JCAMECH

Vol. 51, No. 2, December 2020, pp 411-416
DOI: 10.22059/jcamech.2020.309019.548

Geometrically nonlinear analysis of axially functionally graded beams by using finite element method

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ARTICLE INFO

Article history:
Received: 20 August 2020
Accepted: 03 September 2020

Keywords:
Axially Functionally Graded Beams
Geometrically nonlinear Analysis
Finite Element Method
Total Lagrangian

ABSTRACT

The aim of this paper is to investigate geometrically nonlinear static analysis of an axially functionally graded cantilever beam subjected to transversal load. The considered problem is solved by finite element method with total Lagrangian kinematic approach. The material properties of the beam vary along the longitudinal direction according to the power law function. The finite element model of the beam is considered in the three dimensional continuum approximation for an eight-node quadratic element. The geometrically nonlinear problem is solved by Newton-Raphson iteration method. In the numerical results, the effects of the material distribution on the geometrically nonlinear static displacements of the axially functionally graded beam are investigated. Also, the differences among of material distributions are investigated in geometrically analysis.

1. Introduction

Functionally graded materials (FGM) are special composites whose properties change gradually though direction. In generally, functionally graded materials consist of a mixture of ceramic and metal materials. In the last years, the functionally graded materials have been found in many engineering applications, such as aircrafts, space vehicles and biomedical sectors.

By increasing functionally graded structures, many researchers investigated the mechanical behavior of functionally graded structures in last decade. In the literature, some investigations of mechanical behavior of functionally graded and composite structures are as follows; Agarwal et al. [1] presented the geometrically nonlinear static and vibrations of FGM beams. Ke et al. [2] studied the postbuckling behavior of damaged FGM beams. Kang and Li [3] studied the nonlinear deformation of a FGM cantilever beam with considering work hardening of power law. Su et al. [4] presented post-buckling of FGM Timoshenko beams with piezoelectric layers under temperature and electric effects. Kocatürk et al. [5] presented geometrically non-linear static analysis of a FGM beam by using Total Langragian finite element approximation with Timoshenko beam theory. Soleimani and Saadatfar [6] presented large deflection of FGM beams by using shooting method. Anandrao et al. [7] studied nonlinear vibration and buckling of FGM Timoshenko beams by using finite element method. Askari et al. [8] presented nonlinear oscillations of FGM beams. Anandrao et al. [9] analyzed non-linear stress analysis FGM beams by using Euler-Bernoulli beam theory and finite element method. Machado and Piovan [10] analyzed the nonlinear vibrations of FGM beams under thermal and harmonic transverse loads. Akbaş [11] investigated geometrically nonlinear analysis of edge cracked FG Timoshenko beams by using Total Langragian finite element method. Tung and Duc [12] investigated nonlinear results of thick functionally graded doubly curved shallow panels resting on foundations under different type loads. Akbaş [13] investigated post-buckling of axially FGM beams. Nguyen et al. [14] presented the geometrically nonlinear analysis of FGM planar beam and frame structures by using the finite element method. Mohammadi and Rastgoo [15,16] presented the nonlinear vibration analysis of composite nanoplates with functional-graded cores. Mohammadi et al. [17] investigated Nonlinear free and forced vibration behavior of a porous functionally graded Euler-Bernoulli nanobeam subjected to mechanical and electrical loads. Kocatürk and Akbaş [18], Akbaş [19-24] investigated post-buckling responses of functionally graded and composite beams by using

