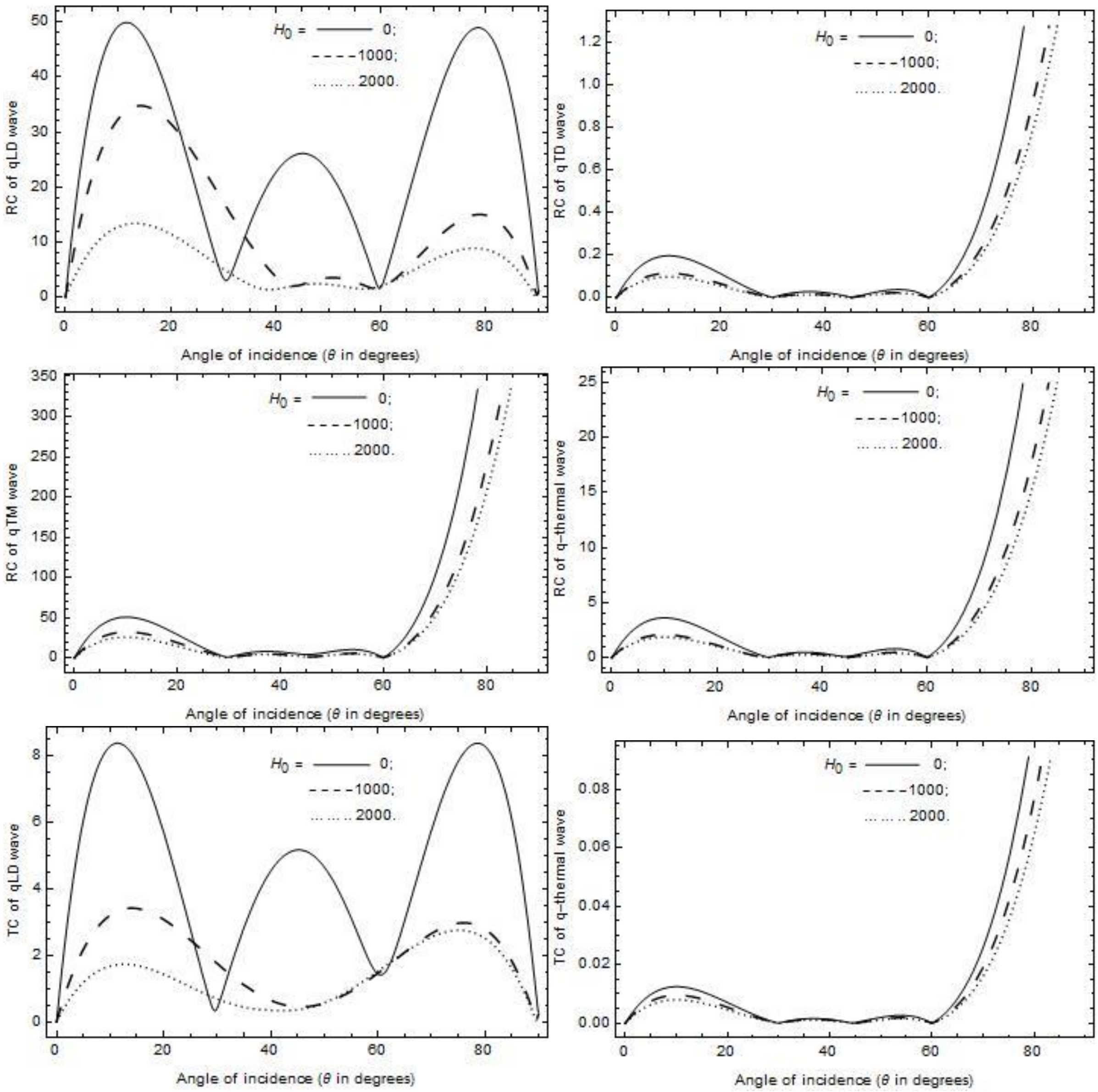

Fig 2: Variations of amplitude ratios Z_i of reflected and transmitted waves versus angle of incidence for distinct values of magnetic field parameter H_0.

