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Nonlocal elasticity theory for static torsion of the bi-directional functionally graded microtube under magnetic field

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ABSTRACT

The microtubes are important structures in nanoelectromechanical systems. In this study a nonlocal model is presented to investigate the static torsion behavior of microtubes made of bi-directional factionally graded material (BDFGM) subjected to a longitudinal magnetic field. Mechanical properties of BDFGM microtube varies in the radial and longitudinal direction according to an arbitrary function. The governing equation is obtained using the principle of minimum potential energy. a sinusoidal distributed torque and uniform magnetic field with clamped boundary condition are considered to capture the effects of nonlocal parameter, FGM indexes and intensity of longitudinal magnetic field on the torsional angle of BDFGM microtube. The numerical solution of generalized differential quadrature (GDQ) is compared with Galerkin method which a reasonable agreement is observed. Result indicates that intensity of the longitudinal magnetic field has important role on the torsional angle of microtubes such that when intensity of longitudinal magnetic field increases the torsional angle of microtubes decreases.

1. Introduction

In recent years, microstructures have been widely used in various industries such as computer industry and medical devices because of their high efficiency. Microtubes have attracted much attention by researchers [1, 2]. The microtubes are microstructures that are extensively used in fabricating microelectromechanical systems (MEMSs) [3]. Therefor study of their mechanical behavior is very necessary. Experimental data showed effect of size on mechanical properties in micro scale so that size effect in microtubes cannot be ignored [4-8]. On the other hand classical continuum mechanics cannot able to consider influence of size therefore to predict mechanical behavior of micro and nano structures some of new theories have been developed such as nonlocal elasticity theory[9-11], couple stress theory [12], strain gradient theory [13] and etc. Nonlocal elasticity theory has received much attention from scientists. Based on nonlocal elasticity theory the stress at a reference point is considered to be a function of the strain at every point in the body. There are numerus studies related to size dependent theory [4-7, 14-30].

Scientists have been always looking to improve the properties of materials. Functionally graded materials (FGM) are advanced composite materials in which material properties continually vary along one or more directions [31-36]. FGMs has more advantages than traditional composites such as improved residual stress distribution, higher fracture toughness and reduced stress intensity factors. Related to FGMs a number of articles have been published [37-39].

Magnetic field effect is important in mechanical study of micro structures such as accelerometers and gyroscopes. Recently the research interest has grown on studying the magnetic properties of microtubes and mechanical behaviour of microtubes subjected to an external magnetic field. Arani et al [40] present an investigation about vibration of double-walled carbon nanotubes (DWCNTs) conveying fluid placed in uniform magnetic field based on nonlocal elasticity theory. Murmu et al [41] studied the influence of a transverse magnetic field on the axial vibration of nanorods based on nonlocal elasticity approach. An nonlinear study was performed by [42] to investigate the vibration of the single-walled carbon nanotubes (SWCNTs) under longitudinal magnetic field. magnetoelastic model of a pinned beam with the thermal loads was performed by Wu [43] to analysis of dynamic instability and vibration motions of a pinned beam. Influences of longitudinal magnetic field on wave propagation in carbon nanotubes embedded in elastic matrix were reported by Wang et al. [44]. Narendar et al. [45] presented the effect of longitudinal magnetic field on wave dispersion characteristics of equivalent continuum structure (ECS) of single-walled carbon nanotubes (SWCNT) embedded in elastic medium using nonlocal Euler–Bernoulli beam theory. Li [46] presented a new analytical approach to investigate torsion of cylindrical nanostructures using nonlocal elasticity theory. Nonlocal scale effect is modeled in nano-beams under torsion by Barretta et al. [47]. In addition Barretta et al performed an exact solution to investigate torsion of functionally graded viscoelastic circular nanobeams based on nonlocal elasticity theory. Murmu et al [48] developed an analytical model for

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studying the effects of a longitudinal magnetic field on the vibration of a magnetically sensitive double-walled carbon nanotube system. Chang [49] performed the nonlinear free vibration analysis of nanobeams under longitudinal magnetic field based on Eringen's nonlocal elasticity theory.

