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Rotating magneto-thermoelastic rod with finite length due to moving heat sources via Eringen's nonlocal model

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ABSTRACT

This paper deals with a new nonlocal model based on Eringen's nonlocal elasticity and generalized thermoelasticity. A study was carried out on magneto-thermoelastic waves in a thermoelastic isotropic conducting finite rod subjected to a moving heat sources permeated by a primary uniform magnetic field and rotating with a uniform angular velocity. The Laplace transform technique has been used to solve the resulting non-dimensional coupled field equations. Expressions for nonlocal thermal stress, temperature, and displacement in the physical domain are obtained using a numerical inversion technique. The effects of nonlocal parameter, rotating, magnetic field and the speed of the heat source on the physical fields are detected and illustrated graphically. The results obtained in this work should be useful for researchers in nonlocal material science, low-temperature physicists, new material designers, as well as to those who are working on the development of the theory of nonlocal thermoelasticity.

1. Introduction

The movement of the heat sources is the subject of transient heat transfer, which applies to problems, especially welding engineering. In the early in the previous century, welding engineers began studying the sources of moving heat, both theoretically and empirically [1]. Depending on material properties and plate geometry, the solutions take three different forms: intermediate, semi-infinite, or thin plate. The distribution of temperature and cooling rates can be determined from theoretical solutions to the problem, allowing engineers to understand the significance of heat sources in the welding process and final product performance.

Studying the effect of the magnetic field on thermoelastic medium is another interesting area. The field of study known as magneto thermoelasticity has many applications in several areas, especially in biomedical engineering, nuclear devices and geomagnetic investigations. Some works related to this area are in [2-9]

The nonlocal theory of elasticity was used to study applications in Nano-mechanics, including the lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics, surface tension fluids, etc. Of all the nanostructures, the mechanical behavior of nanotubes and nanobeams have been most widely investigated. The models of the nonlocal beams expected increasing attention in the early few years. In 1972, Eringen introduced the theory of nonlocal continuum mechanics [10-12], in an effort to deal with the small-scale structure problems. The theories of the nonlocal continuum consider the state of stress at a point as a function of the states of the strain of all points in the body while the classical continuum mechanics assume the state of stress at a certain point uniquely depends on the state of the strain on that same point.

In this theory, the equilibrium laws contain non-local residues of fields and these residues are identified with the constitutive equations that form the basis for some requirements of stability and thermodynamic constraints. Constitutive equations and non-local residues are functional of deformation gradients and the motions of all points of the body.

Inan and Eringen [13] investigated the thermoelastic wave propagation in plates based on the nonlocal theory of thermoelasticity. Wang and Dhaliwal [14] introduced the energy and the work equation in nonlocal generalized thermoelasticity and also proved that the initial and boundary value problems have a unique solution. Zenkour and Abouelregal [15] constructed a new model of nonlocal thermoelasticity with phase lags for beam theory due to a harmonically varying heat considering the thermal conductivity to be variable. Koutsoumaris et. al [16] expressed the nonlocal continuum theory, either integral or differential form which is widely used to explain size effect phenomena in micro and nanostructures. Liew et. al [17] introduced a literature review of recent research

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studies on the applications of nonlocal elasticity theory in the modeling and simulation of grapheme sheets. Rajneesh et. al [18] investigated the transient analysis of nonolocal thermoelastic microstretch thick circular plate with phase lags. Solutions for various problems support this theory [19-27]. There are also some other problems that have been studied in this area as in [28-38].

This research presents a thermoelastic analysis of a rotating finite rod subjected to a moving heat source under Eringen nonlocal theory. An analytic technique is presented to display transient nonlocal thermal stresses in a rotating rod with constant angular velocity. The variations of temperature, displacement and stress distributions along the axial direction are investigated. The effects of the moving heat source speed, nonlocal parameter and the applied magnetic field on all studied fields are considered.

2. Review of Nonlocal Thermoelasticity theory

According to the nonlocal elasticity theory of Eringen [10-12], the stress tensor at arbitrary points x of a nano-material body not only depends on strain tensor at x but also depends on all points of the body. The nonlocal elasticity basic equation for isotropic, elastic and homogeneous materials in the absence of body force is expressed as follows:

$$\boldsymbol{\tau}(\mathbf{x}) = \int_V \alpha\left(|\mathbf{x}'-\mathbf{x}|,\xi\right)\boldsymbol{\sigma}(\mathbf{x})\mathrm{d}V\,(\mathbf{x}') \tag{1}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla(\mathbf{u}^T)\right), \tag{2}$$

$$\boldsymbol{\sigma}(\mathbf{x}) = 2\mu\boldsymbol{\varepsilon} + \lambda(\mathrm{div}\mathbf{u})\mathbf{I} - \gamma\theta\mathbf{I} \tag{3}$$

By employing Eringens' nonlocal formulation, the nonlocal stress tensor $\boldsymbol{\tau}(\mathbf{x})$ can be expressed as:

$$\left(1-\xi^2\nabla^2\right)\boldsymbol{\tau}(\mathbf{x}) = \boldsymbol{\sigma}(\mathbf{x}) \tag{4}$$

which takes into account the size effect on the response of nanostructures.

