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A study on the effect of crack on free vibration of thick rectangular plates with initial geometric imperfection using differential quadrature method

Mohammad Orak[a], Mehdi Salehi[a,]*

[a] *Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran*

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ABSTRACT

In this study, vibration of initially imperfect cracked thick plate has been investigated using the differential quadrature method. The crack modeled as an open crack using a no-mass linear spring. The governing equations of vibration of a cracked plate are derived using the Mindlin theory and considering the effect of initial imperfection in Von-Karman equations. Differential equations are discretized using the differential quadrature method and are converted to a non-standard eigenvalue problem. Finally, natural frequencies and mode shapes of the cracked plate are obtained solving this eigenvalue problem. The accuracy of the proposed approach is verified using the results presented in other references. Various examples of the cracked plate problem have been solved utilizing the proposed method and effects of selected parameters such as crack depth, length and position have been checked. It is demonstrated that increasing the length and the depth of the crack decrease the plate stiffness and natural frequencies. Moreover, the effects of crack location on natural frequencies are more complicated, since they depend on the mode shapes, and when the crack is placed at a node-line, it will not influence the frequencies.

1. Introduction

Plates suffer from various damages during their lifetime, such as the creation and expansion of the cracks. The presence of cracks in the plate affects its mechanical behavior. Since the crack is known as a deterioration factor in structures, investigation of the effect of crack on the behavior of structures has significantly affect the assurance and economic design of various structures. Hence, many researchers have studied the behavior of cracked structures in different fields. One of these areas is to investigate the free vibrations of cracked plates, which is considered in this research. The first step is to develop crack detection methods in the plate with the help of its vibrational properties.

Rice and Levy [1] studied the effect of cracks in the plate by developing a strap model with edge-cutting. Meanwhile they calculated the coefficient of stress intensity for the rectangular cracked plate under the influence of tensile and bending loads. Khadem and Rezaei [2] examined the vibrational behavior of rectangular plate with total cracks using bending spring model. They have always presumed the cracks as open and split the plate along the crack. With the replacement of a linear spring instead of the crack, Israr [3] studied the effect of the existence of middle cracks parallel to one side of the isotropic rectangular plates on the natural frequencies of the plate, and observed that, with increasing the crack's length, the natural frequencies decreased. Israr and Atepor [4] solved the problem of compulsory vibration of the cracked plate, using analytical and numerical methods, and compared obtained results with the experimental results. They extracted the nonlinear vibration equation of cracked plates and solved it using the Galerkin method and multiple scale method.

Osman et al. [5] used differential quadrature method to investigate the stress intensity factor of the third mode of a cracked plate. They split the plate with an arbitrary shape into a set of sub-domains and, using the mapping method, converted the plate into several smaller rectangular plates. Then, by applying a differential quadrature method on each of these plates, and also considering the appropriate boundary conditions, they calculated the third mode stress intensity factor for a plate having angular cracks. Bachene et al. [6-7] also studied the vibrations of a rectangular cracked plate using X-FEM method. In their calculations, they also considered the rotary inertia and transverse shear deformation effects and used the desired method to examine the behavior of the plates having edge and middle cracks. Stahl and Kear [8] investigated the vibrations and buckling of rectangular plates having lateral and middle cracks parallel to one side of the plate. Using dual series, they reduced the problem of the eigenvalue to the second-order Fredholm integral equations, which calculated the values of the natural frequencies and the buckling load of the cracked plates. Nezo [9] examined the free vibrations of a rectangular plate with simple supports. He assumed that the desired cleavage was parallel to one side of the plate. Makvandi et al. [10] studied the vibrations

* Corresponding author. Tel.: +98 913 1265600; e-mail: Mehdi.salehi@pmc.iaun.ac.ir

of buckled cracked rectangular plates using the differential quadrature element method. By using the arc length method and considering the crack as a rotational linear spring, they investigated the effect of different crack parameters on plate behavior, and compared the results with the results of the finite element method and also the experimental results.

As noted earlier, the widespread use of the plates has made it very important to study the behavior of cracked plates. It has been determined from the studies that so far, the vibrational behavior of thick, rectangular cracked plates with initial geometric imperfection has not been investigated. Therefore, in this research, a method is developed to analyze the crack effect on vibrational behavior of thick cracked plates. For this purpose, first, using the fracture mechanics relations, the crack is modeled as a non-massive linear rotary spring. Then, with the assumption that the crack is open, the governing equations are extracted from the problem. These equations form a system of differential equations. The obtained equations are then discriminated using the differential quadrature method. The equations system is the result of a nonstandard eigenvalue problem, after converting it into a standard form and solving it, the frequencies and the mode shapes of the cracked plate are obtained. Investigation of the frequencies and the shape of the obtained modes for different values of the depth, length and location of the cracks showed that increasing the length and depth of the crack decreases the natural frequencies by decreasing the stiffness of the plate, while the effect of the crack position on the frequency depends on the shape of the mode in question.