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finite element method within total Langrangian nonlinearity. Wu et al. [25] presented dynamic investigations of axially functionally graded beams by using the semi-inverse method. Huang and Li [26] investigated free vibration of axially functionally non-uniform graded beams. Hein and Feklistova [27] investigated vibration of axially functionally graded beams with different cross-sections and boundary conditions by using the Haar wavelet series. Alshorbgy et al. [28] presented free vibration analysis of of non-uniform axially or transversally graded beams. Eltaher et al. [29] presented free vibration analysis of functionally graded nanobeams based on nonlocal elasticity theory by using finite element method. Shahba et al. [30] and Shahba and Rajasekaran [31] analyzed free vibration and stability of axially functionally graded beams by using finite element method. Akbaş [32-36] presented free vibration analysis of functionally graded beams with different mechanical cases. Farajpour et al. [37,38] investigated buckling analysis of nano composite plates based on nonlocal theories. Şimşek et al. [39] investiated dynamic analysis of axially functionally graded simply supported beam subjected to moving harmonic load. Huang et al. [40] investigated vibration behaviors of axially functionally graded Timoshenko beams with non-uniform cross-section. Rajasekaran [41,42] presented vibration analysis of axially functionally graded tapered and non-uniform beams by using differential transformation and differential quadrature element methods. Akgöz and Civalek [43] presented vibration responses of axially functionally graded tapered microbeams based on modified couple stress theory. Nguyen [44] studied large displacements of tapered an axially functionally graded cantilever beam. Babilio [45] investigated the dynamics of an axially functionally graded simply supported beam under axial time-dependent load. Akbaş [46] presented free vibration of axially functionally graded beams with thermal effects. Mohammadi et al. [47,48] investigated effects of temperature on the vibration of Graphene Sheets resting on foundation. Akgöz and Civalek [49,50] presented static and vibration analyses of functionally graded microbeams by using the nonlocal theory. Akbaş [51-58] investigated forced vibration of functionally graded beams by using finite element method. Ghayesh [59] analyzed forced nonlinear vibration of axially functionally graded micro beams by using coupled stress theory. Liu et al. [60] studied free vibration of axially functionally graded tapered beams by using the spline finite point method. Çalım [61] presented transient analysis of axially functionally graded beams with variable cross-section. Alimoradzadeh et al. [62] investigated nonlinear vibration analysis of axially functionally graded beams under moving harmonic load. Uzun et al. [63] investigated free vibration of functionally graded nanobeams by using finite element method. Cao and Gao [64] investigated free vibration of non-uniform axially functionally graded beams with the asymptotic development method. Barati et al. [65] presented static torsion of bi-directional functionally graded microtube based on the couple stress theory under magnetic field. Sharma et al. [66] presented the modal analysis of an axially functionally graded beam under hygrothermal effect.

In this study, geometrically nonlinear static analysis of an axially functionally graded cantilever beam subjected to transversal load is investigated by using finite element method with total Lagrangian and three dimensional continuum approximation. The nonlinear problem is solved by incremental displacement-based finite element method in conjunction with Newton-Raphson iteration method.

2. Theory and Formulations

In figure 1, a cantilever beam with length L, width b, thickness h under a non-follower transversal point load Q is shown with Lagrangian coordinate system (X,Y,Z) and Euler coordinate system (x,y,z).

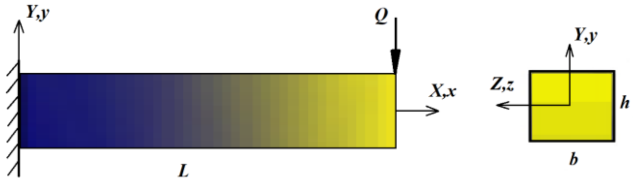

Figure 1. A cantilever axially functionally graded three dimensional beam under a transversal point load (Q).

The material properties (P) of the beam in case of functionally graded material change though longitudinal direction (X) based on following power-law function distribution;

$$P(X) = (P_L - P_R)\left(1-\frac{X}{L}\right)^k + P_R \tag{1}$$

where P_L and P_R are the material properties of the left and the right surfaces of the beam and k is the non-negative power-law exponent which dictates the material variation profile through the axially direction. In Eq. (1), when X=0, $P = P_L$, and when X=L, $P = P_L$. when k=0 material of beam gets homogenous full left side material, and when k=∞ material of beam gets homogenous right material.

In this study, Total Lagrangian finite element equations of three dimensional continuums for an eight-node quadratic element are used for geometrically nonlinear analysis of axially functionally graded three dimensional beams.

The constitutive relation between the second Piola-Kirchhoff stress tensor (S_{ij}) and the Green-Lagrange strain tensor (E_{ij}) can be assumed as follows

$$\{{}_0^1S\} = \begin{Bmatrix} {}_0^1S_{xx} \\ {}_0^1S_{yy} \\ {}_0^1S_{zz} \\ {}_0^1S_{yz} \\ {}_0^1S_{xz} \\ {}_0^1S_{xy} \end{Bmatrix} = \begin{bmatrix} {}_0C_{11} & {}_0C_{12} & {}_0C_{13} & 0 & 0 & 0 \\ {}_0C_{12} & {}_0C_{22} & {}_0C_{23} & 0 & 0 & 0 \\ {}_0C_{13} & {}_0C_{23} & {}_0C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & {}_0C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_0C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & {}_0C_{66} \end{bmatrix} \begin{Bmatrix} {}_0^1E_{xx} \\ {}_0^1E_{yy} \\ {}_0^1E_{zz} \\ 2\,{}_0^1E_{yz} \\ 2\,{}_0^1E_{xz} \\ 2\,{}_0^1E_{xy} \end{Bmatrix} \tag{2}$$