The balance of linear momentum results in the following equation of motion

$$\nabla\cdot\boldsymbol{\tau} + \mathbf{F} = \rho\ddot{\mathbf{u}} \tag{5}$$

where $\mathbf{F}$ represents the external body force vector and ρ is the mass density.

After using (4) and (5), the invariant form of nonlocal equation of motion can be derived as follows:

$$\nabla\cdot\boldsymbol{\sigma} + \left(1-\xi^2\nabla^2\right)\mathbf{F} = \rho\left(1-\xi^2\nabla^2\right)\ddot{\mathbf{u}} \tag{6}$$

Then, the equations of motion can be obtained in terms of the temperature and displacements as

$$(\lambda+\mu)\nabla(\nabla\mathbf{u}) + \mu\nabla^2\mathbf{u} - \gamma\nabla\theta + \left(1-\xi^2\nabla^2\right)\mathbf{F} = \rho\left(1-\xi^2\nabla^2\right)\ddot{\mathbf{u}} \tag{7}$$

One may see that when the internal characteristic length is neglected, i.e., the particles of a medium are considered to be continuously distributed, ξ is zero, and Eq. (4) reduces to the constitutive equation of classical local thermoelasticity.

The modified Fourier's law of heat conduction extended by Lord and Shulman [27] is given by

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial\theta}{\partial t} + \gamma T_0\frac{\partial}{\partial t}(\mathrm{div}\mathbf{u}) - Q\right) = K\nabla^2\theta \tag{8}$$

The Maxwell's electromagnetic field equations for a homogeneous and electrically conducting thermoelastic solid can be retrieved as [39].

$$\mathbf{J} = \nabla\times\mathbf{h},\ \nabla\times\mathbf{E} = -\mu_0\frac{\partial\mathbf{h}}{\partial t},\ \nabla\cdot\mathbf{h} = 0,$$
$$\mathbf{h} = \nabla\times(\mathbf{u}\times\mathbf{H}),\ \mathbf{J} = \sigma_0\left[\mathbf{E} + \mu_0\left(\frac{\partial\mathbf{u}}{\partial t}\times\mathbf{H}\right)\right] \tag{9}$$

When rotated is rotating with an angular, velocity $\mathbf{\Omega}$, we establish the centripetal acceleration and Coriolis increasing speed as a two further terms in the equation of motion, which affects the thermoelastic response. If the thermoelastic medium rotating with uniform angular velocity $\mathbf{\Omega} = \Omega\mathbf{n}$, ($\mathbf{n}$ will be a unit vector demonstrating the direction of the rotation axis) with isotropic, homogeneous electronic and thermoelastic properties, then Eq. (8) can be expressed as[30-33]

$$(\lambda+\mu)\nabla(\nabla\mathbf{u}) + \mu\nabla^2\mathbf{u} - \gamma\nabla\theta + \left(1-\xi^2\nabla^2\right)\mathbf{F} = \rho\left(1-\xi^2\nabla^2\right)\left[\ddot{\mathbf{u}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\dot{\mathbf{u}}\right] \tag{10}$$

For non-rotating media, the angular velocity $\mathbf{\Omega} = 0$ and hence the Centripetal acceleration $\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u})$ and Coriolis acceleration $2\mathbf{\Omega}\times\dot{\mathbf{u}}$ do not appear in the equation of motion (10).

In Eqs. (1)-(10), we have used the following symbolizations:

λ, μ	Lam´e's constants	K	thermal conductivity
α_t	thermal expansion coefficient	$\boldsymbol{I}$	identity tensor
$\gamma = (3\lambda + 2\mu)\alpha_t$	coupling parameter	$\boldsymbol{H}$	initial magnetic field
T_0	environmental temperature	$\boldsymbol{J}$	current density
$\theta = T - T_0$	temperature increment	$\boldsymbol{u}$	displacement vector
T	absolute temperature	$\boldsymbol{F}$	body force vector
C_E	specific heat	Q	heat source
$\boldsymbol{\tau}$	nonlocal stress tensor	τ_0	relaxation time
$\boldsymbol{\sigma}$	local stress tensor	$\boldsymbol{h}$	induced magnetic field
$\boldsymbol{\varepsilon}$	strain tensor	ρ	material density
$\lvert\boldsymbol{x}' - \boldsymbol{x}\rvert$	is the Euclidean distance	$\boldsymbol{E}$	induced electric field
$\alpha(\lvert\boldsymbol{x}' - \boldsymbol{x}\rvert)$	nonlocal Kernel	$oxyz$	Cartesian coordinate
τ_{ij}	local stress tensor	∇^2	Laplacian operator
$\xi = e_0 a/l$	nonlocal parameter	α	fractional derivative of order
a	internal characteristic length	t	the time
l	external characteristic length	μ_0	magnetic permeability
e_0	adjusting constant	σ_0	Electric conductivity

3. Problem Formulation

The problem to be considered is a homogeneous isotropic infinite nonlocal thermoelastic rod that is unstrained and unstressed initially, but has a uniform temperature distribution T_0. We will take the x-axis as the axial direction of the rod. Let $x = 0$ characterizes the plane area over which the moving heat source $Q(x,t)$ is situated and propagating along x direction. The dynamic problem of the rod can be treated as a one-dimensional problem, all the physical variables considered depend only on the space variable x and time variable t. In addition, it is assumed that the rod be rotated along z-axis with a uniform angular speed $\mathbf{\Omega} = (0,0,\Omega)$.