2. Differential Quadrature Method

The differential quadrature method is developed based on the Gaussian derivative method to calculate the derivative of a function based on its value in a limited number of domain points. This method provides an approximation to express the derivation of a function at a point on its domain, in terms of the weight composition of the function values of the points within that domain [11] and the first-order derivative f (x, y) at a point relative to x and y is a linear approximation of the sum of the values of the function over the entire derivative interval as Equation (1).

$$\begin{aligned}
&\frac{\partial^n}{\partial x^n} f\left(x_i, y_j\right) = \sum_{k=1}^{n_x} C_{ik}^{(n)} f\left(x_k, y_j\right) \\
&\frac{\partial^m}{\partial y^m} f\left(x_i, y_j\right) = \sum_{k=1}^{ny} C_{jk}^{(m)} f\left(x_i, y_k\right) \\
&\frac{\partial^{(n+m)}}{\partial x^n \partial y^m} f\left(x_i, y_j\right) = \sum_{k=1}^{n_x} C_{ik}^{(n)} \sum_{l=1}^{n_y} C_{jl}^{(m)} f\left(x_k, y_l\right) \\
&n = 1, \ldots, n_x - 1 \quad , \quad m = 1, \ldots, n_y - 1
\end{aligned} \tag{1}$$

In the above relation f is the function in question, n_x and n_y are the number of grid points along x, y, x_i and y_i are the i'th and j'th points of accuracy on the domain of the function and $C_{ij}^{(n)}$ and $C_{ij}^{(m)}$ are the weight coefficients to obtain the derivatives of order n and m of the function f (x_i, y_i) in (x_i, y_i). In this study, relations 2 and 3 have been used to calculate the differential quadrature weight coefficients [12].

$$C_{ii}^{(1)} = \frac{1}{(x_i - x_j)} \prod_{\substack{k \neq i, j \\ i \neq j}}^{n_x} \frac{(x_i - x_k)}{(x_i - x_k)} \qquad i, j = 1,2,3,\ldots,n_x \tag{2}$$

$$C_{ii}^{(1)} = -\sum_{\substack{j=1 \\ j \neq 1}}^{n_x} C_{ij} \tag{3}$$

The weight coefficients of higher-order derivatives are obtained by using first-order weight coefficients of the recurrence relation as relations 4 and 5.

$$C_{ij}^{(n)} = n\left(C_{ij}^{(1)} . C_{ii}^{(n-1)} - \frac{C_{ij}^{(n-1)}}{(x_i - x_j)}\right), \quad j \neq i \tag{4}$$

$$C_{ii}^{(n)} = n - \sum_{j \neq i}^{N} C_{ij}^{n} \tag{5}$$

One of the effective parameters in the accuracy of the differential quadrature method is the type of the used grid points. Some of the common types of them are:

1. Accuracy points with equal space

$$x_i = \frac{i-1}{N-1}, \; i = 1, 2, 3, \ldots, N \tag{6}$$

2. oots of Chebyshev polynomials

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{2i-1}{2N}\pi\right)\right], \; i = 2, 3, \ldots, N-1 \qquad x_1 = 0, x_N = 1 \tag{7}$$

3. roots of Legendre Polynomials

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{2i-3}{2N-4}\pi\right)\right], \; i = 2, 3, \ldots, N-1 \qquad x_1 = 0, x_N = 1 \tag{8}$$

۴. roots of Chebyshev-Gauss-Lobatto polynomials

$$x_i = \frac{1}{2}\left[1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right], i = 2, 3, \ldots, N-1 \qquad x_1 = 0, x_N = 1 \tag{9}$$

4. Cracked plate modeling

The presence of crack in the structure of the plates reduces the stiffness of the plate. One of the methods of crack modeling in plates is the use of rotary spring model. In this research, the plate is first divided into six segments around the crack, and then the crack is modeled on the boundary between the two adjacent segments by a rotating spring. Then differential equations of plate behavior, boundary conditions, and proper continuity are considered. The equation system is transformed into a system of algebraic equations using the differential quadrature method, by solving which the vibration frequencies of the cracked plate can be calculate. In order to investigate the free vibrations of the cracked plate, a plate is considered having the length a, width b, thickness h, which has a crack of the depth of h_c, length L_c, and location l_c as in Fig. (1). Using the Mindlin theory and taking into account the existence of an initial geometric imperfection in strain-displacement relations, the differential equations of the vibrations of the desired plate are obtained as relations (10) to (14).

$$\begin{aligned}&\frac{Eh}{1-\nu^2}[u_{,xx} + w_{,xx}w_{0,x} + w_{,x}w_{0,xx} + \nu(v_{,xy} + \\ &+w_{,xy}w_{0,y} + w_{,y}w_{0,xy})] + \frac{Eh}{2\ 1+\nu}[u_{,yy} + \\ &+v_{,xy} + w_{0,xy}w_{,y} + w_{0,x}w_{,yy} + w_{,xy}w_{0,y} + \\ &+w_{,x}w_{0,yy}] = \mu\ddot{u}\end{aligned} \tag{10}$$

$$\begin{aligned}&\frac{Eh}{1-\nu^2}[v_{,yy} + w_{,yy}w_{0,y} + w_{,y}w_{0,yy} + \nu(u_{,xy} + \\ &+w_{,xy}w_{0,x} + w_{,x}w_{0,xy})] + \frac{Eh}{2\ 1+\nu}[u_{,xy} + \\ &+v_{,xx} + w_{0,xx}w_{,y} + w_{0,x}w_{,xy} + w_{,xx}w_{0,y} + \\ &+w_{,x}w_{0,xy}] = \mu\ddot{v}\end{aligned} \tag{11}$$