To the best of authors' knowledge, it is obvious that there are strong scientific requirements to develop a good analytic model for the static torsion analysis of BDFG nano-rod under magnetic field. In this study, minimum potential energy is used to drive equation of motion. The effects of some parameters like magnetic field, size scale parameter and inhomogeneity constant are investigated utilizing the GDQ method in detail.

2. Formulation

Schematic of BDFGM microtube under a longitudinal magnetic field is shown in the Fig1.

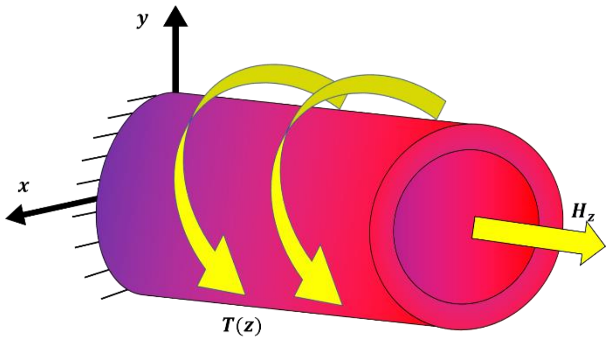

Figure 1. Schematic of microtube made of bi-directional functionally graded material subjected to the longitudinal magnetic field and distributed torque

The material properties of BDFGM microtube varies along longitudinal and radial direction according to

$$G(r)=(G_i-G_o)\left(\frac{r_o-r}{r_o-r_i}\right)^{\beta_G}+G_o \tag{1a}$$

$$G(z)=e^{\alpha_G \frac{z}{L}} \tag{1b}$$

$$\eta(r)=(\eta_i-\eta_o)\left(\frac{r_o-r}{r_o-r_i}\right)^{\beta_\eta}+\eta_o \tag{1c}$$

$$\eta(z)=e^{\alpha_\eta \frac{z}{L}} \tag{1d}$$

where $G(r)$ and $G(z)$ are the shear module in the radial and the longitudinal direction respectively. $\eta(r)$ and $\eta(z)$ are magnetic permeability in the radial and the longitudinal direction respectively. G_i and G_o is the shear modulus in the inner radius and the outer radius of the BDFGM microtube, respectively. η_i and η_o are magnetic permeability coefficients in the inner radius and the outer radius of the BDFGM microtube, respectively. The distribution of BDFGM microtube material properties in the radial and longitudinal direction has been illustrated in Figure 2 and 3 respectively. It is observed that the selection of various quantities for α or β indexes, various distributions of mechanical properties in the radial and longitudinal directions are obtained, respectively.

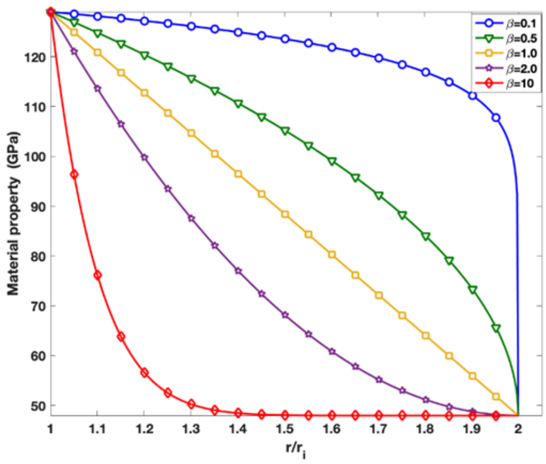

Figure 2. Variation of material properties in the radial direction for different values β

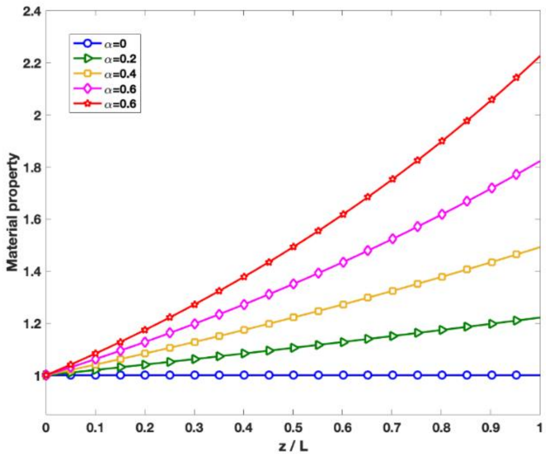

Figure 3. Variation of material properties in the longitudinal direction for different values α