For one-dimensional problems, the components of displacement have the form

$$u_x = u(x,t),\ u_y = u_z = 0 \tag{11}$$

It is assumed that the applying longitudinal magnetic field with constant intensity acts perpendicular to the axial direction of the rod $\mathbf{H}=(0,H_x,0)$.

Lorentz force $\mathbf{F}=\mathbf{J}\times\mathbf{H}$ induced by the applying longitudinal magnetic field $\mathbf{H}$ appearing in the motion Eq. (9) are given by

$$\mathbf{F}=(f_x,f_y,f_z)=-\sigma_0\mu_0H_x^2\left(\frac{\partial u}{\partial t},0,0\right) \tag{12}$$

Equation (4) also gives

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\tau_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}-\gamma\theta \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (10), the following equation of motion can be obtained:

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\left[\rho\frac{\partial^2 u}{\partial t^2}+\rho\Omega^2u+\sigma_0\mu_0H_x^2\frac{\partial u}{\partial t}\right]=(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\gamma\frac{\partial\theta}{\partial x} \tag{14}$$

The heat conduction equation (8) is now given by

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial\theta}{\partial t}+\gamma T_0\frac{\partial^2 u}{\partial t\,\partial x}-Q\right)=K\frac{\partial^2\theta}{\partial x^2} \tag{15}$$

Considering the following dimensionless quantities

$$\begin{gathered}\{x',u'\}=c_0\,\omega_o\{x,u\},\quad \{t',\tau_0'\}=c_0^2\,\omega_o\{t,\tau_0\},\\ \Omega'=\frac{\Omega}{\omega_0},\theta'=\frac{\theta}{T_0},\ \xi'=c_0^2\,\omega_0^2\,\xi,Q'=\frac{Q}{K\,T_0\,c_0^2\,\omega_0^2},\\ \tau_{xx}'=\frac{\tau_{xx}}{\mu},\ c_0=\sqrt{\frac{\lambda+2\mu}{\rho}},\ \omega_0=\frac{\rho_0C_E}{K_0}.\end{gathered} \tag{16}$$

Governing equations (13)-(15) may be finally written as (dropping the primes)

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\tau_{xx}=\beta^2\frac{\partial u}{\partial x}-b\,\theta \tag{17}$$

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\left[\frac{\partial^2 u}{\partial t^2}+\Omega^2u+\varepsilon\frac{\partial u}{\partial t}\right]=\frac{\partial^2 u}{\partial x^2}-\frac{b}{\beta^2}\frac{\partial\theta}{\partial x} \tag{18}$$

$$\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial\theta}{\partial t}+g\frac{\partial^2 u}{\partial t\,\partial x}-Q\right)=\frac{\partial^2\theta}{\partial x^2} \tag{19}$$

Where

$$\beta^2=\frac{\lambda+2\mu}{\mu},b=\frac{\gamma T_0}{\mu},g=\frac{\gamma}{\rho C_E},\varepsilon=\frac{\sigma_0\mu_0H_x^2}{\rho c_0^2\,\omega_o}. \tag{20}$$

The homogeneous initial conditions are taken as

$$u(x,t)\big|_{t=0}=\frac{\partial u(x,t)}{\partial t}\bigg|_{t=0}=0,\quad \theta(x,t)\big|_{t=0}=\frac{\partial\theta(x,t)}{\partial t}\bigg|_{t=0}=0 \tag{21}$$

We will assume the rod has a non-dimensional length L and the displacement $u(x,t)$ and temperature $\theta(x,t)$ needs to satisfy the following four spatial boundary conditions

$$u(x,t)\big|_{x=0}=\frac{\partial u(x,t)}{\partial x}\bigg|_{x=L} \tag{21}$$

$$\frac{\partial\theta(x,t)}{\partial x}=0\quad\text{at } x=0,L \tag{22}$$

The rod is subjected to a moving plane of the heat source of constant strength Q_0, releasing its energy continuously while moving along the positive direction of x-axis with a constant velocity c. This moving heat source is assumed to be the following non-dimensional form [34]

$$Q=Q_0\delta(x-ct)\ \text{on}\ x=0,L \tag{23}$$

4. Solution of the problem in the Laplace transforms domain

The closed form solution of the governing and constitutive equations can be possible by adapting the Laplace transformation technique. Taking the Laplace transform defined by the relation

$$\bar f(x,s)=\int_0^\infty f(x,t)e^{-st}dt \tag{24}$$

to both sides of Eqs. (17)-(19) and using the homogeneous initial conditions (21), one gets the field equations in the Laplace transform space as

$$\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar\tau_{xx}=\beta^2\frac{\mathrm{d}\bar u}{\mathrm{d}x}-b\,\bar\theta \tag{25}$$

$$\alpha_0\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar u=\frac{\mathrm{d}^2\bar u}{\mathrm{d}x^2}-\frac{b}{\beta^2}\frac{\mathrm{d}\bar\theta}{\mathrm{d}x} \tag{26}$$

$$(1+\tau_0s)\left(s\bar\theta+sg\frac{\mathrm{d}\bar u}{\mathrm{d}x}-\frac{Q_0}{c}e^{-\frac{sx}{c}}\right)=\frac{\mathrm{d}^2\bar\theta}{\mathrm{d}x^2} \tag{27}$$

Where $\alpha_0=s^2+\Omega^2+\varepsilon s$.

