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Modified mathematical model for variable fill fluid coupling

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ABSTRACT

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1. Introduction

Fluid couplings are used in industrial transmission systems as a coupling, torque limiter, soft starter and speed control units. These couplings are composed of a pump and a turbine and the power is transmitted through the coupling by the fluid flow between the pump and the turbine Therefore, there is no mechanical contact in these couplings. Modeling turbomachinery with analytical and numerical methods is of great importance in the study of their behavior and optimizing them [1-3]. This model can be used in different applications such as design of the control system in variable speed units, coupling sizing, thermal analysis of the coupling and coupling selection. Although there are suitable off design models for fully filled fluid coupling [4], little work has been done for partially filled fluid couplings [5]. However, their application is quite common in speed control units and also, usually one size of coupling is used for a wider range of power transmission applications by changing the amount of oil in the coupling. Fully filled fluid coupling can be modeled by several approaches [4-7]. In one of the simplest and most efficient methods, coupling losses is calculated both from slip between the pump and the turbine and the sum of frictional, incidence and circulation losses1. Then, by equating these losses, the fluid mass flow rate can be estimated between the pump and the turbine. When the fluid mass flow rate is in hand, other coupling variables such as coupling transmitted power and torque can be calculated easily. Maqableh has extended this method for partially filled fluid couplings [5]. He considered two different flow regime for the fluid flow in the coupling and derived the governing equations for partially filled fluid coupling. Although this model can be tuned to have good performance for partially filled couplings with fixed amount of oil, but more work is

Variable fill fluid couplings are used in the speed control units. Also, variation in coupling oil volume is used in adapting one size of coupling to a wider range of power transmission applications. Available model for the partially filled fluid couplings, has a good performance for couplings with fixed amount of oil but their performance will be degraded if they are used for the variable fill couplings. In this paper, the current model for partially filled fluid couplings is modified to have better performance for variable fill couplings. For this purpose, the circulation loss calculation is modified and also, the effect of oil temperature variations and blade thickness are included in the model. The effect of these modification on the model performance are investigated in couple of simulations. Comparing the simulation results with the available experimental data shows that the suggested modifications can improve the model performance very well.

needed to improve the model accuracy for variable fill couplings and reducing the tuning procedure.

In this paper, the model for partially filled fluid coupling is modified so that it has better performance specially for variable fill couplings and it can be used for wider range of fill angles without any additional adjustment. For this purpose, circulation loss calculation is modified and the effect of oil temperature variations on coupling performance is considered. Also, the blade thickness is included in the model equations. The available data from one of the well-known coupling manufactures is used to evaluate the effectiveness of the proposed modifications on the coupling model.

2. Partially filled fluid coupling

Fluid coupling is composed of a pump and a turbine and the power transmitted by the fluid flow from the pump to the turbine. The Fluid flow in the coupling has two main components, one is the circumferential flow about the coupling axis and the other is the circulation of the fluid in the vortex passing from the pump to the turbine in the plane of the coupling axis (Figure 1). For partially filled fluid couplings, two flow regime can be considered for the vortex flow, the outer centered flow regime and the annular flow regime (Figure 2) [5]. In the current work, outer centered flow regime is assumed and the partially filled coupling equations are derived by considering the blade thickness. Also, almost in all the commercial couplings, the turbine and the pump have the same impeller with straight blade angle. The only difference between their impellers is that there is one more blade in one of them to eliminate resonance at the blade passing frequency.



Figure 1. Fluid coupling and the oil flow path(b)



Figure 2. Coupling configuration, outer centered and annular flow

By the assumption of the outer centered flow regime for the vortex flow and straight blades, the geometric dimensions of the vortex can be calculated from continuity equation and coupling geometry,

$$R_i = \frac{(D'-d')}{2} \tag{1}$$

$$R_{o} = \frac{(D'+d')}{2} \tag{2}$$

$$D + d = D' + d' \tag{3}$$

in which D, D', d and d' are center to center distance of the cups, center to center distance of the fluid vortex, cup diameter and vortex diameter respectively. If one defines oil volume fraction F as,

$$F = \frac{\text{oil volume}}{\text{coupling volume}} = \frac{d^2 D'}{d^2 D}$$
(4)

and then combines eq. (4) with eq. (3), following third order equation can be reached to calculate the vortex diameter (d'),

$$d'^{3}-(D+d) d'^{2}+FD d'^{2}=0$$
(5)

Then, eq. (1) to eq. (3) can be used to calculate other coupling parameters. Now, by the assumption of the linear velocity profile for the vortex flow (Figure 3) and applying the continuity equation the vortex mean radius (Rm)can be calculated by,

$$R_{m} = \sqrt{\frac{R_{o}^{2} + R_{i}^{2}}{2}}$$
(6)

After calculating the geometric dimensions of the vortex, the fluid speed that passes between the pump and the turbine should be calculated. This can be done by equating the loss calculated from the slip between the pump and the turbine and the loss calculated from the sum of frictional, incidence and circulation losses.

