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RESEARCH PAPER

Parametric study of Sandwich Plates with Viscoelastic, Auxetic Viscoelastic and Orthotropic Viscoelastic Core Using a New Higher Order Global-Local Theory

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Abstract
In this paper, sandwich plates with flexible core and composite surfaces as well as viscoelastic and auxetic core are investigated under dynamic loading. A new higher order global-local theory is used for simulation of the dynamic behavior of sandwich plates. The presented theory is able to consider the thickness change of the sandwich plates as well as transverse stresses accurately. It is obvious that considering these parameters are so crucial in study behavior of thick sandwich plates or sandwich plates with soft core. Furthermore, in terms of solving equations, an iterative incremental method based on the formulation of transient nonlinear finite element as well as a real time algorithm was employed to simulate viscoelastic behavior accurately. The results indicate a significant increase in the stiffness of the sandwich plate due to the auxetic properties of the core materials, leading consequently to the reduction of the vibration amplitude and stresses level. Some of the innovations belonging to this paper are: 1) presenting a global-local higher-order theory while considering the changes in the thickness of the sandwich plates; 2) calculating transverse stresses using the three-dimensional elasticity method as well as modifying the results obtained from displacement and inertial effects based on this method; 3) simulating sandwich plate with viscoelastic and auxetic cores; 4) taking orthotropic properties for the viscoelastic core into account.
Keywords: Higher-order global-local theory, Sandwich plate, Soft core, Viscoelastic, Auxetic viscoelastic, Orthotropic viscoelastic.

Introduction

Today, the application of energy dampers in a structure is of utmost importance, leading to a reaction against the structure vibration, thus, dampening it. The use of the viscoelastic damper in a structure has two important effects: 1) increase in depreciated energy, 2) increase in structure stiffness. During an unplanned loading, a large amount of energy is imposed on the structure. This input energy appears in the structure as a kinetic and potential (strain) energy, which should be absorbed or dissipated in some way. If there is no damping in the structure, the structure continues to vibrate infinitely, but due to the structural characteristics, there is some damping in it, which can be used to counteract the vibration of the structure, hence dampening it. The efficiency of the structure can be increased by adding energy absorbers to it. In recent years, extensive studies have been done on composite plates and viscoelastic plates. When the viscoelastic theory was developed by the researchers, the solution method was based on the Laplace transform on which the elastic - viscoelastic correspondence principle was based

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which is, in turn, an interpretation of Volterra prinicple. Laplace transform equations are used for stress-strain relationships as well as strain-displacement ones. Hashin [1] investigated the issues of viscoelasticity with fixed properties. In terms of studying viscoelastic property, Lakes and Wienman [2] embarked on changes in the Poisson's ratio. Eshmatov [3] used the Kelvin-Woot model and considered an exponential function for the relaxation function. Real models based on these assumptions cannot really show the effect of dynamic loading. If the same assumptions are made for nonlinear models, then an integral differential equation is created, which in many cases can be solved by numerical methods such as Runge-Kutta. Alex et al. [4] studied static and dynamic crack propagation in non-homogeneous viscoelastic materials under shear loading. Schovanec and Walton [5] investigated the conditions for crack propagation in a heterogeneous linear viscoelastic object under quasi-static loading and calculated the released energy ratio. Paulino and Jin [6] investigated the principle of the correspondence about functionally graded materials and their continuous modeling issues for functionally graded viscoelastic plates under in-plane shear forces and extensive crack under load. Assie et al. [7] obtained the dynamic response of an impact on a viscoelastic frictionless plate through finite element. A linear viscoelastic solid model was used to describe viscoelastic behavior. Cederbaum and Aboudi [8] studied the response of viscoelastic composite plates under step load. They used the relationships belonging to Boltzmann and Fourier series to find the frequency response. Chen [9] investigated the dynamical and quasi-dynamic response of viscoelastic beams using the Laplace transform theory through finite element method. Llyasov [10] studied the stability of viscoelastic plates under triangular loading. Abdoun et al. [11] investigated the response of the viscoelastic plate under harmonic loading through using numerical methods. The finite element method employed to describe the viscoelastic properties has been explained using a complex relaxation matrix, which was a function of the relaxation frequency. Assie [12] investigated the behavior of viscoelastic composite plates under triangular loading; they used the Richard model to describe the viscoelastic property.

On the other hand, over the past decades, the application of sandwich plates has spread extensively in various industries, such as aerospace, marine, automobile and related industries. Since the three-dimensional analytical solution of sandwich plates has many limitations, the analysis of these structures, based on the theory of plates and shells, is very practical as well as important. The basis of these theories is the removal of the dependence of the model on the thickness and the conversion of three-dimensional models to two-dimensional ones. For this purpose, various methods have been presented such as equivalent single-layer theories, separate layer theories (layered as well as higher order sandwich ones), and theories based on the superposition principle (zigzag and global-local) [13]. Equivalent single layer theories are usually used to analyze the overall behavior of structures. When it comes to separate layer theories, each layer of a plate or shell is perceived as an independent layer and the description of the displacement field is applied for each layer independently. The models obtained on this basis are called layerwise or local theories. The main idea of the theories based on the superposition principle (zigzag and global-local theories) is that, initially, an identical displacement field is considered for all layers of multilayered or sandwich plates (similar to the equivalent single layer theory) and then independent displacement functions are considered for each layer in order to take the different behavior of each layer into account, stemming from their different properties (zigzag effect) [14]. A limited number of analyses in the field of dynamic and static response of sandwich plates with soft core have been published. The papers published by Moreira and Rodrigues[15], Elmalich and Rabinovitch [16], Rabinovitch and Frostig [17] , Cetkovic and Vuksanovic [18], and Wu and Chen [19] are among these papers. Ghaznavi and Shariyat [20] also presented a new higher order global-local theory to investigate the static, frequency, and dynamic behavior of sandwich plates with soft cores and auxetic core. Wan et al. [21] studied vibration of a composite sandwich plate with a viscoelastic core using

Donnell and Kirchhoff's theories. Vibration and damping characteristics of composite sandwich plates with viscoelastic cores were investigated by Zhai et al. [22] using the first-order shear deformation theory. Liu et al. [23] analyzed sandwich plates with viscoelastic cores using a differential quadrature hierarchical model and Carrera's unified formulation. Moita et al. [24] analyzed damped vibration of multilayer sandwich plates with viscoelastic cores. Higher-order and zig-zag formulations were proposed by Filippi et al. [25] to analyze structures with viscoelastic layers. Ren et al. [26] treated damping vibration of sandwich plates with thick viscoelastic cores, using a mixed layerwise theory.

Auxetic materials are relatively new materials that have recently been used and developed in various industries. The main difference between this material and other ordinary materials is that when there is a pressure on one direction, the other directions also contract. Also, if a tensile force is imposed on one direction, the material will experience an increase in length on other directions. A few papers have embarked on the study of the dynamic behavior of sandwich plates with auxetic cores [27-29]. Imbalzano et al. [30] study the resistance efficiency of auxetic and conventional honeycomb cores and metal facets against dynamic loadings. Zhang et al. [31] study mechanical properties of auxetic honeycomb structures. Wang et al. [32] investigate on the optimization of a double-V auxetic sandwich plates under air blast load. Zhang et al. [33] study the nonlinear transient responses of an auxetic honeycomb sandwich plate under impact dynamic loads using Reddy's higher shear deformation theory and Hamilton's principle. They found that the honeycomb sandwich plate with negative Poisson's ratio would be a better choice compared with the positive one for some structures under dynamic loads.

In this paper, a higher order global-local theory together with a three-dimensional elastic correction is presented for dynamical analysis of a rectangular sandwich plate with a viscoelastic soft core and an auxetic viscoelastic core. One of the main advantages of this theory is satisfying the continuity conditions of transverse stresses between layers as well as considering displacement along the thickness. Another advantage is the low cost of computations compared to layerwise theories, while benefiting from a high accuracy of the results. As can be seen in the results section of the paper, the changes in the local components of displacement and stress have been carefully evaluated. Equilibrium equations of elasticity have also been applied to calculate transverse stresses. Given the fact that this theory considers the changes in plate thickness, it enables us to study the exact behavior of thin and thick sandwich plates with soft viscoelastic and auxetic cores. In summary, some of the most important innovations belonging to this paper are: 1) presenting a new higher order global-local theory; 2) the application of a three-dimensional elastic correction method as well as modifying all the results obtained from it; 3) simulating the change in the thickness of the core , which is very crucial when it comes to studying the behavior of sandwich plates with soft core, especially viscoelastic ones, and 4) studying the behavior of the plate with the auxetic viscoelastic core.

The Governing Equations of Dynamic Analysis of Sandwich Plate with Viscoelastic Core

The Description of The Displacement Field of The Sandwich Plate Presented Though Using The Higher Order Global-Local Theory

The origin of the general coordinate system is located on the middle plane of the plate and the z axis is considered to be upside positive. The length and width of the plate along x and y are a and b respectively, and H is the total thickness of the plate. Also, the thickness of the upper layer is considered to be $h1$, the thickness of the core is considered to be $h2$ and the thickness of the lower layer is considered to be $h3$. It should be mentioned that in the whole paper, a thick plate means a plate in which the ratio of length to thickness is $a / H = 4$ and a thin plate means

a one in which *a / H = 10*. In-plane displacement components of the plate are considered a combination of two parts of global – local.

$$\begin{cases} u(x,y,z,t) = u_G(x,y,z,t) + u_L^k(x,y,z,t) \\ v(x,y,z,t) = v_G(x,y,z,t) + v_L^k(x,y,z,t) \end{cases} \quad (k = 1,2,3) \tag{1}$$

In which u_G and v_G are general components of displacement field which are considered as follows:

$$\begin{cases} u_G(x,y,z,t) = u_0(x,y,t) + z\varphi_x(x,y,t) + z^3\lambda_x(x,y,t) \\ v_G(x,y,z,t) = v_0(x,y,t) + z\varphi_y(x,y,t) + z^3\lambda_y(x,y,t) \end{cases} \tag{2}$$

Also, u_L and v_L are local components of the displacement field and include:

$$\begin{cases} u_L^k(x,y,z,t) = u_0^k(x,y,t) + z\varphi_x^{(k)}(x,y,t) \\ v_L^k(x,y,z,t) = v_0^k(x,y,t) + z\varphi_y^{(k)}(x,y,t) \end{cases} \quad (k = 1,2,3) \tag{3}$$

In above equations, index k indicates the number of the layer. Also u_0^k , v_0^k , $\varphi_x^{(k)}$ and $\varphi_y^{(k)}$ respectively indicate the local displacement of the middle plane of each layer and the local rotation of each layer. φ_x and φ_y are the general rotation of the middle plane of the core. λ_x and λ_y are also related to curvature changes in the curve belonging to the distribution of displacement components along the thickness. By applying the conditions of continuity and simplification, equation (4) will eventually be obtained:

$$\begin{cases} u_1 = u_0 + z\varphi_x + z^3\lambda_x(x,y,t) + (z - z_1^u)\varphi_x^{(1)} + z_1^u\varphi_x^{(2)} \\ v_1 = v_0 + z\varphi_y + z^3\lambda_y(x,y,t) + (z - z_1^u)\varphi_y^{(1)} + z_1^u\varphi_y^{(2)} \end{cases} \quad ;\ z_1^u \le z \le z_2^u$$

$$\begin{cases} u_2 = u_0 + z(\varphi_x + \varphi_x^{(2)}) + z^3\lambda_x(x,y,t) \\ v_2 = v_0 + z(\varphi_y + \varphi_y^{(2)}) + z^3\lambda_y(x,y,t) \end{cases} \quad ;\ z_1^l \le z \le z_1^u \tag{4}$$

$$\begin{cases} u_3 = u_0 + z\varphi_x + z^3\lambda_x(x,y,t) + (z - z_1^l)\varphi_x^{(3)} + z_1^l\varphi_x^{(2)} \\ v_3 = v_0 + z\varphi_y + z^3\lambda_y(x,y,t) + (z - z_1^l)\varphi_y^{(3)} + z_1^l\varphi_y^{(2)} \end{cases} \quad ;\ z_2^l \le z \le z_1^l$$