Elimination $\bar\theta$ from Eqs. (25) and (26), one obtains:

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4}-m_1\frac{\mathrm{d}^2}{\mathrm{d}x^2}+m_2\right)\bar u(x)=m_3e^{-\frac{sx}{c}} \tag{28}$$

where the coefficients m_1, m_2 and m_3 are given by

$$\begin{gathered}m_1=\alpha_4+\frac{\alpha_2}{\alpha_1}+\frac{\alpha_3\alpha_5}{\alpha_1},m_2=\frac{\alpha_2\alpha_4}{\alpha_1},m_3=\frac{s\,\alpha_3\alpha_6}{c\,\alpha_1},\\ \alpha_1=1+\xi^2\left(s^2+s\varepsilon+\Omega^2\right),\ \alpha_4=s\left(1+s\tau_0\right),\\ \alpha_2=\left(s^2+s\varepsilon+\Omega^2\right),\alpha_5=sg\left(1+s\tau_0\right),\\ \alpha_3=\frac{b}{\beta^2},\alpha_6=\frac{Q_0}{c}\left(1+s\tau_0\right)\end{gathered} \tag{29}$$

The solution of equations (28) can be represented as

$$\bar u(x)=\sum_{n=1}^{2}\left(C_n\,\mathrm{e}^{-k_nx}+C_{n+2}\,\mathrm{e}^{k_nx}\right)+C_5e^{-\frac{sx}{c}} \tag{30}$$

Where $C_n,(n=1,2,3,4)$ are parameters depending on s to be determined from the boundary conditions.

In Eq. (30) k_1 and k_2 are the roots of the characteristic equation

$$k^4 - m_1 k^2 + m_2 = 0 \tag{31}$$

and the parameter C_5 is given by

$$C_5 = \frac{m_3}{(s/c)^4 - m_1 (s/c)^2 + m_2} \tag{32}$$

In the same way, eliminating $\bar{u}$ between Eqs. (25) and (26),we obtain

$$\left(\frac{d^4}{dx^4} - m_1 \frac{d^2}{dx^2} + m_2 \right) \bar{\theta}(x) = m_4 e^{-\frac{sx}{c}} \tag{33}$$

Where $m_4 = \dfrac{\alpha_2 \alpha_6}{\alpha_2} - \dfrac{s^2 \alpha_6}{c^2}$.

The solution of the differential equation (33) can be written as

$$\bar{\theta}(x) = \sum_{n=1}^{2} \left(\beta_n C_n\, e^{-k_n x} + \beta_{n+2} C_{n+2}\, e^{k_n x} \right) + C_6 e^{-\frac{sx}{c}} \tag{34}$$

where the compatibility between $\bar{\theta}(x)$ and $\bar{u}(x)$ in Eq. (26), gives

$$\beta_n = -\frac{\alpha_1 k_n^2 - \alpha_2}{\alpha_1 k_n},\ n = 1,2,$$
$$\beta_n = \frac{\alpha_1 k_n^2 - \alpha_2}{\alpha_1 k_n},\ n = 3,4, \tag{35}$$
$$C_6 = \frac{m_4}{(s/c)^4 - m_1 (s/c)^2 + m_2}.$$

Using Eqs. (30) and (34) in Eq. (25), the stress component $\bar{\tau}_{xx}(x)$ can be determined as

$$\bar{\tau}_{xx}(x) = \sum_{n=1}^{2} \left(-\gamma_n C_n\, e^{-k_n x} + \gamma_{n+2} C_{n+2}\, e^{k_n x} \right) + C_7 e^{-\frac{sx}{c}} \tag{36}$$

Where

$$\beta_n = \frac{\beta^2 k_n + b\beta_n}{1 - \xi^2 k_n^2},\ n = 1,2,$$
$$\beta_n = \frac{\beta^2 k_n - b\beta_{n+2}}{1 - \xi^2 k_n^2},\ n = 3,4, \tag{37}$$
$$C_6 = \frac{-C_5 \beta^2 (s/c) - bC_6}{1 - \xi^2 (s/c)^2}.$$

After using Laplace transform, the boundary conditions (21) and (22) take the forms

$$\bar{u}(x,s)\big|_{x=0} = \left.\frac{\partial u(\bar{x},s)}{\partial x}\right|_{x=L},$$
$$\frac{d\bar{\theta}(x,s)}{\partial x} = 0 \quad \text{at } x = 0, L \tag{38}$$