$$\begin{aligned}&K_S Gh(\alpha_{,x} + w_{,xx}) + K_s Gh(\beta_{,y} + w_{,yy}) + \frac{Eh}{1-v^2} \\ &[u_{,xx} w_{0,x} + w_{,xx} w_{0,x}^2 + w_{0,xx} w_{0,x} w_{,x} + v \\ &(v_{,xy} w_{0,x} + w_{,xy} w_{0,x} w_{0,y} + w_{0.xy} w_{0,x} w_{,y})] + \\ &+ \frac{Eh}{1-v^2}(u_{,x} w_{,xx}) + \frac{Eh}{1-v^2}[u_{,x} w_{0,xx} + w_{0,xx} \\ &w_{0,x} w_{,x} + v(v_{,y} w_{0,xx} + w_{,y} w_{0,xx} w_{0,y})] + \\ &+ \frac{Eh}{1-v^2}[v_{,yy} w_{0,y} + w_{,yy} w_{0,y}^2 + w_{0,yy} w_{0,y} \\ &w_{,y} + v(u_{,xy} w_{0,y} + w_{,xy} w_{0,x} w_{0,y} + w_{0,xy} \\ &w_{0,y} w_{,x})] + \frac{Eh}{1-v^2}(v_{,y} w_{0,yy} + v(u_{,x} w_{0,yy} + \\ &+ w_{,x} w_{0,yy} w_{0,x}) + \frac{Eh}{2(1+v)}(u_{,xy} w_{0,y} + v_{,xx} \\ &w_{0,y} + w_{,xx} w_{,y} w_{0,y} + w_{,xy} w_{0,x} w_{0,y} + w_{,xx} \\ &w_{0,y}^2) + \frac{Eh}{2(1+v)}(u_{,yy} w_{0,x} + v_{,xy} w_{0,x} + w_{,yy} \\ &w_{0,x}^2 + w_{,xy} w_{0,x} w_{0,y}) + \frac{Eh}{(1+v)}(u_{,y} w_{0,xy} \\ &+ v_{0,x} w_{0,xy} + w_{0,xy} w_{0,x} w_{,y} + v_{0,xy} w_{,x} w_{0,y}) = \mu\ddot{w}\end{aligned} \tag{12}$$

$$\begin{aligned}&\frac{Eh^3}{12(1-V^2)}\left(\alpha_{,xx} + v\beta_{,xy}\right) + \frac{Eh^3}{24(1+V)}\left(\alpha_{,yy} + \beta_{,xy}\right) - \\ &- K_S Gh\left(\alpha + \omega_{,x}\right) = I_X \ddot{\alpha}\end{aligned} \tag{13}$$

$$\begin{aligned}&\frac{Eh^3}{12(1-V^2)}\left(\beta_{,xx} + v\alpha_{,xy}\right) + \frac{Eh^3}{24(1+V)}\left(\alpha_{,yy} + \beta_{,xx}\right) - \\ &- K_S Gh\left(\beta + \omega_{,y}\right) = I_y \ddot{\beta}\end{aligned} \tag{14}$$

Here $\mu = \rho h$ and $I_x = I_y = (\rho h^3) / 12$, where ρ is the density of the desired plate. u, v, and w are the displacements along the axes x, y and z, respectively, and α and β are rotation around the axes x and y respectively. Also, the initial geometric defect of the plate is represented by displacement w_0. E is the modulus of elasticity, ν is the Poisson ratio, G is the shear modulus, and K_s is the shear correction coefficient used to compensate for the parabolic distribution of shear stress versus the assumption of the uniform distribution of shear stress considered in the Mindlin theory.

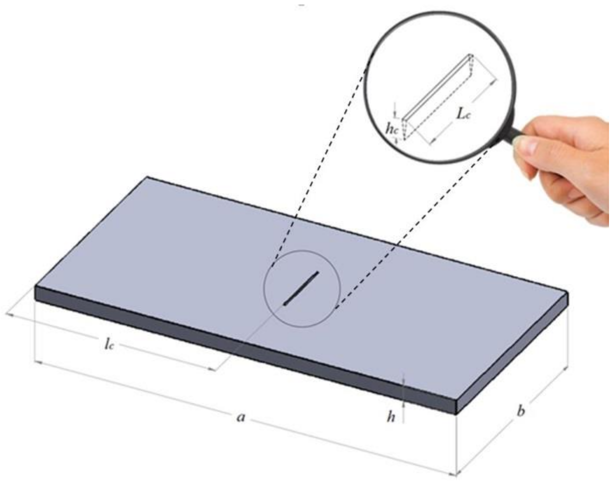

Figure 1. Cracked rectangular plate

Considering the use of the differential quadrature method for solving the equations shown above, the desired plate is first divided into six elements around the crack, as shown in Fig. 2, and then the differential equations governing the plate, as well as the boundary conditions and proper continuity apply to each element. Continuity relations between adjacent elements include the conjunction of all displacements and rotations, as well as all forces and moments and only the slopes of the two elements 2 and 5 are discontinuous in the direction of perpendicular to the path of the crack. This discontinuity is shown in Equation (15).