Fig. 2 shows the variation of amplitude ratios (RC/TC) or Z_i, $i = 1, 2, 3, 4, 5$, 6 of qLD, qTD, qTM qT, qLT and qTT waves respectively versus incident angle with varying magnetic field parameter H_0 and constant thermal relaxation times. The amplitude ratios Z_i, $i = 1, 2, 3, 4, 5, 6$ are decreased for increased magnetic effects and increasing angles. This means that higher amplitude ratios are attained when the magnetic parameters are removed thus showing that the modulation of the waves are been influenced. Also for $\theta > 0^0$, i.e. as the angle increases, mixed behaviors in terms of modulation, decrease and increase in amplitudes are encountered between varying magnetic parameters in the media especially on qLD and qLT waves. Hence higher magnetic effects could lead to vanishing of the waves before grazing angles of incidence.

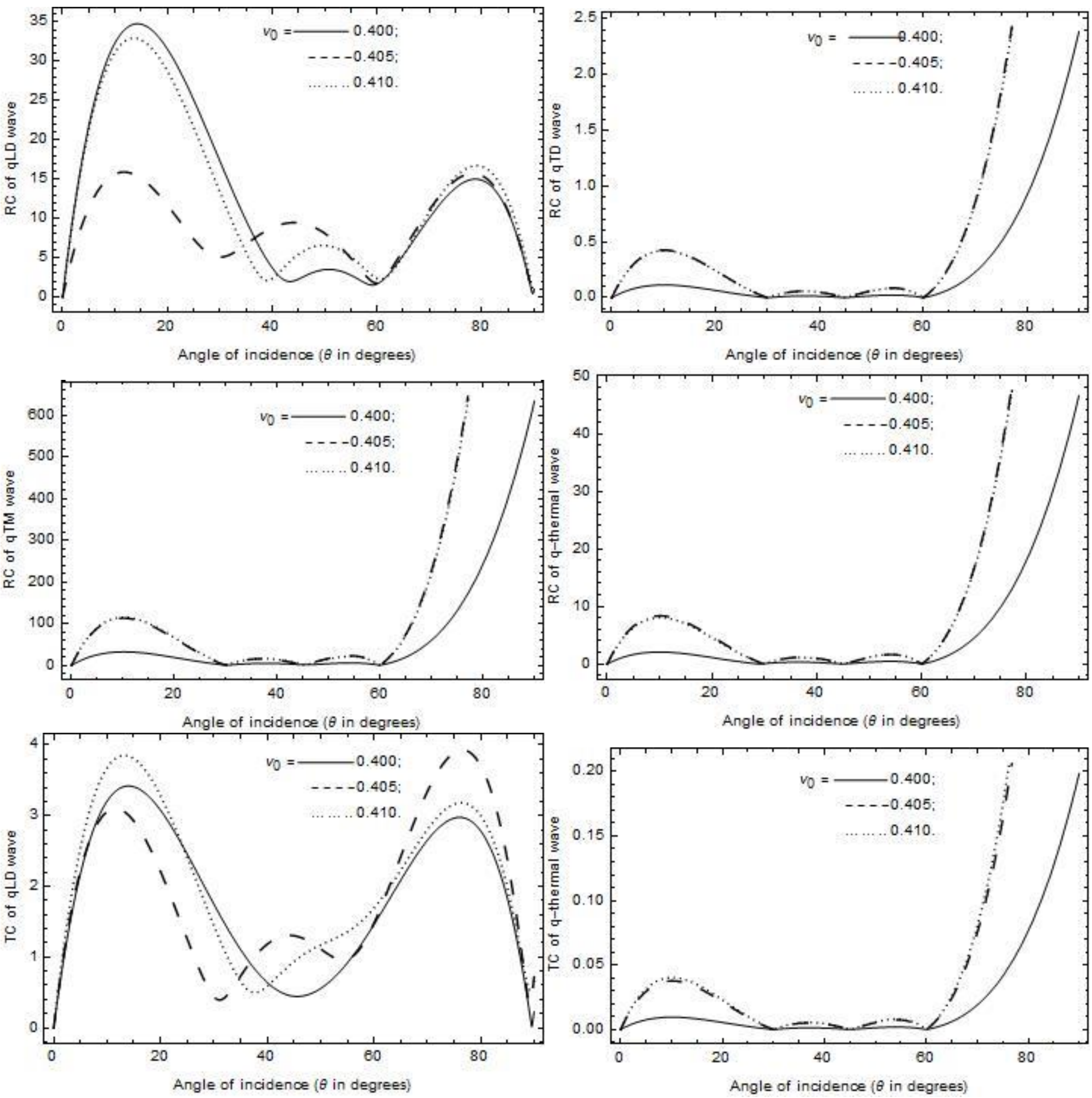

Fig 3: Variations of amplitude ratios Z_i of reflected and transmitted waves versus incidence angle for distinct values of thermal relaxation parameter ν_0 .