Substituting Eqs. (30) and (34) into the above boundary conditions, one obtains four linear equations

$$\sum_{n=1}^{2} \left(C_n + C_{n+2} \right) = -C_5,$$
$$\sum_{n=1}^{2} \left(k_n C_n\, e^{-k_n L} - k_n C_{n+2}\, e^{k_n L} \right) = -\frac{sC_5}{c} e^{-\frac{sL}{c}} \tag{39}$$

$$\sum_{n=1}^{2} \left(\beta_n k_n C_n - \beta_{n+2} k_n C_{n+2} \right) = -\frac{sC_6}{c},$$
$$\sum_{n=1}^{2} \left(\beta_n k_n C_n\, e^{-k_n L} - \beta_{n+2} k_n C_{n+2}\, e^{k_n L} \right) = -\frac{sC_6}{c} e^{-\frac{sL}{c}} \tag{40}$$

The solution of the above system of linear equations gives the unknown parameters $C_n, (n = 1,2,3,4)$. In order to determine the studied fields in the physical domain, the Riemann-sum approximation method is used to obtain the numerical results. The details of these methods can be found in Honig and Hirdes [46].

5. Numerical results

In order to illustrate and compare the analytic results obtained in the previous sections, we now demonstrate a numerical example, which represent the distributions of thermodynamic temperature $\theta(x,t)$, displacement $u(x,t)$, and nonlocal stress component $\tau_{xx}(x,t)$. For the purpose of numerical computations, the material is specified as copper. The relevant material parameters necessary to be known are given in Table 1 [23].

Table1: Mechanical and thermoelastic properties of the rod. ($T_0 = 293\text{K}$):

Material properties	Value
Thermal conductivity $K\,(\text{Wm}^{-1}\text{K}^{-1})$	386
Young' modulus $E\,(\text{GPa})$	128
Density $\rho(\text{Kgm}^{-3})$	8954
Thermal expansion $\alpha_t\,(\text{K}^{-1})$	1.78×10^{-5}
Electric conductivity $\sigma_0(\text{Fm}^{-1})$	$10^{-9}/36\pi$
Magnetic permeability $\mu_0(\text{Hm}^{-1})$	$10^{-7}\times4\pi$
Longitudinal magnetic field $H_x\,(\text{Am}^{-1})$	$10^{-7}/4\pi$
Poisson's ratio ν	0.36
Specific heat C_E (J/KgK)	384.56

The results are represented graphically in Figs. (1-16) at different positions x. The computations were carried out for the wide range of $x, (0 \le x \le 10)$ at small value of time $t = 0.2$. For all numerical calculations ***Mathematica*** programming Language has been used. The field quantities such as the temperature, the strain, the nonlocal stress and the displacement distributions depend not only on the space coordinates x and time t, but also depend on the nonlocal parameter ξ, moving heat source c, magnetic field H_x, and the rotating parameter Ω. Numerical calculation is made for four cases.

5.1. Influence of the rotation

In this case, we take into consideration three different values of the rotation parameter $\Omega = 0,1,5$, while the other parameters have been taken as $\xi = 0.1$, $c = 1$, and the parameter $\varepsilon = 1$. Figures 1-4 are drawn to give a comparison of the obtained results for the displacement $u(x,t)$, strain $e(x,t)$, nonlocal force stress $\tau_{xx}(x,t)$, and temperature distribution $\theta(x,t)$ against positions (x,t) in the cases of absence and presence of the rotation effect.

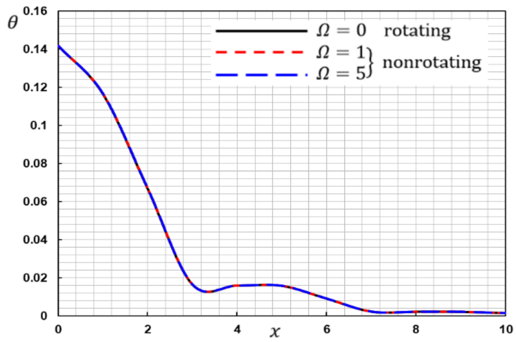

Figure 1: The temperature θ with different rotation parameter Ω .

From all these figures, it is evident that all curves coincident when x tends to infinity, all physical fields satisfy the boundary conditions. Thus, the obtained solution is limited to a finite area of space and does not spread to infinity. This is not in the case of the coupled theory of thermoelasticity, where the solution extends to infinity rapidly, suggesting an infinite velocity in the propagation of waves. The rotating field has noticeable effects on all the profiles of the studied fields.

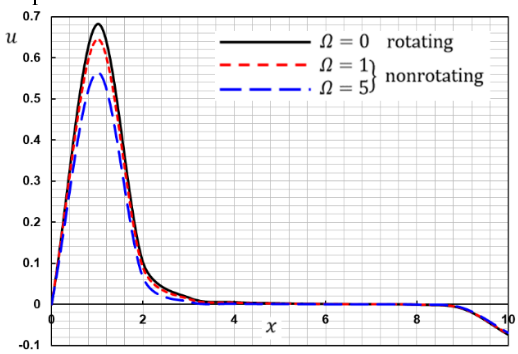

Figure 2: The displacement u with different rotation parameter Ω .