$$\alpha^2 = \alpha^5 + \theta \tag{15}$$

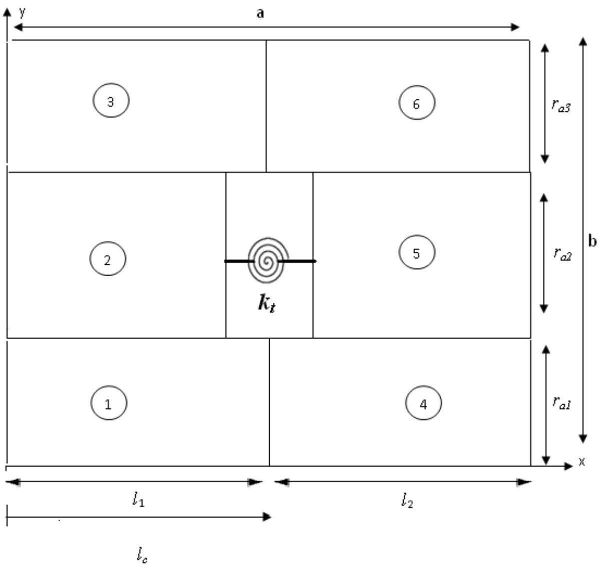

Figure 2. Cracked plate modeling

Due to the reduction of the stiffness of the plate in the crack region, the slope discontinuity is observed on both sides of the crack. Therefore, in order to calculate the additional rotation of the cracked plate, by means of the failure mechanics relations and the definition of the stress intensity coefficient for the first mode of the crack, the amount of this additional rotation can be calculated as the relation (16) [13].

$$\theta = \frac{12\left(1-v^2\right)}{E}\sigma_b\alpha_{bb} \tag{16}$$

Here, σ_b is the nominal bending stress in the direction of perpendicular to the crack and α_{bb} is the softness coefficient of bending, which is presented in relation (17) as a function of the crack parameters [13].

$$\alpha_{bb} = \frac{1}{h}\int_0^{h_c} g_b^2 dh_c \tag{17}$$

In this relation g_b is a dimensionless function of the relative depth of the crack (ξ=h_c/h) in the range ($0 \leq \xi \leq 0.8$), which is taken as the relation (18) [13].

$$g_b = \sqrt{(\pi\xi)\begin{bmatrix} 1.202-1.8872\xi+18.0143\xi^2 \\ -87.3851\xi^2+241.9124\xi^4 \\ -319.9402\xi^5+168.0105\xi^6 \end{bmatrix}} \tag{18}$$

5. Calculation of the natural frequencies of the cracked plate

In this research, the differential quadrature element method is used to calculate the vibration frequencies of the cracked plate. Using this method, the differential vibration equations of the plate and boundary equations and their corresponding continuities are discrete, and a system of equations with eigenvalues as relation (19) is formed, which is a nonstandard eigenvalue problem.

$$\begin{bmatrix} A_{BB} & A_{BI} \\ A_{IB} & A_{II} \end{bmatrix}\begin{Bmatrix} X_B \\ X_I \end{Bmatrix} = -\omega^2\begin{bmatrix} 0 & 0 \\ B_{IB} & B_{II} \end{bmatrix}\begin{Bmatrix} X_B \\ X_I \end{Bmatrix} \tag{19}$$

In this case, X is the displacement vector in the form [u, v, w, α, β]^T. Indices B and I indicate the boundary and internal values, respectively. This equation can be obtained in the form of equation (20). By solving it, the natural frequencies and mode shapes of the plate are obtained.

$$\left[A^*\right]X_I = -\omega^2\left[B^*\right]X_I$$

$$\begin{cases} \left[A^*\right] = \left(A_{II} - A_{IB}\times A_{BB}^{-1}\times A_{BI}\right) \\ \left[B^*\right] = \left(B_{II} - B_{IB}\times A_{BB}^{-1}\times A_{BI}\right) \end{cases} \tag{20}$$

5.1. Results

As it was mentioned in the description of the differential quadrature method, selection of the proper accuracy points greatly affects the convergence and accuracy of the results of the differential quadrature method. In this section, the effects of accuracy points with equal distances, accuracy points using the Chebyshev-Gauss-Lobatto, Chebyshev, and Legendre polynomials are investigated on the first three frequencies of natural vibrations of the plate. The number of accuracy points listed in the following is the number of accuracy points used in each element. The results obtained for a plate of length and width of 1 m and thickness of 0.01 m are presented in Figures 3 to 5. The density, Poisson coefficient and modulus of elasticity of the plate are 7800 kg / m3, 0.3 and 207 GPa, respectively, which are located on the four sides on the joint support. The studied plate has an initial geometric defect, which is considered as a coefficient of the first mode shape of the buckling of the intact plate as in relation (21).

$$\omega_\circ = w_0\left(\sin\frac{n\pi x}{a}\right)\left(\sin\frac{m\pi y}{b}\right) \tag{21}$$