The final reformation of the plate is calculated based on the superposition principle. The purpose of this superposition is to consider simultaneously the effect of the displacement field of each layer along with the effect of the displacement field of the whole plate, in order to study the behavior of each layer separately and accurately, while not only overlooking the general behavior of the plate but also considering its effect on the behavior of each single layer. In other words, with the help of this method, it is possible to compensate for the main drawback of layerwise theories, which is overlooking the general behavior of the plate. Furthermore, it is possible to compensate for the drawback of equivalent single layer theories, such as the third-order shear theory, etc., which is overlooking the behavior of each single layer and studying only the behavior of the whole plate. Thus, it should be mentioned that according to these equations, the sum of the rotation belonging to each layer is not the same and is, therefore, various:

$$\begin{cases} \varphi_x^{*(i)} = \dfrac{\partial u}{\partial z} = \varphi_x + \varphi_x^{(i)} + 3z^2\lambda_x \\ \varphi_y^{*(i)} = \dfrac{\partial v}{\partial z} = \varphi_y + \varphi_y^{(i)} + 3z^2\lambda_y \end{cases} \quad ;\ i = 1,2,3 \tag{5}$$

One of the advantages of this theory is to consider the second - order variations for the W transverse displacement component in the core, as shown in Figure1. Hence, the following function is employed:

$$w=\begin{cases} w_u & ;\ -h_2/2-h_3\le z\le -h_2/2 \\ \mathscr{L}_1(z)w_u+\mathscr{L}_2(z)w_m+\mathscr{L}_3(z)w_l & ;\ -h_2/2\le z\le h_2/2 \\ w_l & ;\ h_2/2\le z\le h_2/2+h_1 \end{cases} \tag{6}$$

w_u, w_l, and w are displacements at the top, bottom and middle of the core respectively, L_1 and L_2 and L_3 are interpolation functions. Thus, the three-layer sandwich plate has a total of 15 independent displacement parameters. These parameters include:

$$u_0, v_0, \varphi_x, \varphi_y, \lambda_x, \lambda_y, \varphi_x^{(1)}, \varphi_y^{(1)}, \varphi_x^{(2)}, \varphi_y^{(2)}, \varphi_x^{(3)}, \varphi_y^{(3)}, w_u, w_m, w_l$$

Finite Element Form of Structural Equations

To solve the problem, the finite element method has been applied with the help of nonlinear rectangular two-dimensional elements. Displacement can be written in the following form using the finite element method:

$$\Phi(x,y,t)=\mathcal{N}(x,y)\Phi^{(e)}(t) \tag{7}$$

In which N and $\Phi^{(e)}$ are the shape function matrix and vector of displacement nodal values respectively. As a result, the displacement field can be written as follows:

$$\psi(x,y,z,t)=\mathrm{H}(z)\mathcal{N}(x,y)\Phi^e(t)=\Gamma(x,y,z)\Phi^e(t) \tag{8}$$

on the other hand, the components of strain and displacement field are:

$$\varepsilon_{xx}^{(i)}=\frac{\partial u_i}{\partial x},\quad \varepsilon_{yy}^{(i)}=\frac{\partial v_i}{\partial y},\quad \varepsilon_{zz}^{(i)}=\frac{\partial w_i}{\partial z},\quad \gamma_{xy}^{(i)}=\frac{\partial u_i}{\partial y}+\frac{\partial v_i}{\partial x}, \tag{9}$$

$$\gamma_{xz}^{(i)}=\frac{\partial u_i}{\partial z}+\frac{\partial w_i}{\partial x},\quad \gamma_{yz}^{(i)}=\frac{\partial v_i}{\partial z}+\frac{\partial w_i}{\partial y},\qquad (i=1,2,3)$$

or in the form of a matrix:

$$\varepsilon=\begin{Bmatrix}\varepsilon^{(1)}\\ \varepsilon^{(2)}\\ \varepsilon^{(3)}\end{Bmatrix}=\begin{bmatrix}D&0&0\\0&D&0\\0&0&D\end{bmatrix}\Psi(x,y,z)=$$
$$\begin{bmatrix}D&0&0\\0&D&0\\0&0&D\end{bmatrix}\Gamma(x,y,z)\Phi^{(e)}(t)=\Lambda(x,y,z)\Phi^{(e)}(t) \tag{10}$$

The components of stress and displacement field are:

$$\sigma=\begin{Bmatrix}\sigma^{(1)}\\ \sigma^{(2)}\\ \sigma^{(3)}\end{Bmatrix}=\begin{bmatrix}C^{(1)}&0&0\\0&C^{(2)}&0\\0&0&C^{(3)}\end{bmatrix}\begin{Bmatrix}\varepsilon^{(1)}\\ \varepsilon^{(2)}\\ \varepsilon^{(3)}\end{Bmatrix}=\hat{\mathcal{Q}}\varepsilon=\hat{\mathcal{Q}}\Lambda(x,y,z)\Phi^{(e)}(t) \tag{11}$$

In which $C^{(i)}$ is the matrix of elastic coefficients in the rotated coordinate system of the plate. Governing equations are obtained using the principle of minimum total potential energy which is, in fact, the sum of the work done by external force and the work from the inertial force. These are shown by the symbols of U and W respectively. Therefore [34]:

$$\delta U-\delta W=0 \tag{12}$$

In which [34]:

$$\delta U=\int_\Omega(\delta\varepsilon)^T\sigma d\Omega=\int_\Omega(\delta\Phi^{(e)})^T\Lambda^T\hat{\mathcal{Q}}\Lambda\Phi^{(e)}d\Omega \tag{13}$$

$$\delta W = \int_A q\,\delta w_u dA - \int_\Omega \rho(\delta\Phi^{(e)})^T \ddot{\Phi} d\Omega \tag{14}$$

In which A, Ω and q are the surface area, volume of the element and the intensity of the external load imposed on the plate respectively. By adding the above equations to the minimum total potential energy principle:

$$(\delta\Phi^{(e)})^T [\int_\Omega \rho\Gamma^T \Gamma \ddot{\Phi}^{(e)} d\Omega + \int_\Omega \Lambda^T \hat{\mathcal{Q}} \Lambda \Phi^{(e)} d\Omega - \int_A q(R\mathcal{N})^T dA] = 0 \tag{15}$$

Since $\delta\Phi^{(e)}$ is an arbitrary, non-zero vector, the governing equation of a simple sandwich plate will be:

$$[\int_\Omega \rho\Gamma^T \Gamma d\Omega]\ddot{\Phi}^{(e)} + [\int_\Omega \Lambda^T \hat{\mathcal{Q}} \Lambda d\Omega]\Phi^{(e)} = \int_A q(R\mathcal{N})^T dA \tag{16}$$

or in a contracted form, it will be:

$$M\ddot{\Phi}^{(e)} + K\Phi^{(e)} = F \tag{17}$$

Modeling of Viscoelastic Behavior Using Finite Element Simulation and Real Time Method

In elastic materials, Hooke's law is sufficient to define a material's behavior. In fact, this law states that stress is fully related to strain and introduces the ratio of stress to strain as the elastic modulus of the material. For full viscous materials, stress is related to the strain rate and the ratio of stress to strain rate is known as viscosity. Materials that have both of these properties are known as viscoelastic materials. In viscoelastic materials, the response of the material is affected by its loading rate. As an example, the more time it takes to reach the final stress of the material, the larger will be the strain response of the material. For this reason, viscoelastic materials record a history of their response and somehow have a memory [35]. This property leads to new governing equations between the tensor belonging to stress and strain of the material. As already mentioned, one of the methods for determining these governing equations in terms of linear viscoelastic materials is the application of Boltzmann's superposition principle. When the load in the form of strain is imposed on the material, this principle is as follows:

$$\varepsilon(t) = \sum_{j=1}^{n} \Delta\varepsilon_j = \int_0^t d[\varepsilon(s)] \tag{18}$$

In which s is supposed to be the arbitrary time interval between time 0 and t, in which strain is constant along its length. According to Hooke's law, increasing the strain up to t can be written as follows:

$$\sigma(t) = \sum_{j=1}^{n} \Delta\sigma_j(t - s_j) = \sum_{j=1}^{n} E(t - s_j)\Delta\varepsilon_j \tag{19}$$

Considering the appropriate ranges, the following governing equation can be obtained:

$$\sigma(t) = \int_0^t E(t-s)\frac{\partial\varepsilon(s)}{\partial s} ds \tag{20}$$

The generalized Maxwell solid Model is commonly used to simulate viscoelastic solid materials [36, 37]. This model is basically a combination of spring and damper. This model can be written in the form of the Prony Series as follows:

$$E(t) = E_\infty + \sum_{j=1}^{N} E_j e^{(\frac{-t}{\tau_j})} \tag{21}$$

In which N, E_j, E_∞ and τ_j are respectively the number of elements belonging to the Maxwell model, the elastic modulus, the long term modulus once the material is totally relaxed, and the relaxation time, which is equal to $\frac{\eta}{E_\infty}$. By inserting the above equation in equation (20) and separating their viscoelastic and elastic terms from each other, it is possible to write [35]:

$$\begin{aligned}\sigma(t) &= \int_0^t E_\infty \frac{\partial \varepsilon(s)}{\partial s} ds + \int_0^t \sum_{j=1}^{N} E_j e^{(\frac{-t}{\tau_j})} \frac{\partial \varepsilon(s)}{\partial s} ds \\ &= E_\infty \varepsilon(t) + \sum_{j=1}^{N} \int_0^t E_j e^{(\frac{t-s}{\tau_j})} \frac{\partial \varepsilon(s)}{\partial s} ds = \sigma_0(t) + \sum_{j=1}^{N} \mathscr{H}_j(t)\end{aligned} \tag{22}$$

By defining the time step $\Delta t = t_{n+1} - t_n$ in which t_{n+1} and t_n indicate the time of the current step and the time of the previous step respectively, and also by considering $\varepsilon(t) = \frac{\sigma(t)}{E_\infty}$, it is possible to define a return equation for the stress variable. By converting the differential equation into discrete time intervals, we can write [38]:

$$\mathscr{H}_j^{n+1} = e^{(\frac{-\Delta t}{\tau_j})} \mathscr{H}_j^{n} + \Upsilon_j \int_{t_n}^{t_{n+1}} e^{(\frac{-t_{n+1}-s}{\tau_j})} ds \, \frac{\sigma_0^{n+1} - \sigma_0^{n}}{\Delta t} \tag{23}$$

In which $\Upsilon_j = E_j / E_\infty$ is the normalized elastic modulus of j element. If the above element is integrated numerically, then we can write the above equation in the form of a returnable matrix [38]:

$$\mathscr{H}_j^{n+1} = e^{(\frac{-\Delta t}{\tau_j})} \mathscr{H}_j^{n} + \Upsilon_j \frac{1 - e^{(-\frac{\Delta t}{\tau_j})}}{\frac{\Delta t}{\tau_j}} (\sigma_0^{n+1} - \sigma_0^{n}) \tag{24}$$