The components of the constitutive tensor can be written in terms of Young's modulus E and Poisson's ratio υ and their dependence on X coordinate are given by Eq. (1) as follows:

$$\begin{aligned} {}_0C_{11} = {}_0C_{22} = {}_0C_{33} &= \frac{E(X)(1-\nu(X))}{(1+\nu(X))(1-2\nu(X))} \\ {}_0C_{12} = {}_0C_{13} = {}_0C_{23} &= \frac{E(X)\nu(X)}{(1+\nu(X))(1-2\nu(X))} \\ {}_0C_{66} = {}_0C_{55} = {}_0C_{44} &= \frac{E(X)}{2(1+\nu(X))} \end{aligned} \tag{3}$$

The Green-Lagrange strain tensor is presented within three-dimensional solid continuum as follows;

$$\{{}_0^1E\}=\begin{Bmatrix} {}_0^1E_{xx} \\ {}_0^1E_{yy} \\ {}_0^1E_{zz} \\ 2{}_0^1E_{yz} \\ 2{}_0^1E_{xz} \\ 2{}_0^1E_{xy} \end{Bmatrix}=\begin{Bmatrix} \frac{\partial u}{\partial X}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial X}\right)^2+\left(\frac{\partial v}{\partial X}\right)^2+\left(\frac{\partial w}{\partial X}\right)^2\right] \\ \frac{\partial v}{\partial Y}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial Y}\right)^2+\left(\frac{\partial v}{\partial Y}\right)^2+\left(\frac{\partial w}{\partial Y}\right)^2\right] \\ \frac{\partial w}{\partial Z}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial Z}\right)^2+\left(\frac{\partial v}{\partial Z}\right)^2+\left(\frac{\partial w}{\partial Z}\right)^2\right] \\ \frac{\partial v}{\partial Z}+\frac{\partial w}{\partial Y}+\left(\frac{\partial u}{\partial Y}\frac{\partial u}{\partial Z}+\frac{\partial v}{\partial Y}\frac{\partial v}{\partial Z}+\frac{\partial w}{\partial Y}\frac{\partial w}{\partial Z}\right) \\ \frac{\partial u}{\partial Z}+\frac{\partial w}{\partial X}+\left(\frac{\partial u}{\partial X}\frac{\partial u}{\partial Z}+\frac{\partial v}{\partial X}\frac{\partial v}{\partial Z}+\frac{\partial w}{\partial X}\frac{\partial w}{\partial Z}\right) \\ \frac{\partial u}{\partial Y}+\frac{\partial v}{\partial X}+\left(\frac{\partial u}{\partial X}\frac{\partial u}{\partial Y}+\frac{\partial v}{\partial X}\frac{\partial v}{\partial Y}+\frac{\partial w}{\partial X}\frac{\partial w}{\partial Y}\right) \end{Bmatrix} \quad (4)$$

where the displacement fields of the finite element are expressed in terms of nodal displacements as follows:

$$u=(\psi_1\cdot u_1+\psi_2\cdot u_2+\psi_3\cdot u_3+\psi_4\cdot u_4+\psi_5\cdot u_5+\psi_6\cdot u_6+\psi_7\cdot u_7+\psi_8\cdot u_8) \quad (5a)$$

$$v=(\psi_1\cdot v_1+\psi_2\cdot v_2+\psi_3\cdot v_3+\psi_4\cdot v_4+\psi_5\cdot v_5+\psi_6\cdot v_6+\psi_7\cdot v_7+\psi_8\cdot v_8) \quad (5b)$$

$$w=(\psi_1\cdot w_1+\psi_2\cdot w_2+\psi_3\cdot w_3+\psi_4\cdot w_4+\psi_5\cdot w_5+\psi_6\cdot w_6+\psi_7\cdot w_7+\psi_8\cdot w_8) \quad (5c)$$