Figures 1–4 show that the rotation parameter Ω acts to increase the nondimensional nonlocal stress $\tau_{xx}(x,t)$, whereas acts to decrease displacement distribution $u(x,t)$. In all cases, the displacement attains maximum values and gradually decreases continuously small negative values. The changes in the nonlocal parameter do not feel any effects on the temperature and the strain, and it remains unchanged with the changes in the nonlocal parameter. It is noticed the displacement and nonlocal stress shows an increase in nature to increase or decrease amplitude with respect to the distance x due to the presence of the rotation terms. This view applies to many authors as well as the author [47].

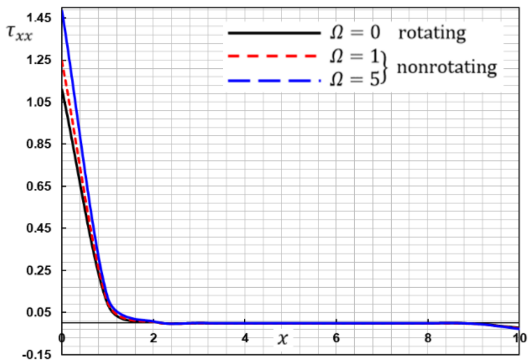

Figure 3: The nonlocal stress τ_{xx} distributions with different rotation parameter Ω .

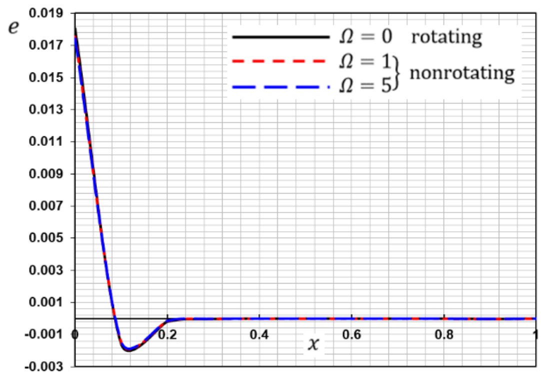

Figure 4: The strain distributions e with different rotation parameter Ω .

5.2. The effect of moving heat source velocity parameter

The first case investigating how the non-dimensional displacement, temperature, and nonlocal stress vary with different values of the moving heat source velocity c when the other parameters remain constant (see Figures 5−8).

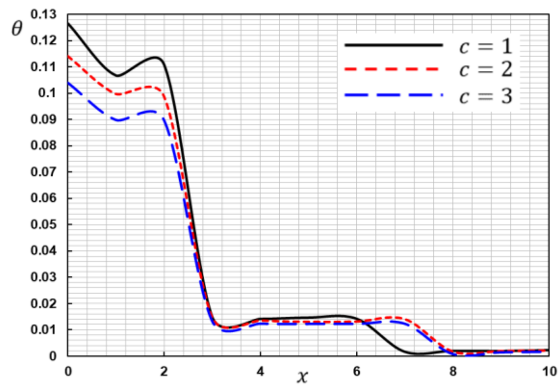

Figure 5: The temperature θ with different values of the moving heat source velocity c .

From the figures 5−8, it is observed that the nature of variations of all the field variables for moving heat source velocity parameter is significantly different. It can be observed that the heat source velocity c has a great effect on the displacement, temperature, strain and nonlocal stress distributions. The rate of increment of nonlocal stress $\tau_{xx}(x,t)$ is very slow with the speed of the heat source. For a fixed value of x , the displacement is decreased with increasing value of heat source velocity c , and these variations are fairly obvious.

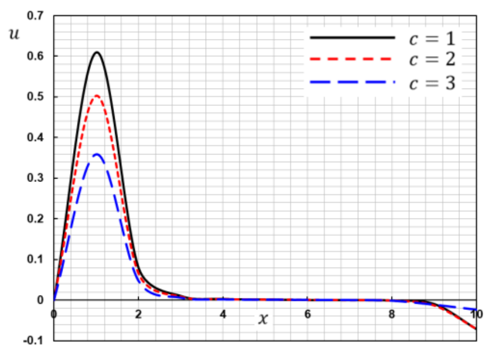

Figure 6: The displacement u with different values of moving heat source velocity c .

Also, the temperature decreases as the moving heat source velocity increases. The thermodynamic temperature, is oscillatory in nature, but the amplitude of the oscillation is decreasing with increasing distance from the heat source is exactly the same as that of [45].

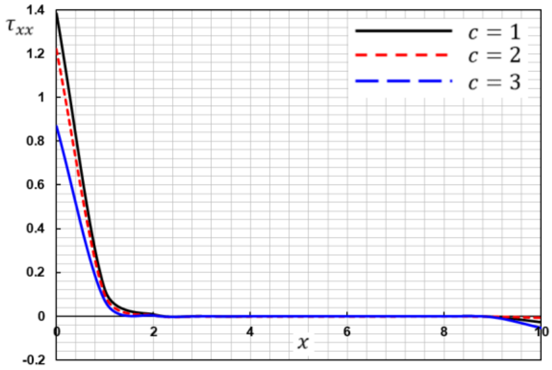

Figure 7: The nonlocal stress τ_{xx} with different values of moving heat source velocity c .