With the help of the above equation, the stress of viscoelastic material can be defined as follows:

$$\sigma^{n+1} = \sigma_0^{n+1} + \sum_{j=1}^{N} \mathscr{H}_j^{n+1} \tag{25}$$

To write equations in the form of a finite element, the same path of linear elastic materials is employed. The matrix form of the relationship between stress and strain in these materials is as follows [38]:

$$\begin{aligned}&\sigma_0^{n+1} = \hat{\mathcal{Q}} \varepsilon^{n+1} \\ &\varepsilon = \mathcal{D}\boldsymbol{\delta} = \hat{\mathcal{Q}} \Lambda(x, y, z)\Phi \\ &\delta U = \int_\Omega (\delta\varepsilon)^T \sigma d\Omega = \int_\Omega (\delta\Phi^{(e)})^T \Lambda^T \hat{\mathcal{Q}} \Lambda \Phi^{(e)} d\Omega; \\ &f_{\text{int}} = \int_\Omega \Lambda^T \hat{\mathcal{Q}} \Lambda d\Omega]\Phi = K\Phi\end{aligned} \tag{26}$$

In which $\hat{\mathcal{Q}}$ is the elastic modulus matrix of the material. The above equation can be rewritten in the following form:

$$\begin{aligned}\sigma^{n+1} &= \hat{\mathcal{Q}}_{infinity} \mathcal{D}\Phi^{n+1} + \sum_{j=1}^{N} [e^{\left(-\frac{\Delta t}{\tau_j}\right)} \mathscr{H}_j^{n} + \gamma_j \mathrm{A}_j (\hat{\mathcal{Q}}_{infinity} \mathcal{D}\Phi^{n+1} - \hat{\mathcal{Q}}_{infinity} \mathcal{D}\Phi^{n}) \\ &= \hat{\mathcal{Q}}_{infinity} \mathcal{D}\left(1 + \sum_{j=1}^{N} \gamma_j \mathrm{A}_j\right)\Phi^{n+1} + \sum_{j=1}^{N} e^{\left(-\frac{\Delta t}{\tau_j}\right)} \mathscr{H}_j^{n} - \hat{\mathcal{Q}}_{infinity} \mathcal{D}\left(\sum_{j=1}^{N} \gamma_j \mathrm{A}_j\right)\Phi^{n}\end{aligned} \tag{27}$$

In which $\hat{\mathcal{Q}}_{infinity}$ is the stable modulus matrix of the material. Also A_j includes:

$$\mathrm{A}_j = \frac{1-e^{\left(-\frac{\Delta t}{\tau_j}\right)}}{\frac{\Delta t}{\tau_j}} \tag{28}$$

Equation (26) can be rewritten for linear viscoelastic materials using the above definition [38]:

$$f_{\text{int}} = K_T \Phi^{n+1} + \mathscr{H}_{hist}^{n+1} - K_{hist} \Phi^n \tag{29}$$

In which Φ^{n+1} and Φ^n are the displacement vectors of nodes in the current time step and the previous time step; also, K_T and K_{hist} include [38]:

$$K_T = \mathcal{D}^T \hat{\mathcal{Q}}_{infinity} \mathcal{D} V \left(1 + \sum_{j=1}^{N} \Upsilon_j \mathrm{A}_j\right) \tag{30}$$

$$K_{hist} = \mathcal{D}^T \hat{\mathcal{Q}}_{infinity} \mathcal{D} V \sum_{j=1}^{N} \Upsilon_j \mathrm{A}_j \tag{31}$$

Vector $\mathscr{H}_{hist}^{n+1}$ is the vector of history of material behavior in the current time step:

$$\mathscr{H}_{hist}^{n+1} = V\, \mathcal{D}^T \sum_{j=1}^{N} e^{\left(-\frac{\Delta t}{\tau_j}\right)} \mathscr{H}_j^n \tag{32}$$

It is possible to calculate $\mathscr{H}_j^{n+1}$ for each element using the following equation [38]:

$$\mathscr{H}_j^{n+1} = e^{\left(-\frac{\Delta t}{\tau_j}\right)} \mathscr{H}_j^n + \Upsilon_j \mathrm{A}_j\, \hat{\mathcal{Q}}_{infinity}\, \mathcal{D}\left(\Phi^{n+1} - \Phi^n\right) \tag{33}$$

Finally, the structural equation for linear viscoelastic analysis can be rewritten in the following form:

$$\begin{gathered} \mathbf{M\ddot{u}^{n+1}} + \mathbf{K_T u^{n+1}} = \mathbf{F} \\ \mathbf{F} = [\mathbf{F}_{\mathbf{ext}} - \mathbf{F}_{\mathbf{hist}}] \end{gathered} \tag{34}$$

In which F_{hist} includes:

$$F_{hist} = \mathscr{H}_{hist}^{n+1} - K_{hist}\ u^n \tag{35}$$

Calculation of Transverse Stresses Using Three-Dimensional Elasticity Equilibrium Equations and the Modification of the Results Based on It

Determining transverse stress components especially shear stress components is very important when it comes to the design process of sandwich structures, since as a sample calculation of shear stresses between the layers accurately is necessary for structural estimation in terms of the strength between the layers and the non-separation between the layers. In most existing theories, that is, the equivalent single layer, layerwise, and even zigzag which have been presented for analyzing plates and shells, shear stresses can be calculated with the approach of using governing equations. For this reason, the amount of shear stresses calculated between the layers is not continuous; this, in turn, affects the accuracy of the values calculated for the shear stresses inside the layer. That is why in many cases, including the application of first order shear theory, it is necessary to apply shear correction coefficient, unless the shear stresses are calculated using the three-dimensional equilibrium equations along with the employment of the intended theory of plate and shell [39, 40]. Chu and Kim [41], Wu and Chen [42] and Shariyat

[43] have carried out plenty of research on sandwich plates with stiff layers, which eventually showed that when it comes to calculating shear stresses, even the application of third order shear theory can have noticeable errors compared to the three-dimensional equilibrium equations. Although the presented global-local theory is capable of describing and evaluating transverse variables of displacement field, the shear stresses calculated using the three-dimensional elastic correction method have been modified. As a result, the conditions of continuity between the layers as well as the condition of zero shear stresses at upper and lower free surfaces have been met. In general, according to the three-dimensional elastic equations, it can be written [44]:

$$\begin{aligned} &\sigma_{x,x}+\tau_{xy,y}+\tau_{xz,z}=\rho\ddot{u} \\ &\sigma_{y,y}+\tau_{xy,y}+\tau_{yz,z}=\rho\ddot{v} \end{aligned} \tag{36}$$

Based on the above equation, it can be written [44]:

$$\begin{aligned} &\tau_{xz}^{(1)}=\int_{z}^{z_1^u}\{\rho\ddot{u}^{(1)}-(\sigma_{x,x}^{(1)}+\tau_{xy,y}^{(1)})\}dz\,; && z_1^u\le z\le z_2^u \\ &\tau_{xz}^{(2)}=\int_{z}^{h_2/2}\{\rho\ddot{u}^{(2)}-(\sigma_{x,x}^{(2)}+\tau_{xy,y}^{(2)})\}dz+\tau_{xz}^{(1)}\Big|_{z=h_2/2}\,; && z_1^l\le z\le z_1^u \\ &\tau_{xz}^{(3)}=\int_{z}^{-h_2/2}\{\rho\ddot{u}^{(3)}-(\sigma_{x,x}^{(3)}+\tau_{xy,y}^{(3)})\}dz+\tau_{xz}^{(2)}\Big|_{z=-h_2/2}\,; && z_2^l\le z\le z_1^l \end{aligned} \tag{37}$$

Similarly:

$$\begin{aligned} &\tau_{yz}^{(1)}=\int_{z}^{z_1^u}\{\rho\ddot{v}^{(1)}-(\sigma_{y,y}^{(1)}+\tau_{xy,x}^{(1)})\}dz\,; && z_1^u\le z\le z_2^u \\ &\tau_{yz}^{(2)}=\int_{z}^{h_2/2}\{\rho\ddot{v}^{(2)}-(\sigma_{y,y}^{(2)}+\tau_{xy,x}^{(2)})\}dz+\tau_{yz}^{(1)}\Big|_{z=h_2/2}\,; && z_1^l\le z\le z_1^u \\ &\tau_{yz}^{(3)}=\int_{z}^{-h_2/2}\{\rho\ddot{v}^{(3)}-(\sigma_{y,y}^{(3)}+\tau_{xy,x}^{(3)})\}dz+\tau_{yz}^{(2)}\Big|_{z=-h_2/2}\,; && z_2^l\le z\le z_1^l \end{aligned} \tag{38}$$

After calculating the modified shear stresses, the result of the correction performed is applied to the whole plate calculations. For this purpose, the modified shear stresses for all nodes are embedded in the stress matrix of the whole plate; hence, the modified stress matrix of the plate is obtained. Then, using the following equation, it is possible to calculate a new node displacement for the whole plate, which was derived from the modified stress matrix.

$$\begin{aligned} &\sigma_0^{n+1}=\hat{\mathcal{Q}}\Lambda(x,y,z)\Phi \\ &\sigma^{n+1}=\hat{\mathcal{Q}}_{infinity}\mathcal{D}\left(1+\sum_{j=1}^{N}\gamma_j\mathrm{A}_j\right)\Phi^{n+1}+\sum_{j=1}^{N}e^{\left(-\frac{\Delta t}{\tau_j}\right)}\mathscr{H}_j^n-\hat{\mathcal{Q}}_{infinity}\mathcal{D}\left(\sum_{j=1}^{N}\gamma_j\mathrm{A}_j\right)\Phi^n \end{aligned} \tag{39}$$

It should also be noted that before calculating a new displacement based on the above equation, the matrix $\mathscr{H}_j^{n+1}$ must also be modified according to equation 40:

$$\mathscr{H}_j^{n+1}=e^{(\frac{-\Delta t}{\tau_j})}\mathscr{H}_j^n+\Upsilon_j\frac{1-e^{(-\frac{\Delta t}{\tau_j})}}{\frac{\Delta t}{\tau_j}}(\sigma_0^{n+1}-\sigma_0^n) \tag{40}$$

By implementing the above changes, the modified displacement vector (Φ^{n+1}) is eventually calculated. According to the numerical integration method, such as the Newmark method, velocity and acceleration at the end of each time step are related to the velocity, acceleration and displacement of the beginning of the time step, which are depicted through the following relationships:

$$\ddot{\mathbf{\Phi}}_{n+1} = a_1(\mathbf{\Phi}_{n+1} - \mathbf{\Phi}_n) - a_2\dot{\mathbf{\Phi}}_n - a_3\ddot{\mathbf{\Phi}}_n \tag{41}$$

$$\dot{\mathbf{\Phi}}_{n+1} = \dot{\mathbf{\Phi}}_n + a_4\ \ddot{\mathbf{\Phi}}_n + a_5\ \ddot{\mathbf{\Phi}}_{n+1} \tag{42}$$

Where the subscripts n and $n + 1$ indicate the beginning and end instants of the considered time step and $a1 – a5$ are constants of the numerical method. Therefore, using the above equations, the velocity and acceleration are corrected based on the modified displacement derived from Equation 39. In other words, at the beginning of the next step, when it comes to calculations, the stress, displacement, velocity, and acceleration corrected based on the results of the three-dimensional elasticity have been employed.

$$\ddot{\mathbf{\Phi}}_{n+1}\Big|_{Corrected} = a_1(\mathbf{\Phi}_{n+1}\Big|_{Corrected} - \mathbf{\Phi}_n) - a_2\dot{\mathbf{\Phi}}_n - a_3\ddot{\mathbf{\Phi}}_n \tag{43}$$

$$\dot{\mathbf{\Phi}}_{n+1}\Big|_{Corrected} = \dot{\mathbf{\Phi}}_n + a_4\ \ddot{\mathbf{\Phi}}_n + a_5\ \ddot{\mathbf{\Phi}}_{n+1}\Big|_{Corrected} \tag{44}$$

It should be noted, however, that in the above equations, $\mathbf{\Phi}_n$, $\dot{\mathbf{\Phi}}_n$ and $\ddot{\mathbf{\Phi}}_n$ belonging to the end of the previous time step have been modified, they have been corrected. In fact, presented approach, correct all the displacements, velocity, and acceleration that used as initial conditions for the next time step. The correction procedure is shown in Figure 1. In previous papers which aimed to correct shear stress, only the results of shear stresses were corrected [45-47]. As was mentioned before, however, in this paper the effect of this correction has also been imposed on velocity and acceleration, which, in turn, indicates the superiority as well as the novelty of the proposed algorithm when it comes to solving problems, including analysis of sandwich plates with viscoelastic core.