These total and incremental displacement fields are presented as follows:

$$u=\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}=\begin{Bmatrix} \sum_{j=1}^{8} u_j\psi_j \quad X,Y,Z \\ \sum_{j=1}^{8} v_j\psi_j \quad X,Y,Z \\ \sum_{j=1}^{8} w_j\psi_j \quad X,Y,Z \end{Bmatrix}=\Psi \quad \Delta \quad (6)$$

$$\bar{u}=\begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix}=\begin{Bmatrix} \sum_{j=1}^{8} \bar{u}_j\psi_j \quad X,Y,Z \\ \sum_{j=1}^{8} \bar{v}_j\psi_j \quad X,Y,Z \\ \sum_{j=1}^{8} \bar{w}_j\psi_j \quad X,Y,Z \end{Bmatrix}=\Psi \quad du \quad (7)$$

where

$$[\psi]=\begin{bmatrix} \psi_1 & 0 & \psi_2 & 0 & \psi_3 & 0 & \psi_4 & 0 & \psi_5 & 0 & \psi_6 & 0 & \psi_7 & 0 & \psi_8 & 0 \\ 0 & \psi_1 & 0 & \psi_2 & 0 & \psi_3 & 0 & \psi_4 & 0 & \psi_5 & 0 & \psi_6 & 0 & \psi_7 & 0 & \psi_8 \end{bmatrix} \quad (8a)$$

$$\{\Delta\}^T=\{u_1\ v_1\ u_2\ v_2\ u_3\ v_3\ u_4\ v_4\ u_5\ v_5\ u_6\ v_6\ u_7\ v_7\ u_8\ v_8\} \quad (8b)$$

$$\{du\}^T=\{\bar{u}_1\ \bar{v}_1\ \bar{u}_2\ \bar{v}_2\ \bar{u}_3\ \bar{v}_3\ \bar{u}_4\ \bar{v}_4\ \bar{u}_5\ \bar{v}_5\ \bar{u}_6\ \bar{v}_6\ \bar{u}_7\ \bar{v}_7\ \bar{u}_8\ \bar{v}_8\} \quad (8c)$$

where ψ_i are the shape functions (Akbaş [13]). Eight-node three dimensional finite element is displayed in figure 2.

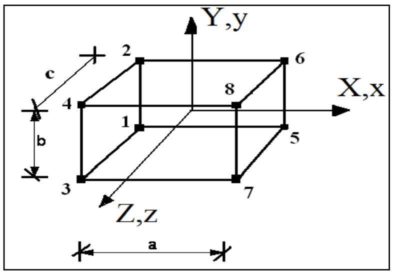

Figure 2. Eight-node three dimensional finite element.

The nonlinear finite element equation of the total Lagrangian finite element model of three dimensional continua for an eight-node quadratic element is presented as follows (Akbaş [13]):

$$\begin{bmatrix} [K^{11L}]+[K^{11N}] & [K^{12L}] & [K^{13L}] \\ [K^{21L}] & [K^{22L}]+[K^{22N}] & [K^{23L}] \\ [K^{21L}] & [K^{32L}] & [K^{33L}]+[K^{33N}] \end{bmatrix}\begin{Bmatrix} \{\bar{u}\} \\ \{\bar{v}\} \\ \{\bar{w}\} \end{Bmatrix}=\begin{Bmatrix} \{{}_0^2F^1\} \\ \{{}_0^2F^2\} \\ \{{}_0^2F^3\} \end{Bmatrix}-\begin{Bmatrix} \{{}_0^1F^1\} \\ \{{}_0^1F^2\} \\ \{{}_0^1F^3\} \end{Bmatrix} \quad (9)$$

where K^{ijL} and K^{ijN} are the components of linear and nonlinear for tangent stiffness matrix. $\bar{u}$, $\bar{v}$, $\bar{w}$ are incremental dispşacements vector and F^i is the load vector. The detail expressions of these martix and vectors can be read in Akbaş [13].

In the solution of nonlinear equations of the problem, Newton-Raphson iteration method is implemented and for *i* th iteration and *n+1* th load increment, the solution form is presented as follows;

$$d\,\mathbf{u}_n^i=(\mathbf{K}_T^i)^{-1}\mathbf{R}_{n+1}^i \quad (10)$$

where K_T^i, R_{n+1}^i and du_n^i are tangent stiffness matrix, residual vector and solution increment vector, respectively. The iteration limit of eq. (10) is selected as following form;