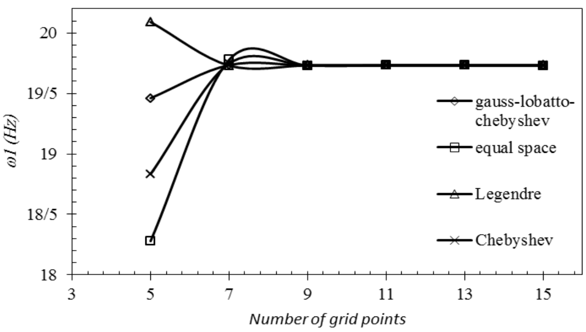

Figure 3. variations of first frequency of plate against the number of nodes

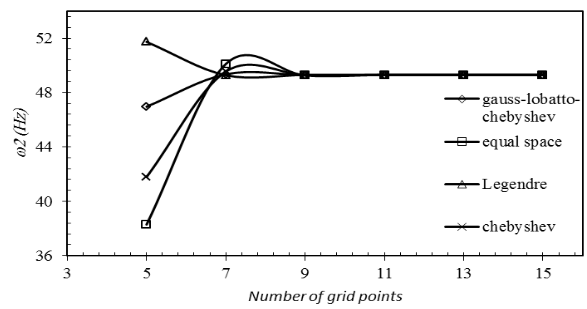

Figure 4. variations of second frequency of plate against the number of nodes

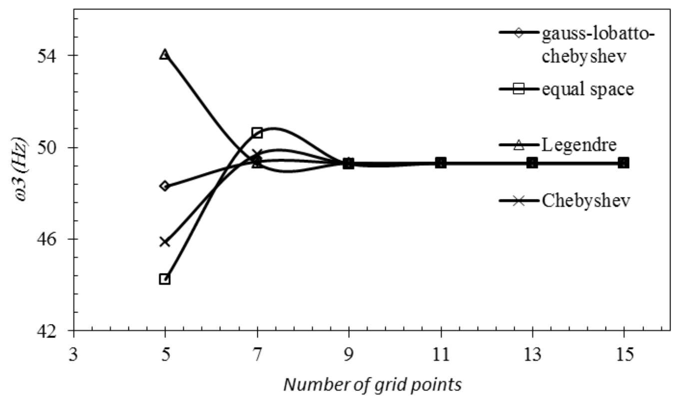

Figure 5. variations of third frequency of plate against the number of nodes

As shown in Figures 3 to 5, the convergence and accuracy of the frequencies obtained using the Gauss-Lobatto-Chebyshev accuracy points are better than the other points. Therefore, in order to calculate the frequencies of the cracked plate, these accuracy point types are used. It is observed that by increasing the number of used points, the relative error rate decreases significantly such that, using the eleven points of accuracy, the relative error value is very small.

In the following, in order to investigate the capability of the proposed method to extract the frequencies of the thick plate, a plate of length and width of 1 m and thickness of 0.1 m is considered. The density, Poisson coefficient and modulus of the elasticity of the plate are 7800 kg / m3, 0.3 and 207 GPa, respectively, and the desired plate is located on the four sides on the simply support. The effect of the number of accuracy points on the convergence of the first to the third frequencies of the plate is presented in Figures 6 to 8. The accuracy points used here are Gauss-Lobatto-Chebyshev accuracy points.

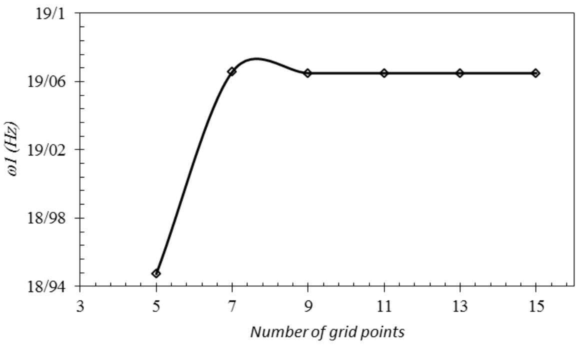

Figure 6. variations of first frequency of plate against the number of nodes

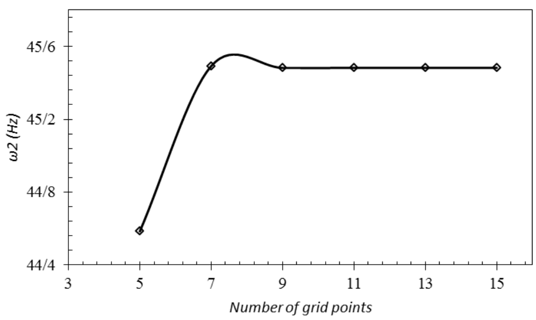

Figure 7. variations of second frequency of plate against the number of nodes

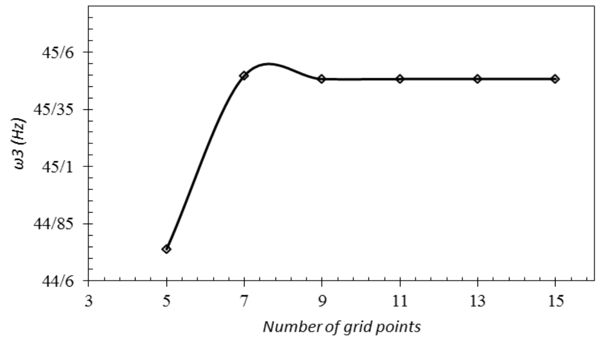

Figure 8. variations of third frequency of plate against the number of nodes

By investigation of Figures 6 through 8, it can be seen that, proper results can be obtained using 11 Gauss-Lobatto-Chebyshev accuracy points. In order to verify the validity of the applied modeling method, in the following, the results are given in Table 1 in comparison with the results presented in reference [14].