Numerical Results

This section presents the results of a dynamic analysis of a sandwich plate with viscoelastic core. The loading has been carried out in such a way as to initially apply a pressure on the upper face of the sandwich plate (at a very short period of time), and then the load has been suddenly removed. As a result of imposing the load, the plate began to oscillate. The behavior of thin and thick sandwich plates with viscoelastic core under the mentioned vibration has been parametrically evaluated. Various parameters, such as the elastic modulus of the core, the long term modulus once the material is totally relaxed, and the relaxation time and the core thickness have been evaluated. Also, the behavior of thin and thick sandwich plates with auxetic viscoelastic core and orthotropic viscoelastic core will be discussed next.

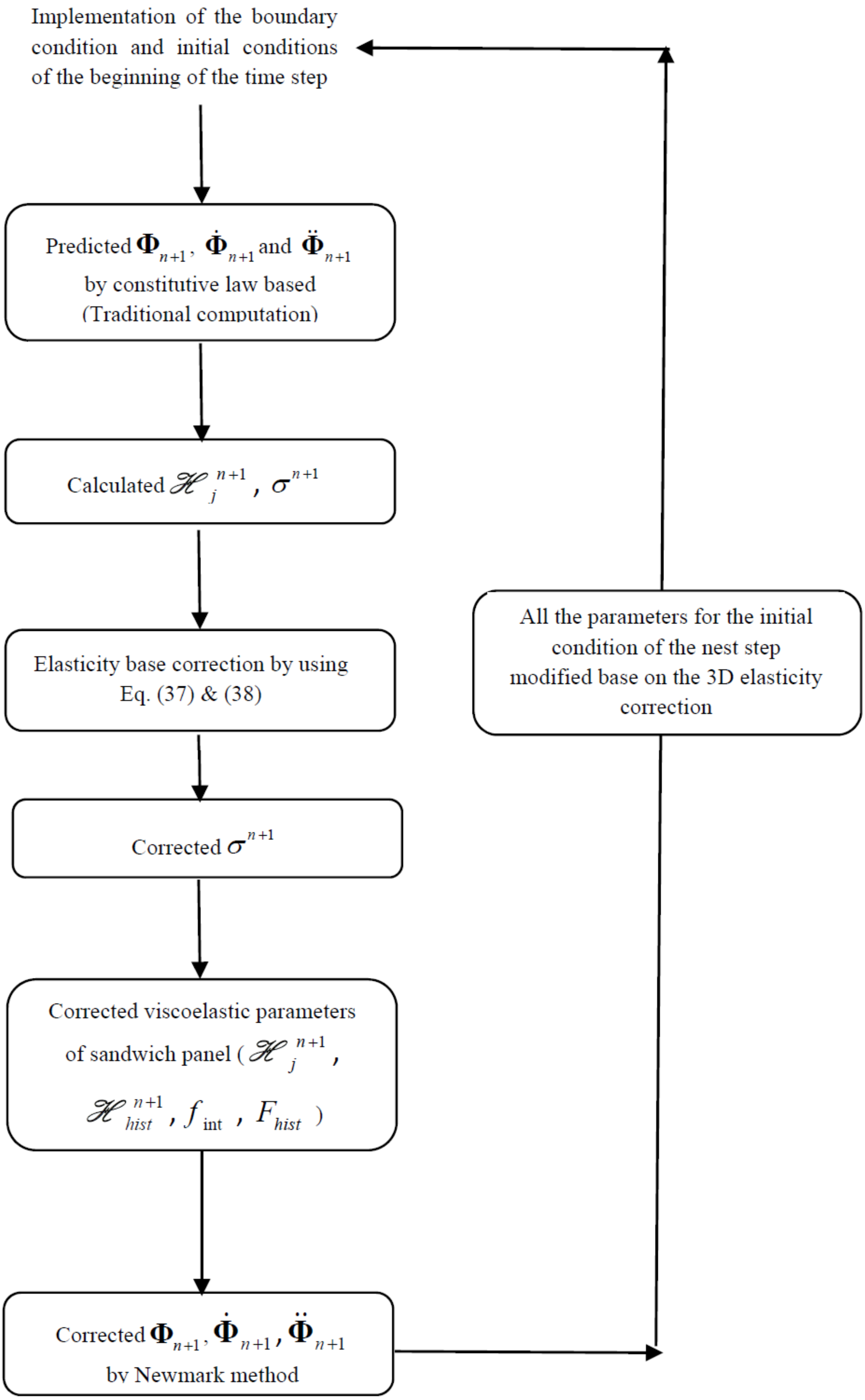


Figure 1. Details of the proposed algorithm for correction of all components of the present displacement-based solution.

Verification of The Results

In this section, before presenting the results of parametric analyses of the sandwich plates with a viscoelastic core, validity of the viscoelastic core behavior is confirmed by deferent references. At first, the same analysis is performed as Wang and Tsai's analysis [48] and the results are compared with the results available in the reference. They studied the dynamic behavior of an isotropic and homogeneous viscoelastic plate under step loading using third order shear deformation theory. They used the three-parameter-solid with the following specifications to analyze the viscoelastic material [48]:

$E_0 = 9.8\times10^7\, N/m^2, E_1 = 2.45\times10^7\, N/m^2, \rho = 2200\, kg/m^3, \nu = 0.35, \eta = 2.744\times10^8\, Ns/m^2$

The obtained results in this research as well as the results in the reference [48] are compared in figure 2. Comparing the results shows a very good agreement between the obtained results and the results provided by Tsai and Wong. The slight difference can be due to the difference in the third order shear deformation theory and presented theory. However, as mentioned, the presented global-local theory has a higher accuracy than the third-order shear deformation theory.

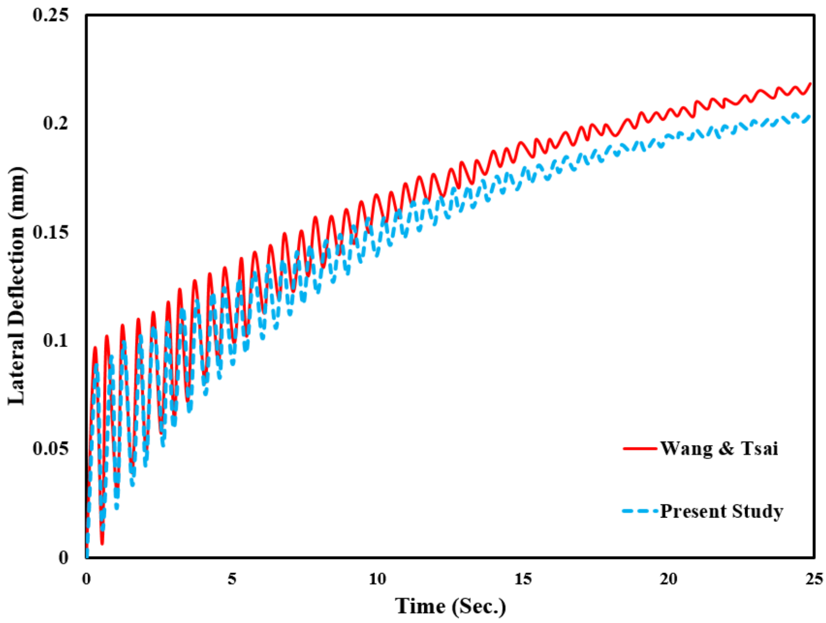


Figure 2. A comparison between time histories of lateral deflection of the central point of an isotropic and homogeneous viscoelastic plate by present approach and Wang and Tsai [48] under a step loading.

Due to the existence of step load and load variations over time, the displacement of the middle point of the elastic plate is approximately twice its static equilibrium point (around which the elastic plate oscillates). In contrast to the elastic plate, when it comes to the viscoelastic plate, the vibration of the plate are damped with the passage of time, due to its damping nature which, in turn, stems from its the presence of viscoelastic material. However, since the plate is exposed to compulsory vibrations due to the existence of external load, with the passage of time, part of the energy transmitted to the plate increases the non-oscillating displacement of the middle point of the plate. Hence, over time, plate behavior will be static and therefore, will not show oscillating behavior. From a phenomenological point of view, the stiffness of the plate is directly affected by the E_0 parameter. At first, the damping element still behaves rigidly and has little effect on the stiffness of the plate and consequently on the answer to the problem. But in a steady state of the plate, the stiffness of the plate is related to $(1/E_0 + 1/E_1)^{-1} = E_0E_1/(E_0 + E_1)$. That is why, with the passage of time, the transverse

displacement of the plate increases. Also, in order to further validate, the natural frequencies and loss factors of sandwich plates have been compared with the results available in valid references [49-51]. The results for the sandwich plate are considered in different modes of boundary condition of all edges clamped (CCCC) and opposite sides clamped (CFCF). The geometrical and material properties of the sandwich plates are given in table 1.

Table 1. The geometrical and material properties of the sandwich plates

Boundary conditions	*Isotropic Face Layers*	*Viscoelastic core layer*
CCCC (Araujo et al.,[49])	$a=0.348$ m, $b=0.3048$ m, $E_1 = E_3 = 68.9$ GPa $\upsilon_1 = \upsilon_3 = 0.3$ $\rho_1 = \rho_3 = 2740$ kg/m^3 $h_1 = h_3 = 0.000762$ m	$G_v = 0.896$ MPa $\upsilon_v = 0.49$ $\rho_2 = 999$ kg/m^3 $\eta_v = 0.5$ $h_2 = 0.000254$ m
CFCF (Huang et al., [50])	$a=0.4$ m, $b=0.4$ m, $E_1 = E_3 = 71$ GPa $\upsilon_1 = \upsilon_3 = 0.3$ $\rho_1 = \rho_3 = 2740$ kg/m^3 $h_1 = 0.003$m, $h_3 = 0.001$ m	$G_v = 0.896$ MPa $\upsilon_v = 0.498$ $\rho_2 = 999$ kg/m^3 $\eta_v = 0.9683$ $h_2 = 0.001$ m
CCCC (Experimental) (Wang et al., [51])	$a=0.6731$ m, $b=0.5207$ m, $E_1 = E_3 = 68.9$ GPa $\upsilon_1 = \upsilon_3 = 0.3$ $\rho_1 = \rho_3 = 2740$ kg/m^3 $h_1 = 0.0008$m, $h_3 = 0.0008$ m	$G_v = 0.896$ MPa $\upsilon_v = 0.49$ $\rho_2 = 999$ kg/m^3 $\eta_v = 0.5$ $h_2 = 0.0000508$ m

Table 2. Natural frequencies and corresponding loss factors of sandwich plate with viscoelastic core under different boundary conditions.