$$\sqrt{\frac{\left[(d\,\mathbf{u}_n^{i+1}-d\,\mathbf{u}_n^i)^T(d\,\mathbf{u}_n^{i+1}-d\,\mathbf{u}_n^i)\right]^2}{\left[(d\,\mathbf{u}_n^{i+1})^T(d\,\mathbf{u}_n^{i+1})\right]^2}}\leq\zeta_{tol} \quad (11)$$

where

$$\mathbf{u}_{n+1}^{i+1}=\mathbf{u}_{n+1}^i+d\mathbf{u}_{n+1}^i=\mathbf{u}_n+\Delta\mathbf{u}_n^i \quad (12a)$$

$$\Delta\mathbf{u}_n^i=\sum_{k=1}^{i}\mathbf{du}_n^k \quad (12b)$$

3. Findings and Discussion

In this section, geometrically nonlinear static deflections and configurations are obtained with different values of the material gradient parameters. The material parameters of these materials are given as follows; at the left side is fully Zirconia (*E*=151 GPa, *ν*=0.2882) and at the right side is fully Aluminum Oxide (*E*70 GPa, *ν*=0.31). The geometry properties of the beam are selected as b = 0.1 m, *h*=0.1 m and *L*= 3m. The number of finite elements are taken as 200 elements in X direction and 10 elements in both Y and Z directions.

The effect of the material distributions on the geometrically nonlinear static displacements of the axially functionally graded beam is presented in figures 3 and 4. In figure 3, the load – the vertical displacements (at the free end of the beam) curves are plotted for different values of the power-law exponents (*k*). In figure 4, the load – the power-law exponent (k) relation is plotted for Q= 400 kN.

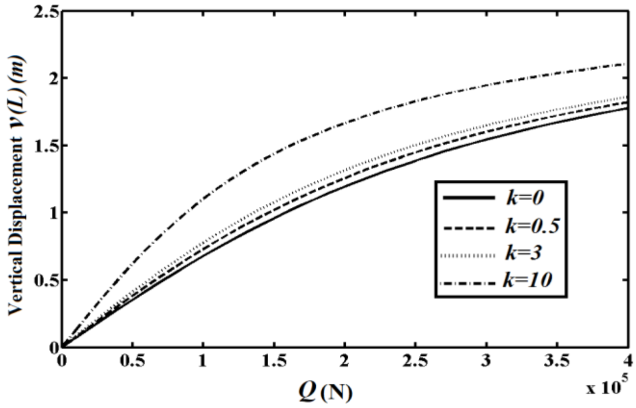

Figure 3. Load- transversal displacement curves for different values of the power-law exponent *k*.

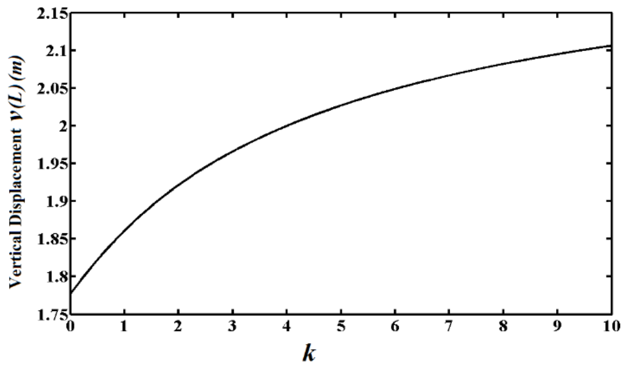

Figure 4. The power-law exponent *k* - transversal displacement curves for different values of the non-follower point load *Q*=400Kn.

It is seen from figures 3 and 4 that increasing in the material power law index *k* causes increase in the vertical deflections for all values of the load (*Q*): Because when the material power law index *k* increase, the material of the beam get close to Aluminum Oxide (right side material) according to Eq. 1 and it is known from the physical properties of the Aluminum Oxide and Zirconia that the Young modulus of Zirconia is approximately two times greater than that of Aluminum Oxide. As a result, the strength of the material decreases. Also, it is seen from Fig. 6 that increase in the material power law index *k*, the curve has an asymptote. In the case of $k=\infty$, the functionally graded material beam is reduced to the homogeneous Zirconia (left side material) beam according to Eq. 1.

In figures 5-8, the effects of the material power law index *k* on the geometrically nonlinear static configuration of the axially functionally graded beam are shown for the non-follower point load *P*=500k*N*.