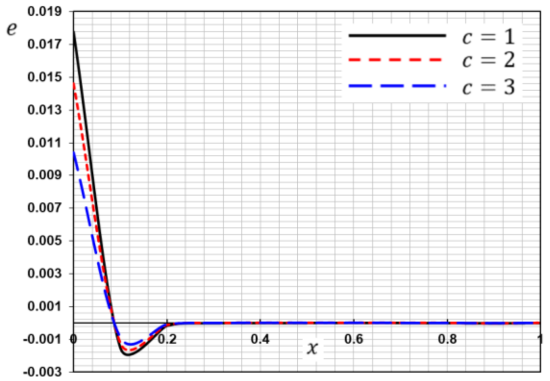

Figure 8: The strain distributions e with different values of moving heat source velocity c .

5.3. The effect of the nonlocal parameter

With a view toward describing the aim of this article, the distributions of temperature, displacement, and nonlocal stress for different values of nonlocal parameter ξ are introduced. In this case, we notice that when the nonlocal parameter ξ vanishing ($\xi = 0$) indicates the old situation (local model of elasticity) while other values indicate the nonlocal theories of elasticity and thermoelasticity.

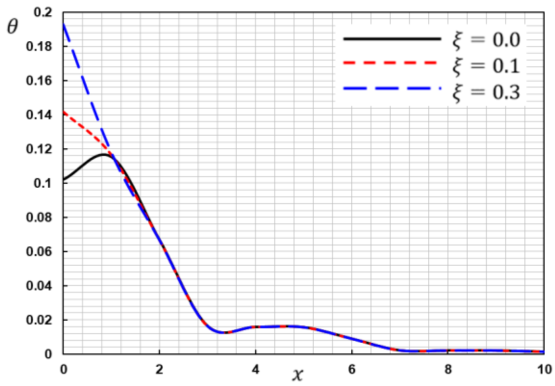

Figure 9: The temperature θ with different values of nonlocal parameter ξ .

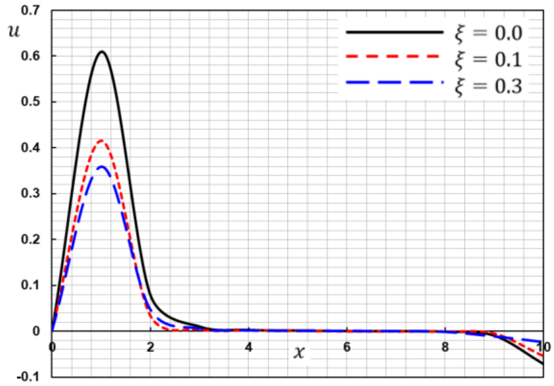

Figure 10: The displacement u with different values of nonlocal parameter ξ .

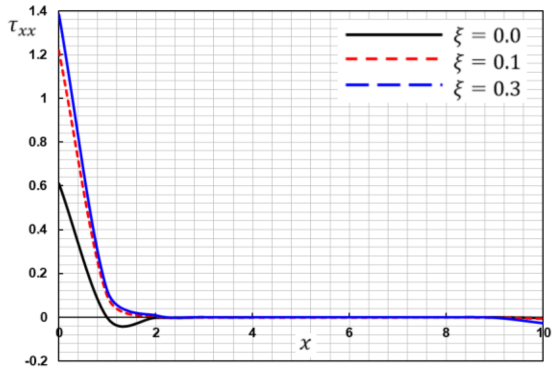

Figure 11: The nonlocal stress τ_{xx} with different values of nonlocal parameter ξ .

Figures (9−12) show that this parameter has a significant effect on all the fields. The waves reach the steady state depending on the value of the nonlocal parameter ξ. The concluding remarks from the Figures can be shortened as follows:

- The changes in the nonlocal parameter do not feel any effects on the temperature, and it remains unchanged with the changes in the nonlocal parameter.
- The temperature $\theta(x,t)$ small, depending on the variation of nonlocal parameter $c = 1$.
- When moving heat source velocity is fixed ($c = 2$), the result of $\xi = 0.1, 0.2$ agreeing well with that for $\xi = 0$, that is, the nonlocal scale parameter has no effect on the temperature.
- However, the nonlocal scale parameter $c = 1$ greatly affects the distributions of displacement $u(x,t)$ and nonlocal stress $\tau_{xx}(x,t)$.
- The displacement $u(x,t)$ decrease when nonlocal parameter $c = 1$ increases.
- The nonlocal effect is a significant factor that could not be ignored in determining stress in sudden nano-scale heating problems source is the same as that of [6,8].

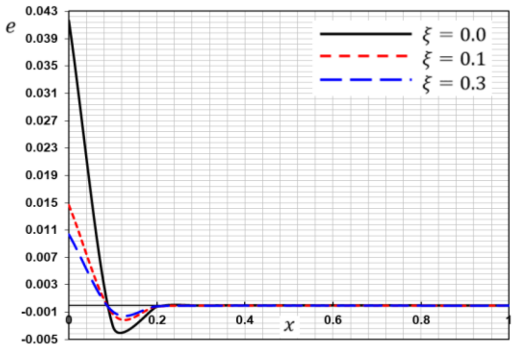

Figure 12: The strain distributions e with different values of nonlocal parameter ξ .