Mode no.	**1**		**2**		**3**	
	Frequency (Hz)	Loss factor	Frequency (Hz)	Loss factor	Frequency (Hz)	Loss factor
CCCC (Araujo et al.,[49])	87.66	0.1886	150.1	0.163	170.99	0.152
Present	87.055	0.1735	145.386	0.1428	177.812	0.153
CFCF (Huang et al., [50])	95.09	0.1315	112.7	0.1274	187.25	0.139
Present	98. 016	0.1861	113.783	0.1302	184.458	0.120
CCCC (Experimental) (Wang et al., [51])	38	0.092	68.5	------	90.3	0.158
Present	38.325	0.073	66.128	0.055	87.427	0.125

The first three natural frequencies, and the corresponding loss factors, are calculated under different boundary conditions of CCCC and CFCF, and are compared with the numerical results obtained by Araujo et al [49] and Huang et al. [50] and experimental result of Wang et al. [51]. The mathematical form of the mentioned boundary conditions for different edges of sandwich panel is:

- ✓ Simply supported edges:
- ✓ x= 0, a : $u_0, w_u, w_m, w_l, \varphi_y, \lambda_y, \varphi_y^{(1)}, \varphi_y^{(2)}, \varphi_y^{(3)} = 0$
- ✓ y= 0,b : $v_0, w_u, w_m, w_l, \varphi_x, \lambda_x, \varphi_x^{(1)}, \varphi_x^{(2)}, \varphi_x^{(3)} = 0$
- ✓ Clamped edge:
- ✓ x = 0, a; y= 0, b: $u_0, v_0, \varphi_x, \varphi_y, \lambda_y, \lambda_x, \varphi_x^{(1)}, \varphi_y^{(1)}, \varphi_x^{(2)}, \varphi_y^{(2)}, \varphi_x^{(3)}, \varphi_y^{(3)}, w_u, w_m, w_l = 0$

All mentioned results is shown in table 2. As can be seen, results obtained from the presented code and proposed algorithm in different boundary conditions are found to be in good agreement with the published ones.

Parametric Study of A Sandwich Plate with Viscoelastic Core

The properties of the viscoelastic material (before the parametric investigation of the problem) are shown in Table 3. The materials presented in the valid reference [52] have been employed for viscoelastic core properties. Also, the properties of the facesheets of sandwich plate have been considered to be orthotropic that made of Boron-Epoxy. These properties include following:

E1= E3=20.7 GPa, E2 =221.0 GPa, G12=G23=5.79 GPa, G13=3.29 GPa, υ12=υ23=0.23, υ31 = 0.45

In order to make a better comparison of the results with each other, the obtained results have become dimensionless using the following:

$$W = w\,\frac{100E_2^{skin}h^3}{a^4 q_0} \quad , \quad S_{xx} = \frac{h^2}{q_0 a^2}(\sigma_{xx})$$

As was mentioned above, the material used in reference [52] was first employed for the viscoelastic properties of the core of sandwich plate, and then the various parameters were changed to fully determine the effect of each parameters on the sandwich plate behavior.

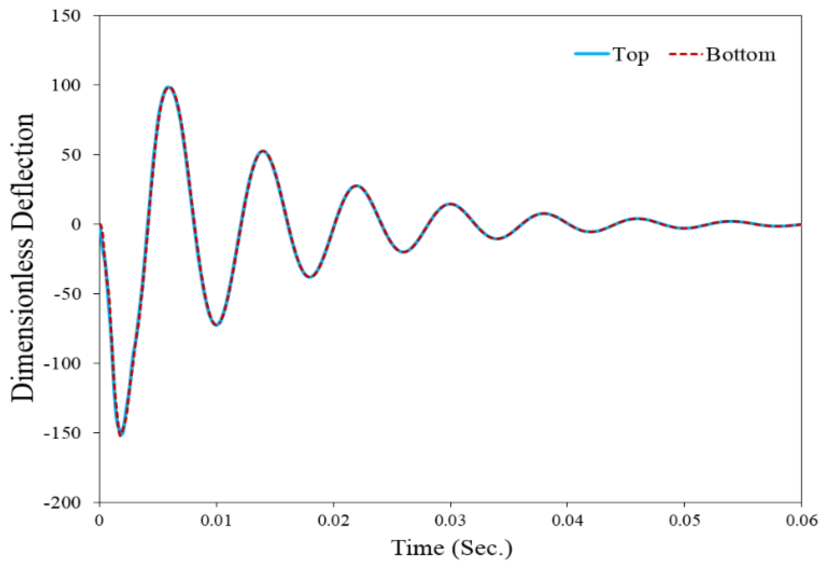
Table 3. Viscoelastic material properties [52]

Property	***Value***
Elastic modulus	100 (Mpa)
Poisson's ratio	0.3
Delayed stiff (E_∞/E_{eq})	0.2
Relaxation time $(\tau=\eta/E_\infty)$	1e-4

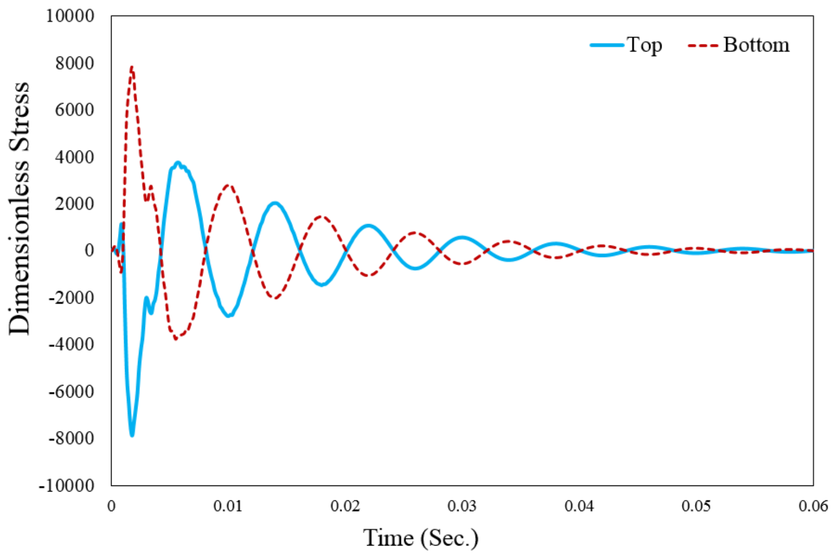
Investigation of The Effect of Delayed Stiffness on The Behavior of Thick And Thin Sandwich Plates

When it comes to the definition of viscoelastic materials, one of the most important parameters is the delayed stiffness (E_∞/E_{eq}). The physical concept of this relationship is the ratio of the long term modulus once the material is totally relaxed to its initial Young's modulus. If the Delayed Stiffness is reduced, the material's ability to reduce energy and to depreciate it increases. To completely study the effect of this parameter on the dynamic behavior of the sandwich plate, the sandwich panel has been analyzed dynamically with three different delayed stiffness which include 0.8, 0.5 and 0.2. These analyses have been carried out both for thick sandwich plates and for thin sandwich plates. Figure 3 depicts the results of displacement and dimensionless stress of thin sandwich plates with the viscoelastic core as well as the ratio of $(E_\infty/E_{eq} = 0.2)$ and also the relaxation time of $\tau = \eta/E_\infty = 0.0001$. The effect of viscoelastic core and its damping on the displacement vibration and stress is quite noticeable. As can be seen the amplitude of the oscillations has decreased considerably for both displacement and stress, and is almost completely depreciated. Figure 4 indicates the same results for a thick plate with a viscoelastic core and with similar properties. The only difference between these two examples is the ratio of length to thickness of the sandwich plate, which changed from 10 for a thin plate to 4 for a thick plate. Other material parameters, including the support conditions and other material properties, are the same in both examples. The displacement behavior of the upper and lower surfaces in this plate is the same, despite being thick. This is due to the stiffness of the viscoelastic core. Also, the stress behavior of the middle point of the upper and lower surfaces is the same and symmetrical, which is similar to that of the thin plate. This reflects the uniform performance of sandwich plates (thin and thick) with the viscoelastic core employed

in the sandwich. When it comes to the thick plate, like the thin one, the energy imposed was depreciated in a short time, and in practice, the displacement range of the middle point of the plate becomes completely zero up to 0.04 seconds and the plate fluctuation disappears.



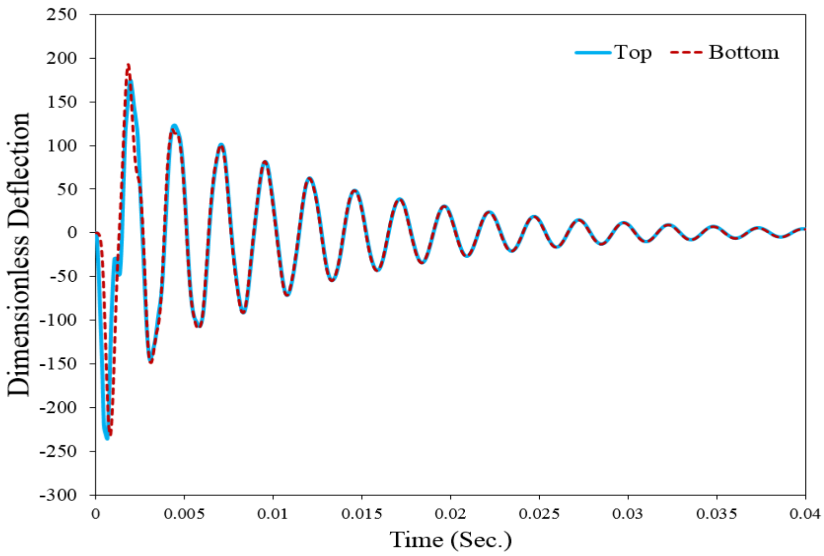
(a)



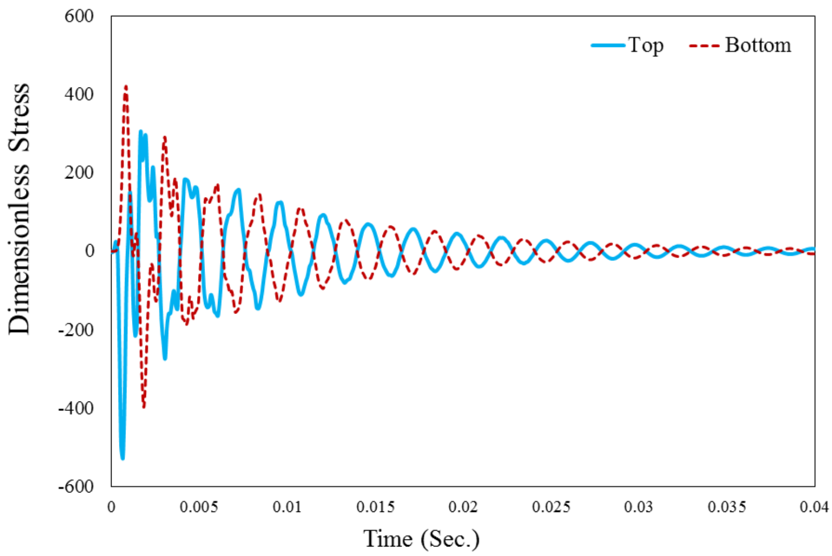
(b)

Figure 3. Thin sandwich plate (a / h = 10) with a viscoelastic core with a ratio of $(E_{\infty}/E_{eq} = 0.2)$ and relaxation time of $\tau = (\eta/E = 0.0001)$: a) dimensionless displacement, b) dimensionless stress.