The displacement field for torsion of microtube expressed as

$$\vec{u}=\vec{\varphi}\times\vec{r}=\begin{Bmatrix} u_1=-y\varphi(z) \\ u_2=x\varphi(z) \\ u_3=0 \end{Bmatrix} \tag{2}$$

where $\vec{\varphi}$ is the torsion angle, $\vec{r}$ is the distance from the center of the microtube cross section, u_1, u_2 and u_3 indicate the displacement in x, y and z direction respectively. When the small deflection is considered, the component of strain reduced to

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial u_j}+\frac{\partial u_j}{\partial u_i}\right)=$$

$$\begin{bmatrix} 0 & 0 & \frac{-y}{2}\frac{d\varphi}{dz} \\ 0 & 0 & \frac{x}{2}\frac{d\varphi}{dz} \\ \frac{-y}{2}\frac{d\varphi}{dz} & \frac{x}{2}\frac{d\varphi}{dz} & 0 \end{bmatrix} \tag{3}$$

Based on nonlocal elasticity theory the stress at a reference point is considered to be a function of the strain at every point in the body and the constitutive equation can be described as follow

$$\left[1-\mu^2\nabla^2\right]\sigma_{ij} = c_{ijkl}\varepsilon_{ij} \tag{4}$$

Where σ_{ij} is the components of stress tensor in the body respectively. μ is the nonlocal parameter which denotes distance between atoms. Due to pure torsion of microtube Eq. (4) simplified as

$$\left[1-\mu^2\frac{d^2}{dz^2}\right]\sigma_{yz} = 2G\varepsilon_{yz} \tag{5}$$

$$\left[1-\mu^2\frac{d^2}{dz^2}\right]\sigma_{xz} = 2G\varepsilon_{xz} \tag{6}$$

Multiplying Eqs. (5,6) by xdA and ydA respectively and integrating the result over the area of the beam cross section and subtracting Eq. (5) from the Eq. (6), the following relation is obtained.

$$\left[1-\mu^2\frac{d^2}{dz^2}\right]T_z = G(z)I_2\frac{d\varphi}{dz}$$

$$T_z = \int_A \left(x\sigma_{yz} - y\sigma_{xz}\right)dA \ ; \ \mathrm{I}_2 = \int_A r^2 G(r)dA \tag{7}$$

The Lorentz force induced by the longitudinal magnetic field can be obtained from Maxwell's relations as follow.

$$\vec{f} = \eta\left(\vec{J}\times\vec{H}\right) \tag{8}$$

$$\vec{h} = \nabla\times\left(\vec{u}\times\vec{h}\right) \tag{9}$$

$$\vec{J} = \nabla\times\vec{h} \tag{10}$$

Denoting $\vec{J}$ as current density, $\vec{h}$ as distributing vector of the magnetic field. According to longitudinal magnetic field in z direction The Lorentz force reduced to

$$\vec{f} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} \eta H_z^2\left(-y\frac{d^2\varphi}{dz^2}\right) \\ \eta H_z^2\left(x\frac{d^2\varphi}{dz^2}\right) \\ 0 \end{bmatrix} \tag{11}$$

The principle of minimum potential energy is expressed as

$$\delta U - \delta W = 0 \tag{12}$$

where δU and δW are the variation of strain energy and external work due to longitudinal magnetic field and external torque respectively.