The torque developed in a turbomachine can be estimated by calculating the rate of change of angular momentum [4]. Consequently, if one assumes that fluid leaves the impeller's blade with the same angle (which is zero for straight blades) the coupling torque (τ) will be,

$$\tau = \int (C_{\theta_2} R - C_{\theta_4} r) d\dot{m}$$

$$= \int_{R_m}^{R_o} \omega_p R^2 \rho (2\pi R - Z_p t) (R - R_m) \omega_{vor} dR$$

$$- \int_{R_i}^{R_m} \omega_t r^2 \rho (2\pi r - Z_t t) (R_m - r) \omega_{vor} dr$$
(7)



Figure 3. Geometric parameters of the coupling (a) and linear velocity distribution in the vortex (b)

in which C_{θ} is the tangential component of the fluid velocity, Z_p is the number of blades in the pump, Z_t is the number of blades in the turbine, t is the blade thickness, ω_p is the angular velocity of the pump, ω_t is the angular velocity of the turbine and ω_{vor} is the angular velocity of the fluid circulation in the vortex. Integrating this equation with the assumption of the linear velocity distribution for the vortex will give,

$$\tau = \mathbf{K}_{1} \,\omega_{\rm vor} \,\omega_{\rm p} \tag{8}$$

where,

$$K_{1} = 2 \pi \rho \left[\frac{R_{o}^{5}}{5} - \frac{R_{o}^{4}R_{m}}{4} + \frac{R_{m}^{5}}{20} - (1-S)(\frac{Rm^{5}}{20} - \frac{R_{i}^{4}R_{m}}{4} + \frac{R_{i}^{5}}{5}) \right] - \rho Z_{p} t \left[\frac{(R_{o}^{4} - R_{m}^{4})}{4} + \frac{R_{m}(R_{o}^{3} - R_{m}^{3})}{3} \right] - (1-S) \rho Z_{t} t \left[\frac{(R_{m}^{4} - R_{i}^{4})}{4} + \frac{R_{m}(R_{m}^{3} - R_{i}^{3})}{3} \right]$$
(9)

and S is the slip factor and it is defined as,

$$S = \frac{\omega_p - \omega_t}{\omega_p} \tag{10}$$

Now, the coupling loss (P_{loss}) can be calculated by the difference between the input work to the pump impeller (P_p) and the output work from the turbine runner (P_t) ,

$$P_{\text{loss}} = P_{\text{p}} - P_{\text{t}} = K_1 \,\omega_{\text{vor}} \,\omega_{\text{p}}^2 \,(1 - S) \tag{11}$$

On the other hand, the coupling loss can be estimated from the sum of frictional (P_f) , incidence (P_i) and circulation losses (P_c) ,

$$\mathbf{P}_{\text{loss}} = \mathbf{P}_{\text{f}} + \mathbf{P}_{\text{i}} + \mathbf{P}_{\text{c}} \tag{12}$$

The frictional loss can be estimated by the well-known loss equation for the pipes. The blade thickness affects the frictional loss by changing the flow passage width. By following the procedure given by [4], this loss can be calculated by,

$$\mathbf{P}_{\mathrm{f}} = K_f \omega_{vor}^{3} \tag{14}$$

where,

$$K_{f} = \frac{\pi \rho f(Z_{p} + Z_{t})}{40} [(R_{m} - R_{i} - D_{L})^{5}(R_{o} - R_{m} - D_{U})^{5}] + \frac{\pi \rho f}{16} [(2(Z_{p} + Z_{t}) D_{L} + 2\pi R_{i} - (Z_{p} + Z_{t})t)(R_{m} - R_{i})^{4} + (2(Z_{p} + Z_{t}) D_{u} + 2\pi R_{0} - (Z_{p} + Z_{t})t)(R_{o} - R_{m})^{4}]$$
(14)

in which f is the friction factor and should be calculated from Moody diagram by considering local Reynolds number, and D_L , D_u are the depth of the lower and upper stream line thickness and they are given by following equations for the outer centered flow regime [5],

$$D_{\rm L} = \frac{R_{\rm m} - R_{\rm i}}{10} \tag{15}$$

$$R_{o} = \frac{(D'+d')}{2} \tag{16}$$

The incidence loss for the impeller with straight blades is given as [4],

$$P_{i} = \int \frac{r^{2}}{2} (\omega_{T} - \omega_{p})^{2} d\dot{m} + \int \frac{R^{2}}{2} (\omega_{T} - \omega_{p})^{2} d\dot{m}$$

$$= \int_{R_{i}}^{R_{m}} \frac{r^{2}}{2} (\omega_{T} - \omega_{p})^{2} \rho (2\pi r - Z_{p}t) (R_{m} - r) \omega_{vor} dr$$