However, it should be mentioned that it is because of this damping effect that the number of oscillation cycles in thick plate is more than those of the thin plate; in other words, when it comes to the thick plate, the number of oscillation cycles up to the full depreciation is almost three times more than those of the thin plate. The studies carried out in these two examples showed that, the stress behavior for the middle points of the plate is in perfect harmony with the transverse displacement behavior of the plate. That is, when the oscillation of the transverse displacement of the plate disappears, the range of in-plane stress variations in the upper and lower surfaces of the plate becomes zero. Given the uniformity of the stress behavior and the displacement of the middle points of the plate, the stress results are not included in the following examples and that the results of the transverse displacement of the middle point of upper and lower surfaces of the sandwich are only briefly presented.



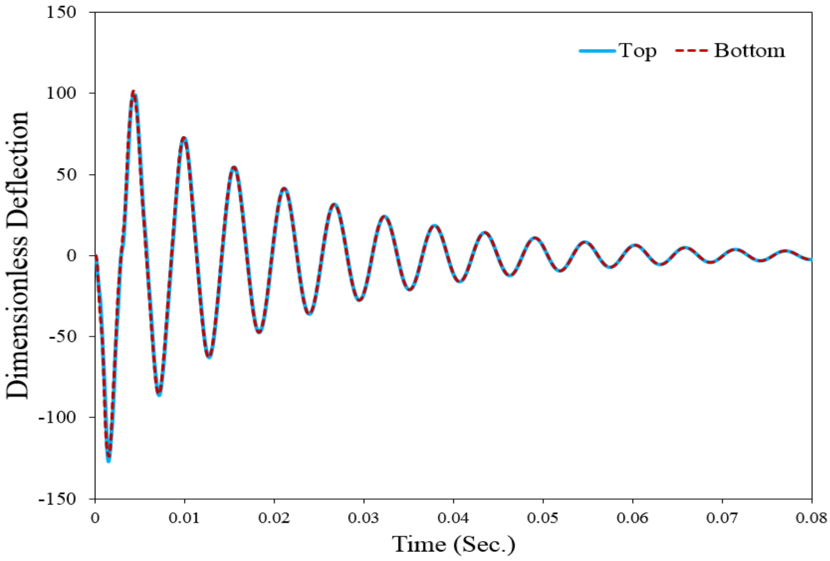
(a)



(b)

Figure 4. Thick sandwich plate (a / h = 4) with a viscoelastic core with a ratio of $(E_\infty/E_{eq} = 0.2)$ and relaxation time of $\tau = (\eta/(E) = 0.0001)$: a) dimensionless displacement, b) dimensionless stress

Figure 5 indicates the dimensionless displacement of the middle point of the plate in the upper and lower layers for both thin plate and thick sandwich plate with a viscoelastic core with the ratio of E_∞/E_{eq} , which equals to 0.5. Other properties of the core are exactly the same as the previous case, and only this ratio belonging to viscoelastic material has changed. Figure 6 also depicts the behavior of the same plates for a case of E_∞/E_{eq} in which the viscoelastic core is 0.8. In general, it can be observed that in all three cases, the frequency belonging to the thick sandwich plate is higher than that of the thin sandwich plate, and at an equal time interval, the number of oscillation cycles belonging to the sandwich plate is more than those of the thin sandwich plate. Furthermore, in both thick and thin sandwich plates, the reduction in the ratio of E_∞/E_{eq} reduces the frequency and increases its damping.



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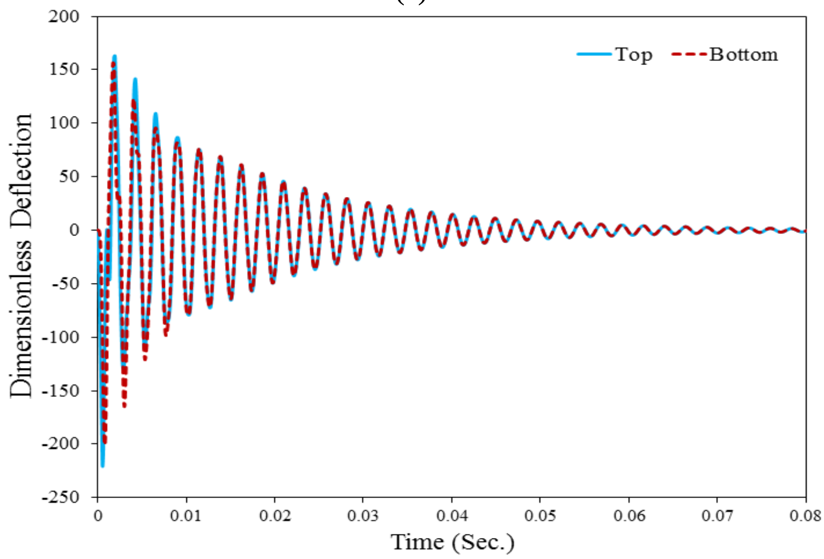


(b)

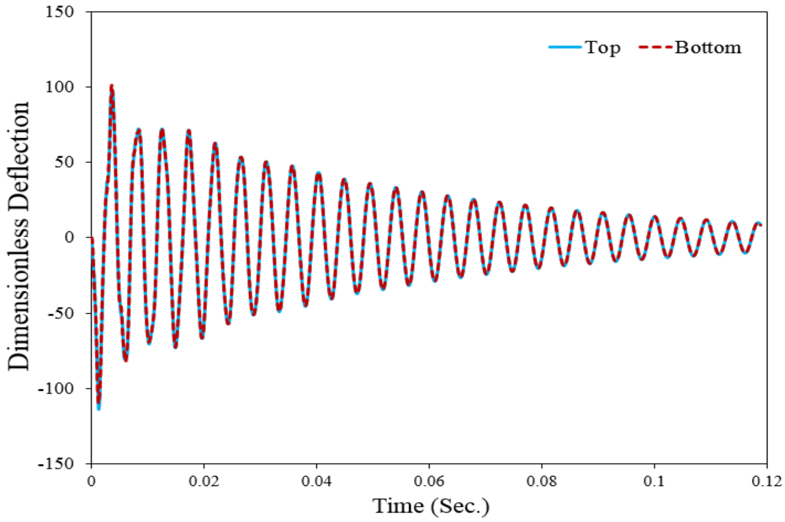
Figure 5. Dimensionless displacement of a sandwich plate (a / h = 10) with a viscoelastic core with a ratio of $(E_{\infty}/E_{eq}=0.5)$ and relaxation time (τ=0.0001): a) thin sandwich plate (a/h=10), b) thick sandwich plate (a/h=4)

For a better comparison of different cases of sandwich panel with each other, a parameter of loss factor has been defined. The loss factor has been calculated for a sandwich panel through measuring the amount of vibration amplitude (x_n) in different cycles (n) and also with the help of the following equation:

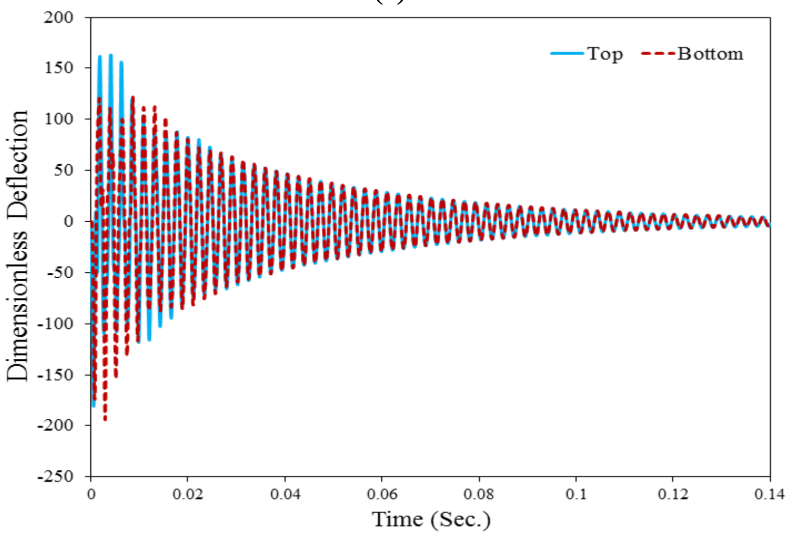
$$\zeta=\frac{1000}{2\pi}\frac{1}{n}\ln\left(\frac{x_1}{x_{n-1}}\right) \tag{45}$$

Table 4. Loss factor for thin and thick sandwich plates with a change of parameter E_{∞}/E_{eq} in a viscoelastic core with a Young's modulus equaling to 100 Mpa and τ equaling to 0.0001

E_{∞}/E_{eq}	***Loss factor***	
	Thin sandwich plate	***Thick sandwich plate***
0.8	12.7	10.4
0.5	38.1	21.1
0.2	84	34.2



(a)



(b)

Figure 6. Dimensionless displacement of a sandwich plate with a viscoelastic core with a ratio of ($E_{\infty}/E_{eq}=0.8$) and relaxation time (τ=0.0001): a) thin sandwich plate, b) thick sandwich plate.

The results of calculating the loss factor for each six example have been presented in Table 4 (three cases of thin sandwich plates and three of thick sandwich plates). For a better comparison, all of the loss factors have been calculated at the same time so that the examples can be compared with each other. The results indicate that thin plates with similar properties have more damping than thick plates. However, the damping differences between thin and thick plates increase significantly as the ratio of E_{∞}/E_{eq} decreases; in other words, when this ratio is 0.8, the ratio of loss factor belonging to the thin plate to the thick plate is 1.22, but this ratio is more than twice when it comes to an example in which a equals to 0.2.As was expected, as the ratio of E_{∞}/E_{eq} decreases, the damping in the both thin and thick plates increases. In general, the frequency of the thin plates has been less than those of the thick plates, and that the oscillations of thin plates have been damped in less number of cycles. For the physical explanation of this phenomenon can refer to the definition of Delayed Stiffness in viscoelastic materials. Young's modulus of viscoelastic materials is composed of two sections of stable and transient Young's modulus. The combination of these two modules is called material equivalent Young’s modulus. The Delayed Stiffness coefficient is basically the ratio of transient Young’s modulus to Young modulus equivalent in the viscoelastic material. With the passage of time

when the energy in the material is damped, the mentioned coefficient increases, and hence, the final and long term material Young's modulus decreases. Therefore, in the same proportion, the amount damping belonging to the material increases. Another noteworthy point is the uniform behavior of the upper and lower surfaces of the plate. Both surfaces of thin and thick sandwich plates demonstrate exactly similar behavior, due to the existence of Young's modulus with a viscoelastic core, which makes the behavior of the sandwich plate uniform even if it is thick. Despite having a high damping, when it comes to models in which the core is $(E_\infty/E_{eq}=0.8)$, the behavior of the upper and lower surfaces are the same and are quite similar and their displacement is uniformly depreciated. This is unlike many examples of thick sandwich plates, especially those with soft cores. To better compare the effect of this parameter on different cases, the results of all three cases of thick and thin sandwich plates are illustrated in Figure 7 along with each other. By comparing these three cases with one another, it can be understood that the decrease in Delayed Stiffness has a direct effect on the increase in damping, and the changes in E_∞/E_{eq} have a significant effect on the dynamic behavior of the structure, which, in turn, indicates the importance of the investigating this parameter, as far as the dynamic analysis of such plates is concerned.