$$\delta U = \int_V \sigma_{ij}\delta\varepsilon_{ij}dV =$$

$$\int_V 2\left(\sigma_{xz}\delta\varepsilon_{xz} + \sigma_{yz}\delta\varepsilon_{yz}\right)dV =$$

$$\int_0^L \left(\delta\left(\frac{d\varphi}{dz}\right)\int_A \left(x\sigma_{yz} - y\sigma_{xz}\right)dA\right) =$$

$$T_z\delta\varphi\Big|_0^L - \int_0^L \frac{dT_z}{dz}\delta\varphi dz \tag{13}$$

$$\delta W = \delta\int_V \left(f_x u_1 + f_y u_2\right) + \int_0^L T(z)\delta\varphi \ \mathrm{dx} =$$

$$\int_0^L \left[D_2 H_z^2 \eta(z)\delta\left(\frac{d^2\varphi}{dz^2}\varphi\right) + T(z)\delta\varphi\right]\mathrm{dz} =$$

$$\int_0^L \Bigg[D_2 H_z^2\left(\frac{d^2}{dz^2}\left(\eta(z)\varphi\right) + \right.$$

$$\left.\eta(z)\frac{d^2\varphi}{dz^2}\right) + T(z)\Bigg]\delta\varphi\mathrm{dz} +$$

$$\left[D_2 H_z^2\left(\eta(z)\varphi\right)\right]\delta\left(\frac{d\varphi}{dz}\right)\Bigg|_0^L +$$

$$\left[-D_2 H_z^2\left(\frac{d}{dz}\left(\eta(z)\varphi\right)\right)\right]\delta\varphi\Bigg|_0^L \quad ;$$

$$D_2 = \int_A r^2\eta(r)dA \tag{14}$$

Substituting Eqs. (13,14) into Eq. (12) with using Eq. (7), the governing equation for torsion of BDFGM is derived as

$$\left[2\mu^2 D_2 H_z^2\eta(z)\right]\frac{d^4\varphi}{dz^4} + \left[6\mu^2 D_2 H_z^2\eta'(z)\right]\frac{d^3\varphi}{dz^3} +$$

$$\left[D_2 H_z^2\left(-2\eta(z) + 7\mu^2\eta''(z)\right) - I_2 G(z)\right]\frac{d^2\varphi}{dz^2} +$$

$$\left[D_2 H_z^2\left(-2\eta'(z) + 4\mu^2\eta'''(z)\right) - I_2 G'(z)\right]\frac{d\varphi}{dz} +$$

$$\left[D_2 H_z^2\left(-\eta''(z) + \mu^2\eta^{(4)}(z)\right)\right]\varphi =$$

$$T(z) - \mu^2 T''(z)$$

and boundary conditions:

$$D_2 H_z^2\eta(z)\varphi = 0 \ or \ \delta\frac{d\varphi}{dz} = 0 \ @ \ 0, L$$

$$D_2 H_z^2 \frac{d}{dz}\left(\eta(z)\varphi\right)+T(z)=0$$
$$or\ \ \delta\varphi=0\ @\ 0,L \qquad (15)$$

3. Solution method

For Galerkin method approximate solution introduced bellow

$$\varphi=\sum_{i=1}^{N} a_i\varphi_i$$
$$\varphi_i=1-\cos\left(\frac{2i\pi x}{L}\right) \qquad (16)$$

where the shape function φ_i, is introduced to satisfy the boundary conditions, and a_i is the unknown coefficient to be determined. As N, the number of terms in the approximate solution, increases, the accuracy rapidly increases. Applying Galerkin method the following relations are achieved.

$$\begin{aligned}&\int_0^L\Big\{\left[2\mu^2 D_2 H_z^2\eta(z)\right]\sum_{i=1}^{N} a_i\frac{d^4\varphi_i}{dz^4}+\\ &\left[6\mu^2 D_2 H_z^2\eta'(z)\right]\sum_{i=1}^{N} a_i\frac{d^3\varphi_i}{dz^3}+\\ &\left[D_2 H_z^2\left(-2\eta(z)+7\mu^2\eta''(z)\right)-\right.\\ &\left.-I_2 G(z)\right]\sum_{i=1}^{N} a_i\frac{d^2\varphi_i}{dz^2}+\\ &\left[D_2 H_z^2\left(-2\eta'(z)+4\mu^2\eta'''(z)\right)-\right.\\ &\left.I_2 G'(z)\right]\sum_{i=1}^{N} a_i\frac{d\varphi_i}{dz}+\\ &\left[D_2 H_z^2\left(-\eta''(z)+\mu^2\eta^{(4)}(z)\right)\right]\varphi-\\ &T(z)+\mu^2 T''(z)\Big\}\varphi_j\ dz=0,\quad j=1,\ldots,N\end{aligned} \qquad (17)$$