$$+ \int_{R_{m}}^{R_{i}} \frac{R^{2}}{2} (\omega_{T} - \omega_{p})^{2} \rho (2\pi R - Z_{t}t) (R - R_{m}) \omega_{vor} dR$$

$$= \omega_{vor} \omega_{p}^{2} S^{2} (K_{i1} + K_{i2})$$
(17)

where,

$$K_{i1} = \pi \rho \left(\frac{R_{m}^{5}}{20} - \frac{R_{i}^{4}R_{m}}{4} + \frac{R_{i}^{5}}{5}\right) - \frac{\rho Z_{p} t}{2} \left(\frac{R_{m}^{4}}{12} - \frac{R_{i}^{3}R_{m}}{3} + \frac{R_{i}^{4}}{4}\right)$$

$$K_{i2} = \pi \rho \left(\frac{R_{0}^{5}}{5} - \frac{R_{0}^{4}R_{m}}{4} + \frac{R_{m}^{5}}{20}\right)$$

$$- \frac{\rho Z_{t} t}{2} \left(\frac{R_{m}^{4}}{12} - \frac{R_{0}^{3}R_{m}}{3} + \frac{R_{0}^{4}}{4}\right)$$
(18)

The circulation loss can be estimated by the minor loss equation for pipes. The circulation path in the coupling can be modeled with four simple right angle bends following each other. The pressure loss for each bend can be calculated by,

$$dp_{\rm B} = K \frac{C_{\rm m}^{2}}{2} d\dot{m}$$
(19)

in which, K is an empirically determined loss coefficient and C_m is the meridional component of the fluid velocity. For example, for lower quadrant of the pump the circulation loss will be,

$$dP_{B_{1}} = \int_{R_{i}}^{R_{m}} K \frac{(R_{m} - r)^{2} \omega_{vor}^{2}}{2} \rho (2\pi r - Z_{p} t) (R_{m} - r) \omega_{vor} dr \qquad (20)$$
$$= K_{B_{1}} \omega_{vor}^{3}$$

where,

$$K_{B1} = K \pi \rho \left(\frac{R_{m}^{5}}{20} + \frac{R_{i}^{5}}{5} - \frac{R_{m}^{3}R_{i}^{2}}{2} - \frac{3R_{m}R_{i}^{4}}{4} + R_{m}^{2}R_{i}^{3}\right) - \frac{\rho Z_{p} t}{2} \left(\frac{R_{m}^{4}}{4} - R_{m}^{3}R_{i} + \frac{3R_{m}^{2}R_{i}^{2}}{2} - R_{m}R_{i}^{3} + \frac{R_{i}^{4}}{4}\right)$$
(21)

If one uses similar procedure for other quadrants of the flow path, the circulation loss will be given by,

$$P_{\rm B} = \sum_{i=1}^{4} K_{Bi} \omega_{vor}^3 \tag{22}$$

in which,

$$K_{B2} = K \pi \rho \left(\frac{R_{m}^{5}}{20} + \frac{R_{o}^{5}}{5} - \frac{R_{m}^{3}R_{o}^{2}}{2} - \frac{3R_{m}R_{o}^{4}}{4} + R_{m}^{2}R_{0}^{3}\right) - \frac{\rho Z_{p} t}{2} \left(\frac{R_{m}^{4}}{4} - R_{m}^{3}R_{o} + \frac{3R_{m}^{2}R_{o}^{2}}{2} - R_{m}R_{o}^{3} + \frac{R_{o}^{4}}{4}\right)$$
(23)
$$K_{B3} = K \pi \rho \left(\frac{R_{m}^{5}}{20} + \frac{R_{i}^{5}}{5} - \frac{R_{m}^{3}R_{i}^{2}}{2} - \frac{3R_{m}R_{i}^{4}}{4} + R_{m}^{2}R_{i}^{3}\right) - \frac{\rho Z_{t} t}{2} \left(\frac{R_{m}^{4}}{4} - R_{m}^{3}R_{i} + \frac{3R_{m}^{2}R_{i}^{2}}{2} - R_{m}R_{i}^{3} + \frac{R_{i}^{4}}{4}\right)$$

$$K_{B3} = K \pi \rho \left(\frac{R_{m}^{5}}{20} + \frac{R_{i}^{5}}{5} - \frac{R_{m}^{3}R_{i}^{2}}{2} - \frac{3R_{m}R_{i}^{4}}{4} + R_{m}^{2}R_{i}^{3}\right)$$
$$- \frac{\rho Z_{t} t}{2} \left(\frac{R_{m}^{4}}{4} - R_{m}^{3}R_{i} + \frac{3R_{m}^{2}R_{i}^{2}}{2} - R_{m}R_{i}^{3} + \frac{R_{i}^{4}}{4}\right)$$

Now, by equating the loss given by eq. (11) and eq. (12), the fluid circulation velocity can be calculated(ω_{vor}). After the fluid circulation velocity is calculated, the transmitted torque and other coupling variable can be estimated from respective equations.