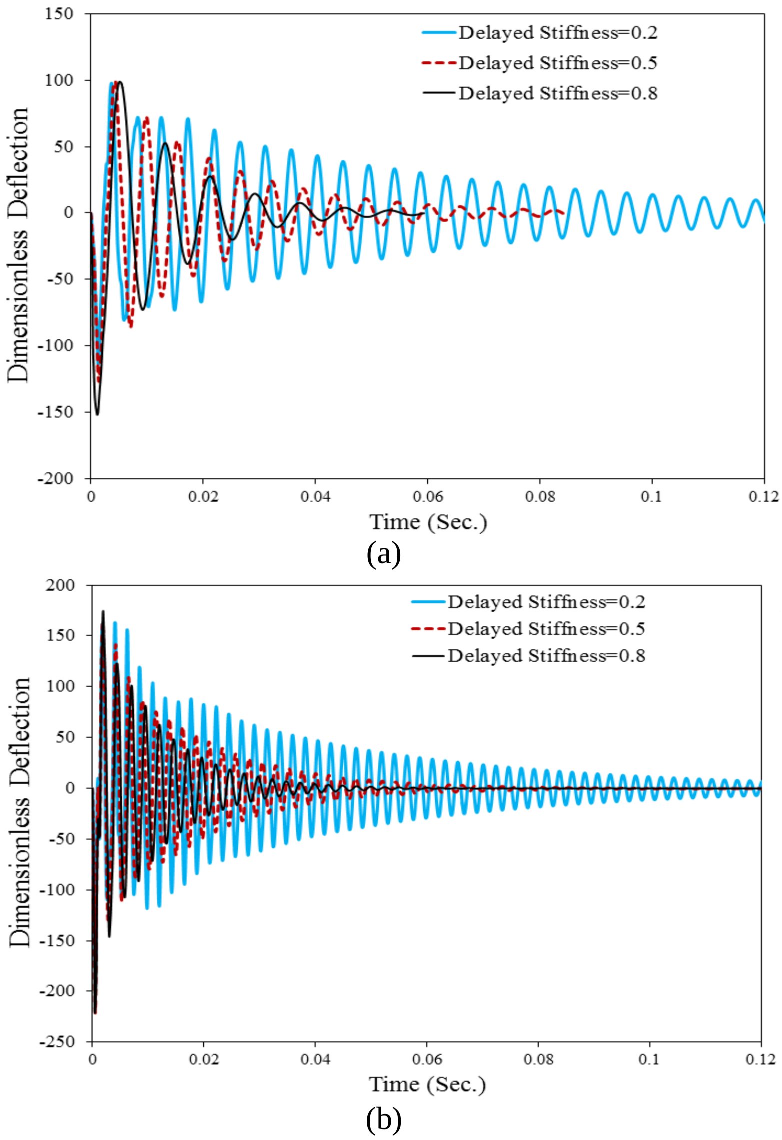


Figure 7. The effect of the ratio of E_∞/E_{eq} with a viscoelastic core on dynamic behavior : a) thin sandwich plate (a / h = 10), b) thick sandwich plate (a / h = 4)

Another important parameter which affects the behavior of viscoelastic materials is its relaxation time $(\tau = (\eta / (E))$. In addition to having a direct relationship with the damping of the structure as well as the parameters of damping elements belonging to viscoelastic, the relaxation time is also associated with the response velocity of the viscoelastic material. In order to better evaluate the effect of this parameter, both thin and thick sandwich plates with τ equaling to 0.0001, 0.003 and 0.1 were analyzed, and the results were then compared with each other. In all three cases, other properties of the viscoelastic core have been considered in accordance with Table 3. Figure 8 depicts the behavior of the thin and thick sandwich plate, in which the relaxation time of the viscoelastic core has changed. As can be seen, for a case in which the relaxation time of the viscoelastic core is equal to 0.003, the oscillating energy of both structures is depreciated up to 0.08 seconds. But the number of oscillations up to the full depreciation of the transverse displacement oscillation of the middle point of the plate for a thick plate is much more than that of a thin plate. A comparison of the results presented in figure 8 indicates as τ increases, the plate damping decreases. The effect of this decrease is in a way that it not only increases the number of oscillation cycles but also increases the time required to dampen plate oscillations. This applies to both thick and thin plates.

In order to carry out a better and more precise investigation of this issue, the loss factors have been calculated for the above examples and are depicted in Table 5. As was expected, as τ increases, the loss factors decrease. When it comes to thin plates, this decrease in loss factors is more significant than thick sandwich plates. As τ decreases, the damping time increase. It, therefore, takes more time to carry out the analyses. For example, for a thin sandwich plate with a viscoelastic core in which τ equals to 0.0001, displacement oscillations are almost completely depreciated and disappeared up to 0.04 seconds, but for the same plate in which τ equals to 0.1, the analysis continued up to 0.4 seconds, and displacements have not been completely dampened yet.

Table 5. Loss factors for thin and thick sandwich plates with the changes in the relaxation time

τ	***Loss factor***	
	thin sandwich plates	***thick sandwich plates***
0.0001	84	34.2
0.003	36.5	21.1
0.1	25.9	12.2

The reason behind this is the definition of the relaxation time of the viscoelastic material. As already mentioned, $\tau = (\eta / (E))$. In fact, with the increase in the relaxation time, the depreciative parameter of the damping element in the Maxwell model is much bigger, which makes the role of dashpot more significant when it comes to the standard linear model. On the other hand, given the definition of Prony series, the Young's modulus of viscoelastic material is equal to:

$$E(t) = E_2 + E_1 e^{-t/\tau_r} \tag{45}$$

From the relationship between the changes in Young's modulus and the relaxation time, it can be understood that when $t = \tau$, material equivalent Young’s modulus is equal to E_{eq}/e which has a direct effect on the amount of displacement and stiffness. Thus, as the relaxation time of the viscoelastic core increases, the amount of time required to reduce material equivalent Young’s modulus increases, so it takes more time for the core to reach its long term modulus.

Therefore, the damping time of the structure also increases as the relaxation time of the viscoelastic core increases.

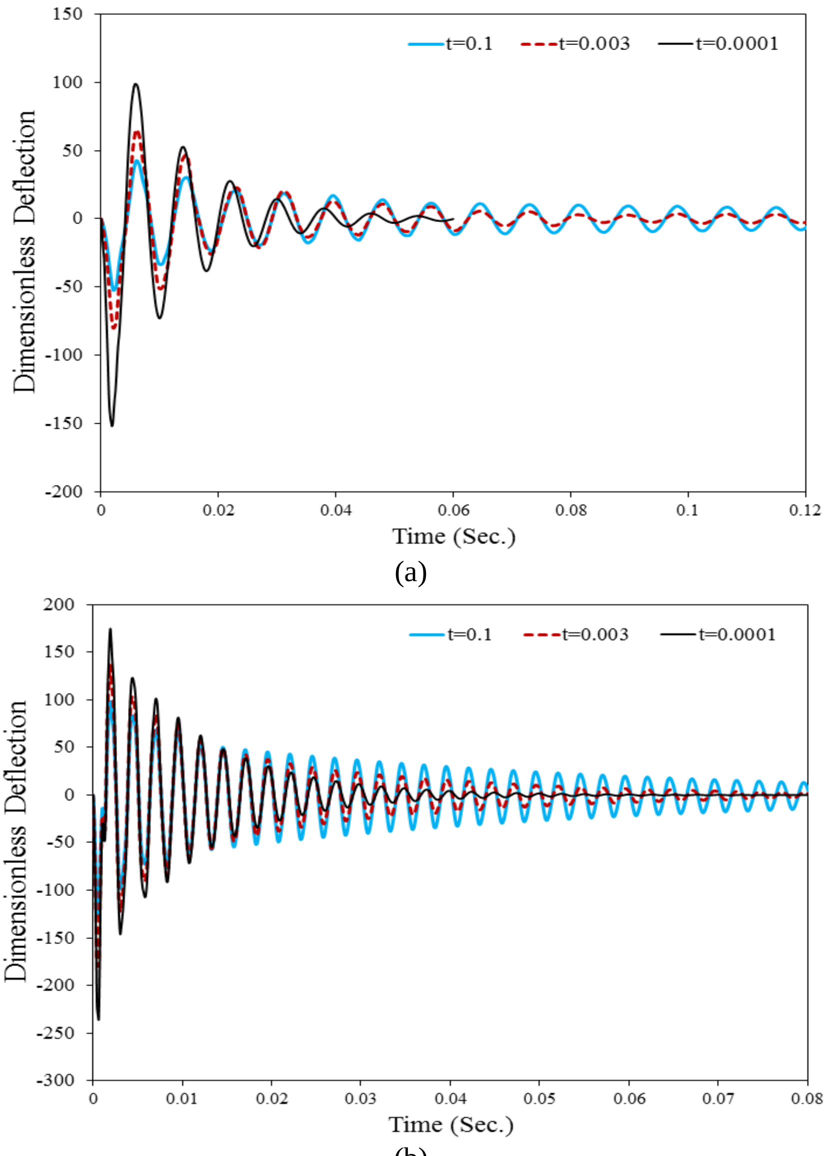
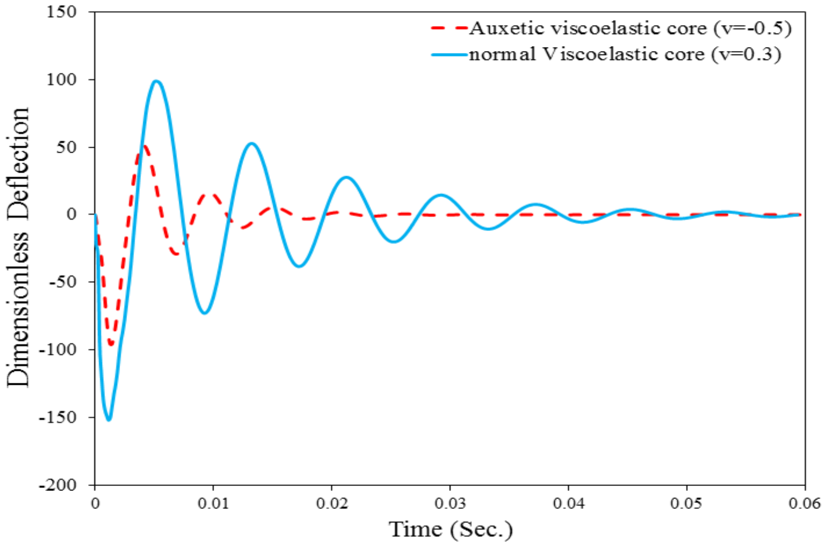


Figure 8: The effect of τ ratio of viscoelastic core on dynamic behavior of sandwich plate: a) Thin sandwich plate (a / h = 10); b) thick sandwich plate (a / h = 4).