According to the GDQ method the various order derivative φ are defined as a linear combination of weighted coefficients of φ.

$$C_{ij}^{(1)}=\frac{\prod_{k=1}^{N}(x_i-x_k)}{(x_i-x_j)\prod_{k=1}^{N}(x_j-x_k)}$$

$$C_{ii}^{(1)}=-\sum_{k=1}^{N} C_{ik}^{(1)}$$

$$C_{ij}^{(n)}=n\left(C_{ii}^{(n-1)}C_{ij}^{(1)}-\frac{C_{ij}^{(n-1)}}{x_i-x_j}\right)$$

$$C_{ii}^{n}=-\sum_{k=1}^{N} C_{ik}^{(n)},\quad \text{i,j=1,2,...,N},\quad i\neq j \qquad (18)$$

Using the GDQ method the boundary condition for clamped microtube based on nonlocal theory is simplified as

$$\varphi_1=0\qquad,\qquad \varphi_N=0$$
$$\sum_{j=1}^{N} C_{1j}^{(1)}\varphi_j\qquad,\qquad \sum_{j=1}^{N} C_{Nj}^{(1)}\varphi_j \qquad (19)$$

The domain of the equation is divided into N points using the Chebyshev polynomials as follows.

$$z_i=\frac{L}{2}\left(1-\cos\left(\frac{i-1}{N-1}\right)\pi\right),\qquad \text{i=1,2,...,N} \qquad (20)$$

By applying the boundary conditions, the governing equation is simplified to the N numbers of algebraic equations below.

$$[A]\{\varphi\}=\{T\} \qquad (21)$$

4. Results and discussion

This section investigates the torsional angle of BDFGM microtube based on nonlocal theory under uniform longitudinal magnetic field and sinusoidal distributed torque $T(z)=10^{-3}\sin(\pi z/L)$. In addition a comparison between the GDQ and Galerkin methods is performed. To this aim following data in table 1 are considered

Table 1. Geometry and material properties of BDFGM microtube

Material property		Geometry property	
$G_i\,(Gpa)$	129	$r_i\,(\mu m)$	20
$G_0\,(Gpa)$	48	$r_o\,\mu m$	40
$\eta_i\,(H/m)$	10^{-1}	$L\,(\mu m)$	100
$\eta_i\,(H/m)$	10^{-3}		

In the figure 4 the ratio of φ/φ_{max} respect to the length of microtube is presented by GDQ method and Three precision of Galerkin approach.

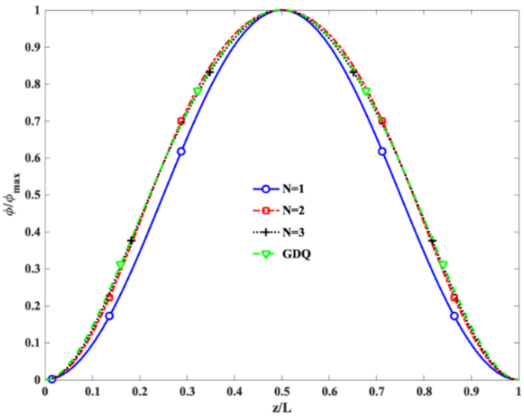

Figure 4. Comparison of the GDQ method with Galerkin method for $\alpha=0$, $\beta=1$, $\mu=20\mu m$, $H_z=1MT$

The convergency of the GDQ numerical method is shown in the figure 5. It is found that by selecting the number of discretization points higher than 50 Convergency is achieved.