Table 1. Comparison of frequencies obtained using differential quadrature method and results presented in other references

	The thickness ratio to the length of the plate (h/a)	First frequency	Second frequency	Third frequency
Reference [14]	0/01	19/7392	49/3480	49/3480
Reference [14]		19/7319	49/3027	49/3027
Differential quadrature		19/7320	49/3032	49/3032
Reference [14]	0/1	19/0584	45/4478	45/4478
Differential quadrature		19/0650	45/4827	45/4827

5.2. The effect of crack depth on the natural frequencies of plate vibrations

Figures 9 through 11 show the variations of the first three initial frequencies for different values of the crack depth. The results are presented for thick plate (h / a = 0.1) for cracks of relative length of 0.4 and relative locations of 0.5.

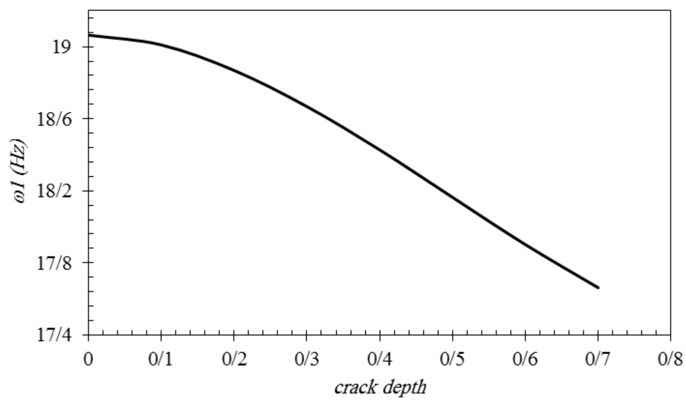

Figure 9. variations of first frequency of plate against crack depth $\left(h/a = 0/1\right)$

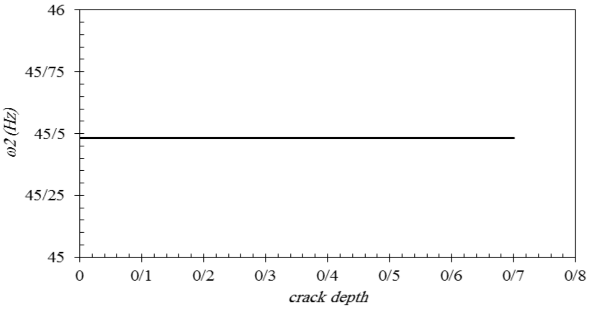

Figure 10. variations of second frequency of plate against crack depth $\left(h/a = 0/1\right)$

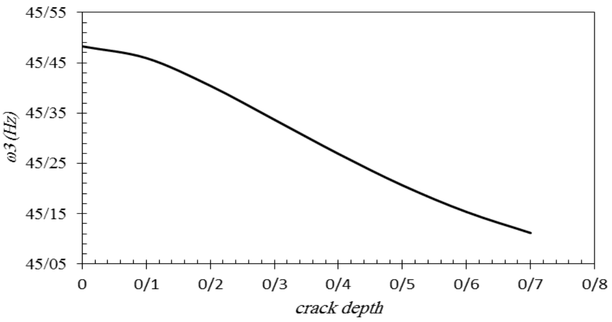

Figure 11. variations of third frequency of plate against crack depth $\left(h/a = 0/1\right)$

As can be seen, with the increase of the crack depth, the first and third frequencies decrease, as the crack depth decreases the stiffness of the plate and thus the natural frequencies decrease. On the other hand, the second frequency remains unchanged. Since the studied crack is located in the center of the plate $(l_c / a = 0.5)$ and on the node line of the second mode, so, the second mode is not basically affected by the presence of the cracks.

5.3. The effect of the crack location on the natural frequencies of plate vibrations

Figures 12 through 14 show the variations of the first three initial frequencies for different values of the crack depth. The results are presented for the thick plate (h / a = 0.1) for cracks of relative length of 0.4 and relative depth of 0.7.