Furthermore, Fig. 3 represents the variation of amplitude ratios (RC/TC) or Z_i , $i = 1, 2, 3, 4, 5, 6$ of qLD, qTD, qTM qT, qLT and qTT waves respectively against incident angle with variations in thermal relaxation parameter ν_0 and constant magnetic field parameter H_0 . Hence, it's obvious that the amplitude ratios Z_i , $i = 2, 3, 4, 6$ are increased for increasing thermal relaxation parameter ν_0 and increasing angles. This means that higher amplitude ratios are attained in the presence of thermal relaxation parameter ν_0 and thus showing increase in modulation of the waves. Also, for a reduced thermal relaxation parameter ν_0 , the wave's modulations and amplitudes of Z_i , $i = 2, 3, 4, 6$ vanishes faster near grazing angle of incidence. Nevertheless, Z_i $i = 1, 5$ of the reflected quasi-longitudinal and quasi-longitudinal transmitted waves in both the solid and liquid materials respectively, depicts somewhat mixed behaviors of increase and decrease, as the angle increases for constant magnetic effects and τ_0 . Moreover, for $0^0 \le \theta \le 38.5^0$, Z_i $i = 1$, increases for reduced thermal relaxation parameter ν_0 and subsequently increases in amplitudes for $\theta > 38.5^0$, while Z_i $i=1$ and its modulations are clearly displayed in mixed effects of increment and decrement as the angle increases.