5.4. The effect of the applied magnetic field

This case illustrates how the field quantities vary with the different values of the applied magnetic field parameter ($\varepsilon = 0,1,2$) with constant $c = 2$, and $\xi = 0.1$. The numerical results are obtained and presented graphically in Figs. (13-16).

Also, we can conclude that:

- We can see the significant effect of the applied magnetic field ε on the displacement $u(x,t)$ and The nonlocal stress $\tau_{xx}(x,t)$.

- The increase in the value of the parameter ε causes a decrease in the values of the lateral displacement which is very obvious in the peak points of the curves. This indicates that the magnetic field acts to damp the thermal expansion deformation of the rod. The results are exactly the same as those reported in [21]

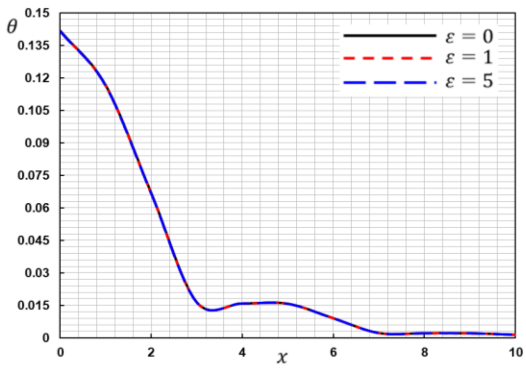

Figure 13: The variation of temperature θ with the applied magnetic field.

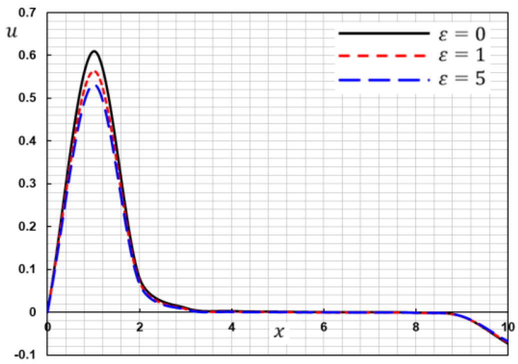

Figure 14: The variation of displacement u with the applied magnetic field.

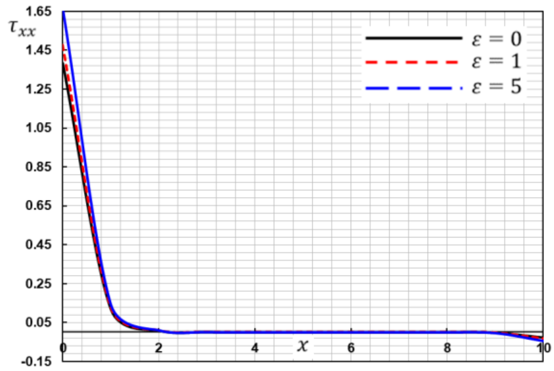

Figure 15: The variation of nonlocal stress τ_{xx} with the applied magnetic field.

- The increasing on the value of the parameter ε causes increases in the values of the nonlocal stress field which is very obvious in the starting points of the curves.
- The applied magnetic field has no effect on the variation of temperature is the same as given in [21].

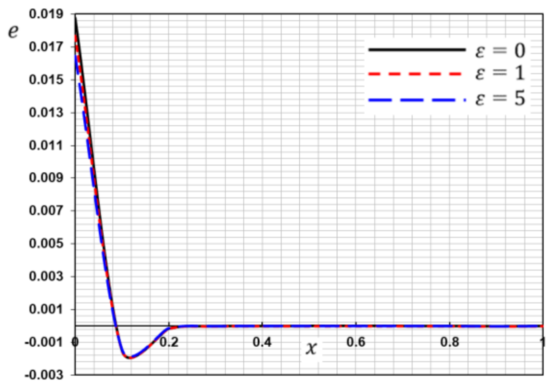

Figure 16: The variation of strain distributions e with the applied magnetic field.

6. Conclusions

In this work, nonlocal generalized thermoelasticity based on Eringen's nonlocal elasticity is proposed. The paper presents an analytic solution for thermoelastic homogeneous rotating finite rod subjected to a periodic source is presented. The nonlocal governing equations of the problem were transferred by applying the Laplace transform and then were solved numerically by using Taylor's expansion series.

The results are displayed graphically to illustrate the effect of nonlocal parameter, magnetic field, the speed of the heat source and the rotating. Also, the results indicate that the field quantities such as the temperature, the strain, the nonlocal stress, and the displacement distributions depend not only on the space coordinate x and time t , but also depend on the nonlocal parameter ξ , moving heat source, magnetic field H_x , and the rotating parameter Ω .

The results obtained in this work should be useful for researchers in nonlocal material science, low-temperature physicists, new materials designers, as well as to those who are working on the development of the theory of nonlocal thermoelasticity.

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