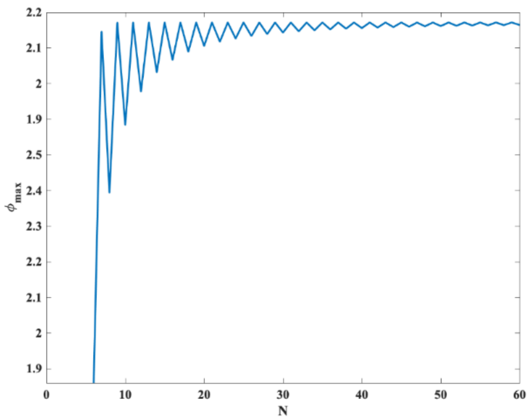

Figure 5. Convergency of the GDQ method for $\alpha = 0$, $\beta = 0$, $\mu = 20\mu m$, $H_z = 5MT$

Magnetic field, FGM indexes and small-scale effect have important role on the static torsion behavior of BDFGM microtubes that should be carefully realized. Nonlocal effect in conjugation with longitudinal magnetic field is studied in figure 6. The results show that by increasing of the longitudinal magnetic field for all values of the small-scale parameter and all FGM indexes reduces the maximum angel of BDFGM microtube. In addition the variation of the torsion angle of microtube is related to the nonlocal parameter such that at high intensity of magnetic field the effect of size is negligible. For values smaller than $H_z = 0.4\ T$ Increasing the nonlocal parameter increases the torsional angle of microtube. In contrast for values higher than $H_z = 0.4\ T$ Increasing the nonlocal parameter decrease the torsional angle of microtube.

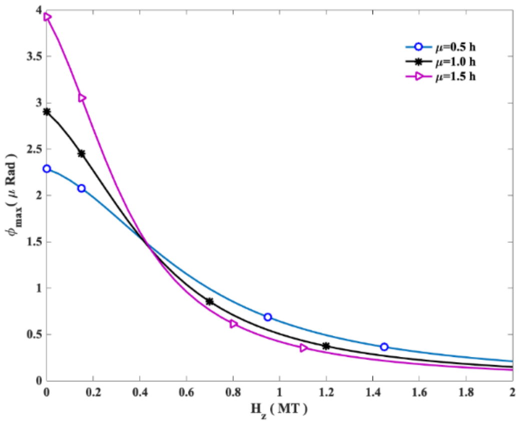

Figure 6. Variation of maximum torsional angle respect to longitudinal magnetic field for different μ, $\alpha = 0$, $\beta = 0$

In figure 7 and 8 maximum torsional angle of BDFGM microtube respect longitudinal magnetic field is plotted in different α_G and β_G respectively. the effects of α_G and β_G on the torsional angle of BDFGM microtube is negligible with increasing of magnetic field intensity. increasing of α_G decrease torsional angle of BDFGM microtube in contrast increasing of β_G increase torsional angle of BDFGM microtube.

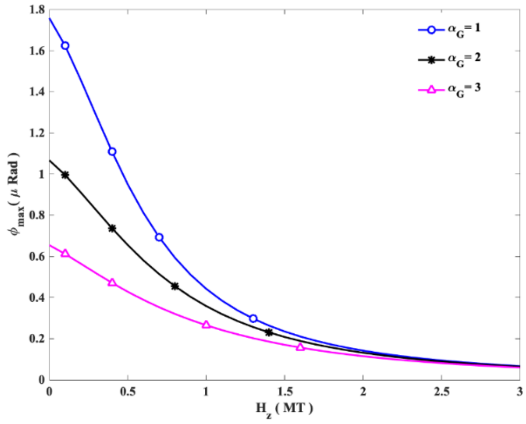

Figure 7. Variation of maximum torsional angle respect to longitudinal magnetic field for different α_G, $\alpha_\eta = 0$, $\beta = 0$, $\mu = 20\mu m$