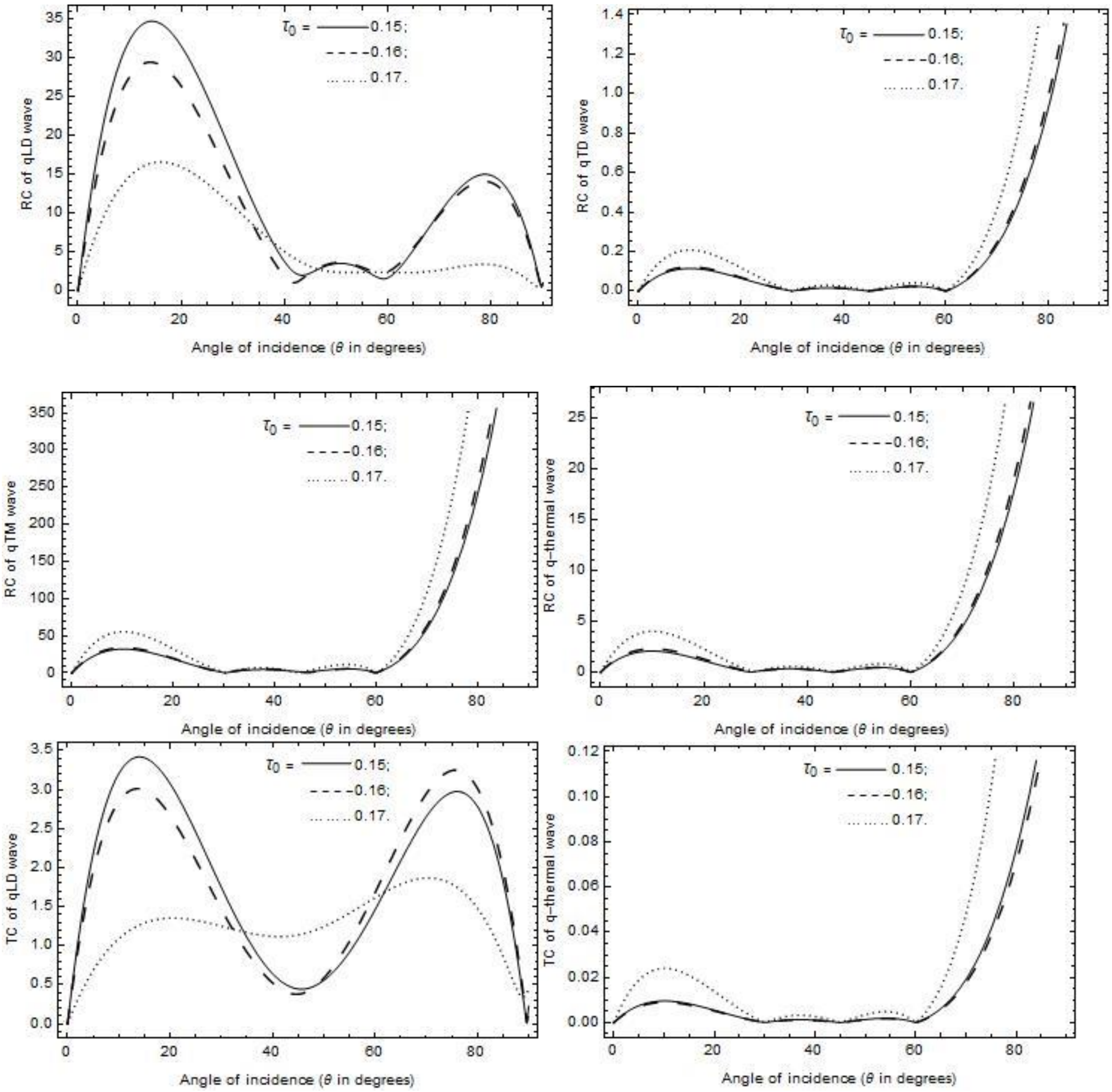

Fig 4: Variations of amplitude ratios Z_i of reflected and transmitted waves versus incidence angle for distinct values of thermal relaxation parameter τ_0

Furthermore, Fig. 4 represents the variation of amplitude ratios (RC/TC) or Z_i, $i=1,2,3,4,5,6$ of qLD, qTD, qTM, qT, qLT and qTT waves respectively against incident angle with varying thermal relaxation parameter τ_0 and constant magnetic field parameter H_0. Note that the modulation and amplitude ratio of Z_i, $i=1$, has a consistent decrease for increased thermal relaxation parameter τ_0 between $0^0 \le \theta \le 35^0$ and with mixed behaviours for $35^0 \le \theta \le 63^0$, as it subsequently returns in a consistent decrease for $\theta > 63^0$. Z_i, $i=2,3,4$ yielded a somewhat consistent increment for higher values of thermal relaxation parameter τ_0 and with vanishing characterization as the angle of incidence increases. We observed that the modulation and amplitude ratio of Z_i, $i=5$, also has a consistent decrease for increased thermal relaxation parameter τ_0 between the normal angle of incidence and $\theta \leq 35^0$, and with mixed behaviors occurring for $\theta \geq 35^0$ i.e., modulations and hence the amplitude ratio increases and decreases along these angles of incidence. In a similar manner, for $\theta > 0^0$, Z_i, $i=6$ showed mixed behaviors and with considerable increments for increased τ_0 and constant magnetic effects with the propensity to vanish as the angle increases before the grazing angle of incidence.

5. Conclusion

This investigation was centered on the propagation, reflection and transmission of magneto-thermo-elastic plane waves in micropolar fibre-reinforced solid and liquid interface using Green and Lindsay theory of thermo-elasticity and with no insulation at the interface of the resulting media. coupled waves; quasi-longitudinal displacement (qLD) wave, quasi-transverse displacement (qTD) wave, quasi-transverse microrotational (qTM) wave and quasi-thermal waves were observed traveling in the solid medium while two waves; quasi- Longitudinal transmitted qLT and quasi-thermal transmitted (qTT) waves are found to propagate in the liquid medium, owing to non-insulation of the interface. The physical characterization of the study hinges on thermal effects using G-L theory under the influence of magnetic fields. Thus, the combined effects of these physical characterizations have remarkable degrees of influences on the modulation of the waves in the media as well as its corresponding amplitudes as observed from the numerical simulated graphs. While magnetic field decreases the reflection and transmission coefficients of waves in the modeled problem; in both the solid and the liquid medium in generally, the reverse is the case for thermal relaxation times effects, notwithstanding their mixed behaviors encountered in both effects within some given range of values of incident angles as the waves propagates. We can deduce that reflection and transmission cannot occur for some incident angles in the media. Cases found in literatures are similar to this study if we neglect thermal or magnetic parameters, thus yielding micropolar fiber reinforced investigation.

Therefore, it is worth noting to state that this research work should be of great importance to researchers in new materials or designers in material sciences, new researchers in the field and experimental based examination involving modulation, reflection and transmission of magneto-thermo-elastic plane waves in micropolar fibre-reinforced solid and liquid interactions and also in mechanization fields similar to seismology or earthquake analysis.

Conflict of interest

The authors declare that there is no conflict of interest.

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