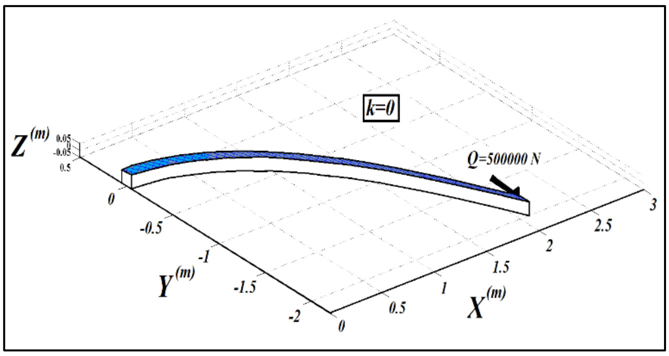

Figure 5. Geometrically nonlinear static deflection configuration of the axially functionally graded beam for *k*=0.

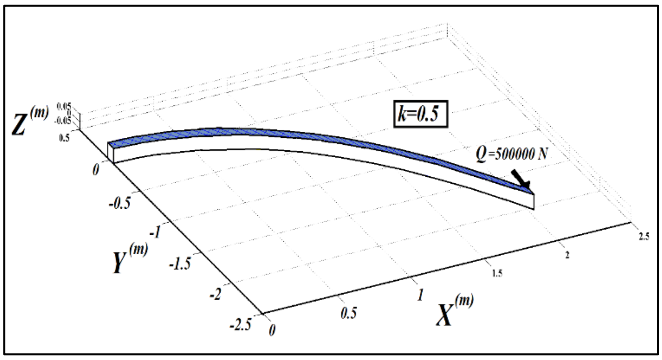

Figure 6. Geometrically nonlinear static deflection configuration of the axially functionally graded beam for *k*=0.5.

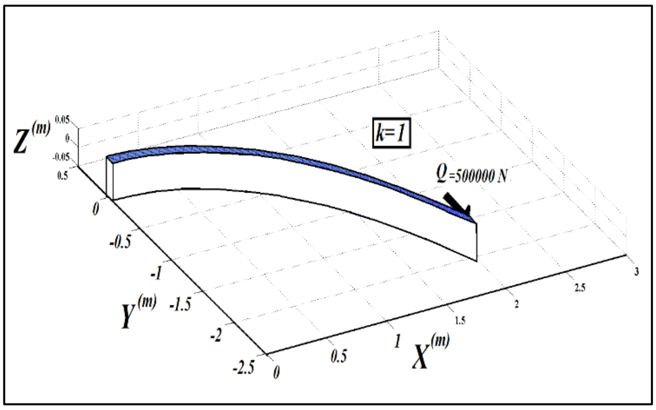

Figure 7. Geometrically nonlinear static deflection configuration of the axially functionally graded beam for *k*=1.

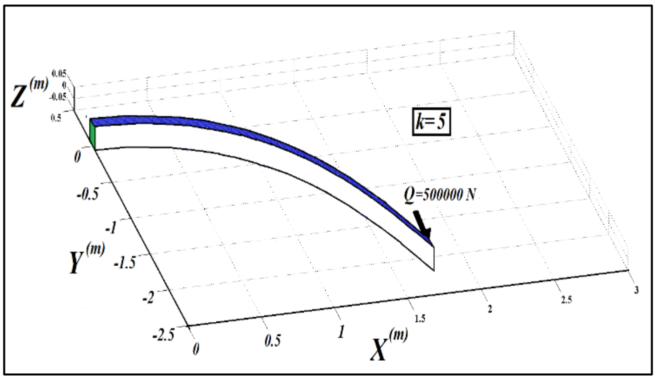

Figure 8. Geometrically nonlinear static deflection configuration of the axially functionally graded beam for *k*=5.

It is seen from figures 5-8 that displacements increase as the power-law exponent (k) increases. This is because as seen from Fig. 2, increase in the power-law exponent (k) leads to decrease in the elasticity modulus and the bending rigidity.

4. Conclusions

Geometrically nonlinear static analysis of an axially functionally graded cantilever beam subjected to a point load are investigated by using the total Lagrangian finite element model of three-dimensional continuum model. The formulations of the geometrically nonlinear analysis of the axially functionally graded beam are derived for total Lagrangian finite element model of three-dimensional continuum. The material properties of the beam vary along the longitudinal direction according to the power law function. In the numerical results, effects of material gradient parameter on the geometrically nonlinear static responses of the axially functionally graded beam are investigated. It is observed from the results that the power-law exponent *k* plays very important role on the responses of the geometrically nonlinear behaviour of the axially functionally graded beam.

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