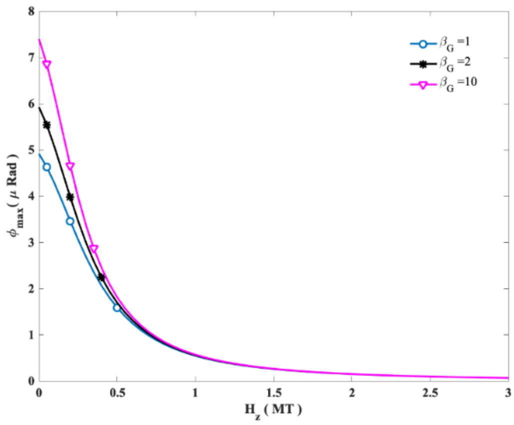

Figure 8. Variation of maximum torsional angle respect to longitudinal magnetic field for different β_G, $\alpha = 0$, $\beta_\eta = 0$, $\mu = 20\mu m$

By using the GDQ approach as the numerical method the maximum torsional angle for BDFGM microtube has been computed for different α_η and β_η in figure 9 and 10. It is observed that maximum torsional angle of BDFGM microtube is decreasing as intensity of longitudinal magnetic field increases. Torsional angle of BDFGM microtube is not sensitive to change of α_η and β_η at low and high intensity of longitudinal magnetic field. The maximum torsional angle of BDFGM microtube is decreasing as α_η increases in contrast the maximum torsional angle of BDFGM microtube is increasing as β_η increases.

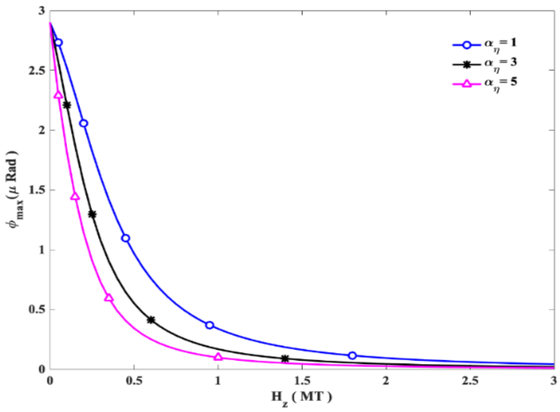

Figure 9. Variation of maximum torsional angle respect to longitudinal magnetic field for different α_η, $\alpha_G = 0$, $\beta = 0$, $\mu = 20\mu m$

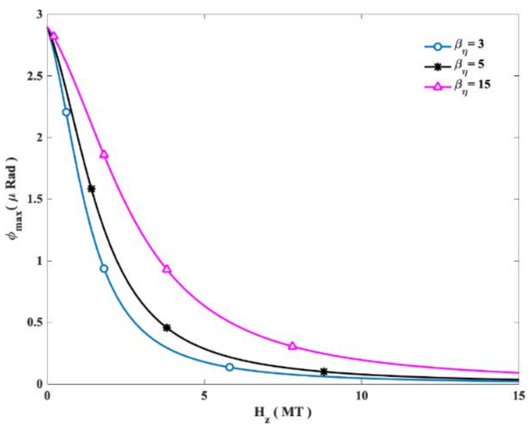

Figure 10. Variation of maximum torsional angle respect to longitudinal magnetic field for different β_η, $\alpha = 0$, $\beta_G = 0$, $\mu = 20\mu m$

5. Conclusion

In this paper the static behavior of BDFGM microtube investigated. The size-effect considered based on nonlocal elasticity theory. Using the GDQ method effects of nonlocal parameter, intensity of magnetic field and FGM indexes on the torsional angle of microtube discussed. Result indicated that intensity of longitudinal magnetic field had important role on the torsional angle of microtubes such that when intensity of longitudinal magnetic field increased the torsional angle of microtubes decreased as well as for values smaller than $H_z = 0.4\ T$ increasing the nonlocal parameter increased the torsional angle of microtube. In contrast for values higher than $H_z = 0.4\ T$ increasing the nonlocal parameter decreased the torsional angle of microtube. In addition the effect of α_G and β_G on the torsional angle of BDFGM microtube was negligible with increasing of magnetic field intensity. Effects of α_η and β_η on the torsional angle of BDFGM microtube was negligible at low and high intensity of longitudinal magnetic field.

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