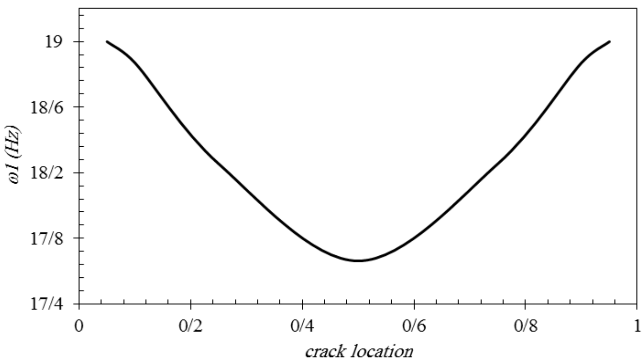

Figure 12. variations of first frequency of plate against crack location $\left(h/a = 0/1\right)$

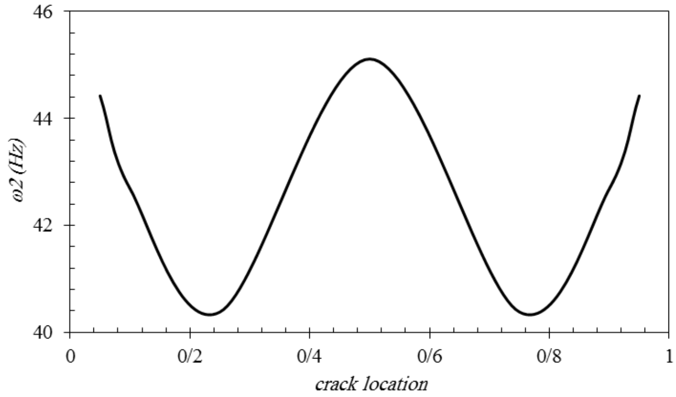

Figure 13. variations of second frequency of plate against crack location $\left(h/a = 0/1\right)$

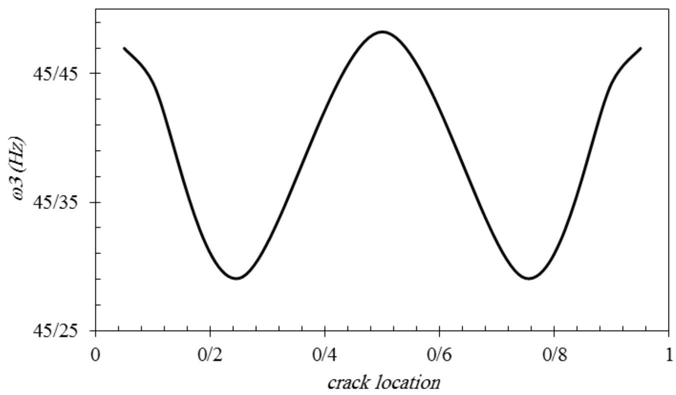

Figure 14. variations of third frequency of plate against crack location $\left(h/a = 0/1\right)$

In Figures 12 to 14, with the maintenance of the relative length and depth of the crack, the effect of the relative location of the crack on the first to third natural frequencies is investigated. It can be seen that if the crack is located on a node line of a particular mode, the natural frequency of the mode is not changed. The reason for this is the zeroing of the bending torque and, consequently, the zeroing of the discontinuity of the slope on the two sides of the crack, as a result of which, the presence of the crack does not have any effect on the natural frequency. It is also observed, by careful consideration of the above figures that the most frequent variations in the natural frequencies of each mode are in the point of the plate, the curvature of which is the maximum.

5.4. The effect of the crack length on the natural frequencies of plate vibrations

In Figures 15 to 17, by keeping constant the depth and relative location of the crack, the effect of the crack length on its natural frequencies has been investigated. As is clear from the figures, by increasing the crack length, the natural frequencies of the plate decrease, due to the reduction of plate stiffness. Here, locating of the crack on the node line causes the crack to be ineffective on the natural frequency of the mode.

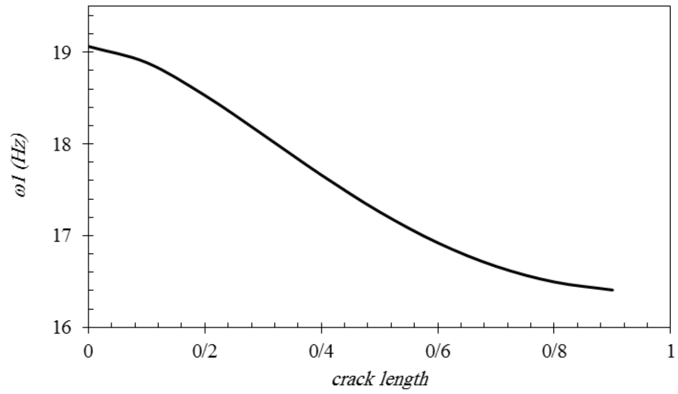

Figure 15. variations of first frequency of plate against crack length $\left(h/a = 0/1\right)$

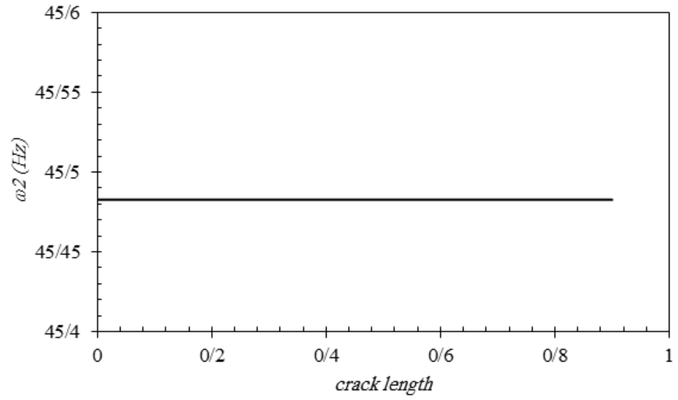

Figure 16. variations of second frequency of plate against crack length $\left(h/a = 0/1\right)$