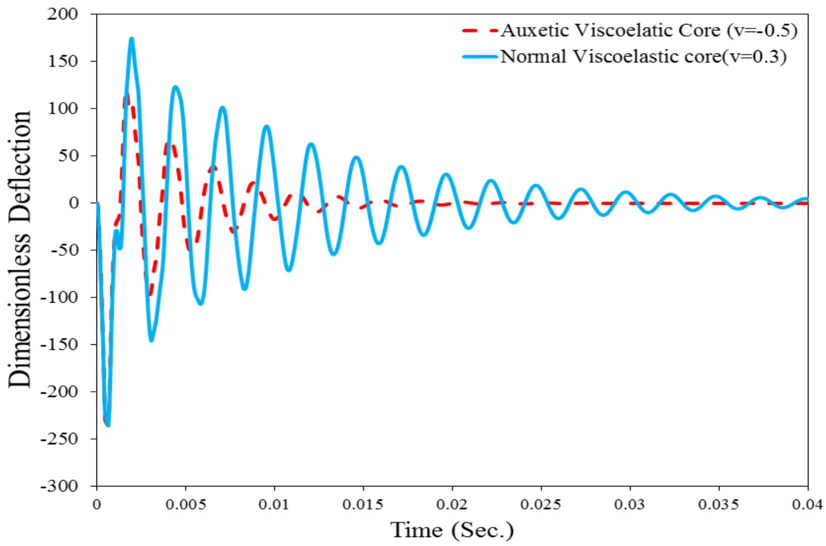
Dynamic Analysis of Thin and Thick Sandwich Plates with Auxetic Viscoelastic Core

The main reason for the attractiveness of auxetic materials is their different behavior during deformation. Auxetic materials are rarely found in nature, while in recent years they have been made in various forms, such as honeycombs, foams and porous materials, and so on. The first attempts to construct foams were made by Lake in 1987. One of the innovations in the current paper is the application of auxetic viscoelastic foam to make sandwich sheets. Hence, this section presents the dynamic behavior of a sandwich plate with auxetic viscoelastic core in both cases of thin and thick. In this case, the core, besides having the viscoelastic properties, is also auxetic. Figure 9 depicts the behavior of thin and thick sandwich plates with an auxetic viscoelastic core. In these examples, the properties mentioned in Table 1, derived from reference [49], have been used to define the viscoelastic core parameters. The difference is that the Poisson's ratio of the core, like auxetic materials, is considered to be "-0.5". Figure 9

illustrates the results of the analysis of thin and thick plates with an auxetic viscoelastic core and a normal viscoelastic core. By comparing these figures, it can be understood that the auxeticity of the viscoelastic core has greatly increased the damping of the plate. This applies to both thick and thin sandwich plates. In other words, the damping time of thin plates decreases from 0.05 seconds to 0.025 seconds and when it comes to the thick sandwich plates, it also decreased from 0.04 seconds to 0.018 seconds. It should be mentioned that the effect of the auxetic core on the damping of a sandwich plate with a viscoelastic core is completely the opposite of a sandwich plate with surfaces reinforced with SMA wires [27, 44].



(a)



(b)

Figure 9: dimensionless transverse displacement of the center of a sandwich plate with an auxetic viscoelastic core: a) Thin sandwich plate (a / h = 10); b) thick sandwich plate (a / h = 4).

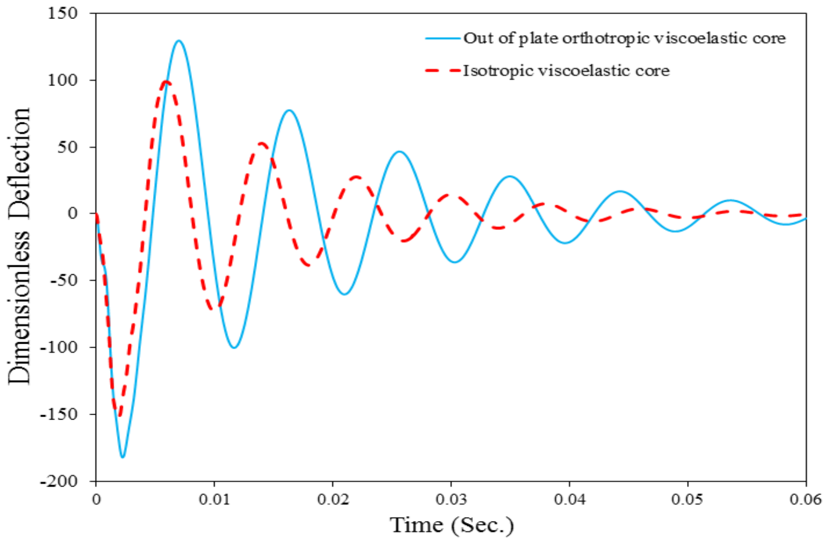
In the examples of surfaces reinforced with SMA, the auxetic core decreases the damping of the plate [27, 44] while in the examples of the plates with a viscoelastic core, the auxetic core increases the damping of the plate. When it comes to samples in which sandwich plates have a damping property due to the existence of SMA wires on the surfaces, the damping property is produced in the plate due to the changes in the phases of SMA wires. The phase changes in the

wires and the formation of the hysteresis cycle due to this phase change are directly related to the stress level of the wires and, consequently, the stress level of the sandwich face sheets. Since making the plate core auxetic decreases the amplitude of the oscillation of the surfaces, and consequently the stress level of the surfaces and consequently the stress of SMA wires, the damping of the whole sandwich plate decreases due to making the core auxetic. On the other hand, compared to the examples belonging to SMA, in this examples, damping is not directly related to the stress level of the surface or the core, due to the difference existing between SMW and auxetic viscoelastic in the nature of the damping and its origin. On the other hand, when it comes to sandwich plates with embedded SMA wires in facesheets, in general, the auxetic property neutralizes the upper surface as well as the load imposed on the structure and hence, reduces the whole vibration domain of the plate [44], while in the sandwich plates with viscoelastic core, the auxetic property of the core not only reduces the oscillation amplitude, but also increases the damping and depreciates this vibration as soon as possible.

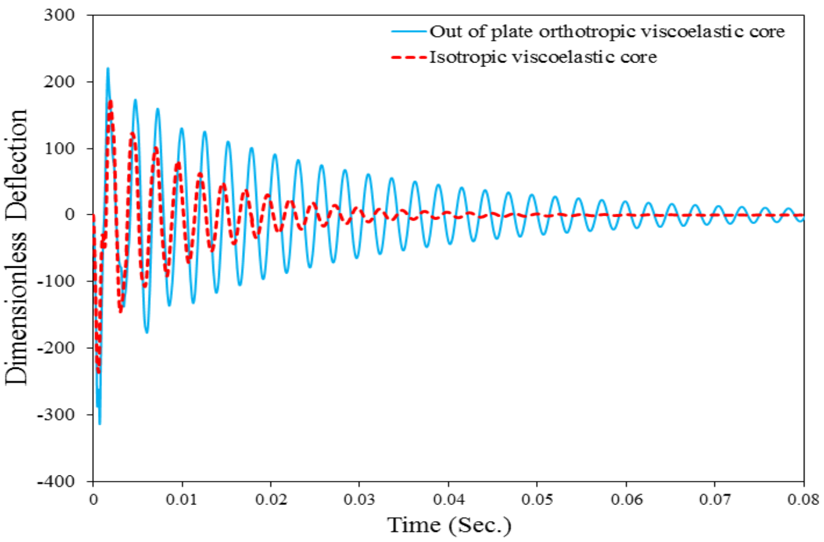
Dynamic Analysis of Thin and Thick Sandwich Plates with Orthotropic Viscoelastic Core

Another important parameter is the consideration of the orthotropic behavior of the core and its effect on the dynamic behavior of the structure. Figure 10 shows the behavior of thin and thick sandwich plates with an orthopedic viscoelastic core compared to a normal viscoelastic core. In this example, it is assumed that the transverse modulus of the core is 2 / 3 of in-plane Young modulus of it. In most of the foams employed to make sandwich plates, the difference between transverse properties and in-plane properties have been observed and reported [50]; hence, the study of this parameter as well as the simulation of orthotropic viscoelastic behavior is absolutely crucial when it comes to the dynamic analysis of different sandwich plates.

As the core becomes out of plate orthotropic, the oscillation amplitude has increased to some extent in both thick and thin sandwich plates. On the other hand, the damping of the structure has also reduced and the plate needs more time to eliminate the effect of the applied load. These figures indicate the importance of considering the transverse orthotropic behavior of sandwich plates with an orthotropic viscoelastic core, because the consideration of orthotropic viscoelastic behavior generally affects the dynamic response of the plate; therefore, in order to obtain a proper judgment of the behavior of such structures, it is necessary to pay attention to the orthotropic effects of the core (either in-plate orthotropic or out of plate orthotropic). It should be noted, however, that the effect of the orthotropic nature of the core on a thick sandwich plate is much greater than that of a thin sandwich plate, and when it comes to thick plates, the damping of the structure has been affected more.



(a)



(b)

Figure 10. The effect of the transversal orthotropic nature of the viscoelastic core compared to the isotropic viscoelastic core on the dimensionless transverse displacement of the middle point of the plate: a) thin sandwich plate (a / h = 10); b) thick sandwich plate (a / h = 4)

Conclusions

In this research, a nonlinear dynamic analysis of thin and thick sandwich structures with a core of viscoelastic, auxetic viscoelastic and orthotropic viscoelastic has been performed. In order to consider the effect of the viscoelastic core, a standard linear model has been employed, which is one of the most suitable models for the analysis of solid viscoelastic materials. This has been carried out through real time method. Finally, in addition to proposing an algorithm of integro-differential-equation, the obtained equations have been solved through using the two-dimensional finite element method and with the help of Picard's iterative incremental method, along with the Newark method. As was mentioned before, since in this paper we face an iterative transient finite element for thick and thin orthotropic sandwich plates with a deformable core, it is necessary to use a theory that not only does it benefit from a high accuracy, but also has a low computation volume, so that if a lot of repetitions are needed to converge the equation, it will not take unreasonable amount of time. Hence, it is impossible and impractical to employ three dimensional finite element method or even two-dimensional

methods with high variables for the analysis of such an equation. For this reason, a higher order global-local theory is applied to analyze the above mentioned equation. The advantage of using higher order global-local theory for such an equation is its ability to investigate the various types of inter-laminar as well as inner layer changes with a high precision (due to the existence of local field belonging to each layer). Another feature of this theory is the independence of the variables of the equation from the number of layers, which increases the amount of time spent on solving them; as a result, it is widely applied in various analyses. Given the thickness of the sandwich plate and the flexibility and the viscoelasticity of the core, solving the equation - while assuming the same changes in W, and not considering changes in the thickness of the sandwich plate - will result in incorrect results. That is why in this paper, the changes in the thickness of the core have been carefully modeled and investigated through using parabolic functions, which is another feature of the current research. Some of the most important results obtained from this study are as follows:

- ✓ When it comes to determining the transverse shear stresses of the sandwich plates, in particular thick sandwich plates with a soft core, a three-dimensional elasticity correction method along with higher order global-local theory are an efficient method with a high accuracy; it not only meets the continuity condition of variations in transverse stress between layers, but also evaluates these variations with a very good precision.
- ✓ The four times increase in the ratio of the long term Young's modulus of viscoelastic core to its equivalent modulus has increased the loss factor of the sandwich plate to 6.6 for the thin plates and 3.2 for the thick ones.
- ✓ Increasing the relaxation time of the viscoelastic core from 0.0001 to 0.1 leads to an increase in the loss factor of the plate to 0.3 in both thick and thin plates. It also increases more than ten times the time needed to depreciate the whole oscillations of the plate.
- ✓ The auxetic nature of the viscoelastic core increases the damping of the sandwich plates significantly. This also applies to thick sandwich plates and thin sandwich plates, in a way that the damping time of the thin plate decreases from 0.05 seconds to 0.025 seconds and in terms of thick sandwich plates, the damping time also decreases from 0.04 seconds to 0.018 seconds.
- ✓ As the viscoelastic core becomes out of plate orthotropic, in both thick and thin sandwich plates, the oscillation amplitude increases remarkably. On the other hand, the damping of the structure is also reduced and the plate needs more time to eliminate the effect of the imposed load.
- ✓ The auxetic nature of the core increases the load-bearing capacity of the structure.
- ✓ The damping effect of a viscoelastic core starts from the beginning of the loading, and continues to the moment of absorbing the entire energy of the plate, regardless of the stress level of the structure and plate geometry as well as the load imposed on the plate.
- ✓ The existence of an auxetic core increases the frequency of the sandwich plate.

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