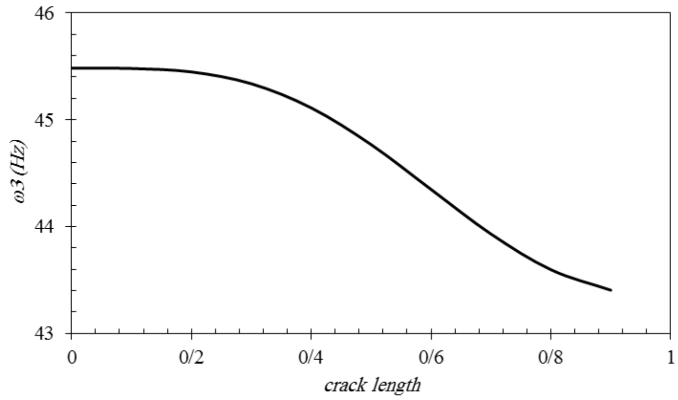

Figure 17. variations of third frequency of plate against crack length $\left(h/a = 0/1\right)$

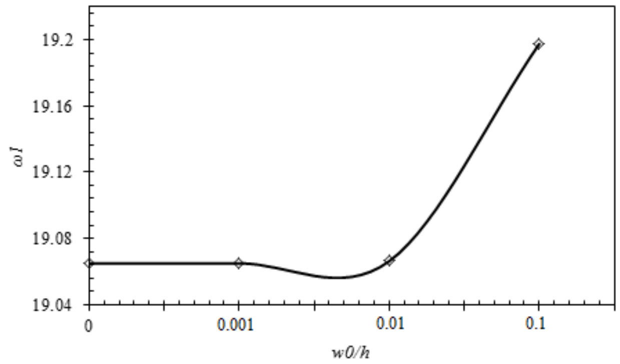

Figure 18. variations of first frequency of plate against initial geometric imperfection amplitude

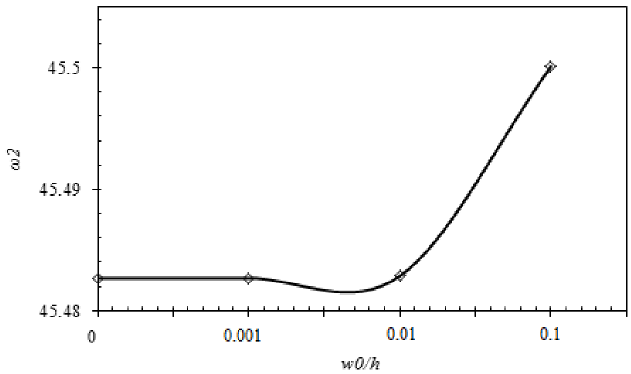

Figure19. variations of second frequency of plate against initial geometric imperfection amplitude

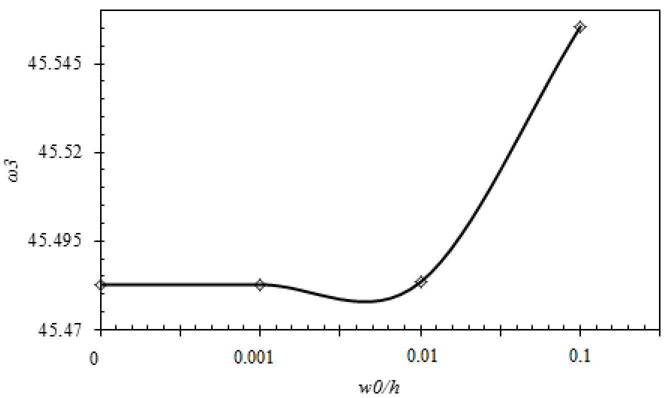

Figure 20. variations of third frequency of plate against initial geometric imperfection amplitude

5.5. Investigation of the effect of geometric defect domain on natural frequencies

In this section, the effect of increasing the domain of the geometric defect on the first three natural frequencies of the thick plate (h / a = 0.1) has been investigated. By investigation of Figures 18 through 20, it is observed that with increasing the domain of the initial geometric defect due to increased plate stiffness, the natural frequencies increase.

6. Conclusion

In this study, the effect of crack on vibrations of thick plates was investigated. The crack was considered open and was modeled with the help of a torsional spring. The solution of the differential equation system was extracted using a differential quadrature method and solving the resulted eigenvalue system. Comparison of the obtained results with the results of other references showed the correctness and accuracy of the proposed method. The influence of different parameters such as crack depth, crack location, crack length and domain of the geometric defect of the plate on natural frequencies was studied. It was observed that with the increase in the length and relative depth of the crack, due to reduced plate stiffness, the natural frequencies would decrease.

While the effect of the crack location on the natural frequencies is dependent on the shape of the mode studied, and if there is a crack on the node line, the presence of the crack does not affect the behavior of the plate. It was also observed that with increasing the amplitude of the initial geometric imperfection due to increased plate stiffness, the natural frequencies increase. The method presented in this study, could be used for crack identification in thick rectangular plates using natural frequencies of plate which would be presented in another paper of authors.

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