3. Variable fill coupling

In the previous section, the model for fully filled fluid coupling is modified for the partially filled fluid couplings and the governing equations are driven by the assumption of the new flow regime. For couplings with fixed amount of oil, different model parameters can be tuned so that it can estimate different coupling variables with a reasonable accuracy. However, the simulation results show that the model will be accurate only around that fill angle and if the change in the coupling oil volume is large, the model results will deviate from experimental data. This issue is more crucial when the fluid coupling is used as a speed control unit and therefore, the controller regulates the oil volume in the coupling to reach the required speed ratio. Consequently, the equations should be modified to have better accuracy for the change in the oil volume. As it is shown in previous section, the fluid coupling model is developed based on the loss calculation in the coupling. The total loss equation (eq. (11)) is calculated from slip between the pump and the turbine and therefore, it does not depend on the flow regime and the oil volume and the same equation can be used for the variable fill couplings. On the other hand, equations for calculation of frictional, incidence and circulation losses should be studied in more details. The frictional loss is estimated by the loss equation for the pipes and since the equivalent hydraulic diameter, Reynolds number, friction factor and stream length are evaluated in each iteration, it can be used for variable fill coupling with a reasonable accuracy. Incidence loss mostly depends on the relative velocity between the pump and the turbine when it is assumed that the fluid leaves the impeller with the blade angle. Therefore, it is not affected by the flow regime and the oil volume in the coupling. The circulation loss for the coupling is modeled with four simple right angle bends and it is calculated by eq. (19). In this equation, K is an empirically determined loss coefficient and it depends on the relative radius of the bend. By variations in oil volume the vortex radius changes in the coupling and fixed loss coefficient is not accurate anymore. To solve this problem in the modified model, the diagram given by White [8] for estimation of the loss coefficient in the right angle bends is used (Figure 4). Consequently, in the modified model, the relative radius is calculated and then the loss coefficient is estimated from Figure 3 in each iteration. The effect of this modification on the model accuracy will be more understandable when the simulation results shows that about 50-60% of the coupling loss is related to the circulation losses. It should be noted that for bends which are used in series, the value of K is not simply additive and the loss coefficient for each bend should be considered to be 80% of the coefficient read from diagram [4].

Another important issue that should be addressed in the variable fill coupling modeling is the variation of the oil temperature. This is because the oil temperature variation will change the oil density and viscosity and consequently, this would affect loss in the coupling which is the basis for coupling modeling. The oil temperature in the coupling is proportional to the transmitted power and coupling slip. Therefore, when the

amount of the oil varies in the coupling, the transmitted power of the coupling will change and consequently, the oil temperature will alter [9]. This issue will be important for coupling without oil circulation and limited slip only. Because, in couplings that are used as a speed reduction unit which have large slip values there is oil circulation circuit that controls the oil temperature.

The oil temperature can be calculated from the steady state heat balance for the fluid coupling,

heat dissipated in the coupling =

heat transferred from coupling to ambient (22)

$$SP_p = F_H (T_{oil} - T_{air})$$

where T_{air} is the ambient temperature and F_H is the empirical heat transfer coefficient which is a function of coupling rotational speed and diameter [10]. To study the oil temperature effect on the coupling performance couple of simulations are done in the following section.



Figure 4. The empirical circulation loss coefficient as a function of bend relative radius [8]

4. Simulation results

To evaluate the performance of the proposed model, couple of simulations are done for a sample coupling. For this purpose, the available data from one of the well-known fluid coupling manufactures, Rexnord Corporation, is used. In their catalog, the transmittable power by the coupling is given in different fill angles (Figure 5). Also in another table they provide the relationship between the oil volume in the coupling and the fill angle. The simulations are done for 20-inch diameter coupling (coupling size=1480) and in different fill angles. For this purpose, a code is developed in the Matlab software and the equations are solved simultaneously with Newton-Raphson method.

55

2.7

1170

83

Table 1. The effect of modified circulation loss coefficient on model accuracy

Catalog data					Model		Modified model			
Input power [hp]	slip(%)	speed [RPM]	fill angle[°]	power [hp]	oil temp.[°C]	error [%]	power [hp]	oil temp.[°C]	error [%]	
100	3.8	1170	64	99.99	69.78	-0.01	99.99	70.71	-0.01	
87.5	3.3	1170	68	88.70	64.17	1.37	87.39	64.59	-0.13	
75	3	1170	73	78.49	60.51	4.65	75.56	60.50	0.75	
67.5	2.9	1170	77	71.98	58.81	6.64	68.22	58.60	1.06	
60	2.7	1170	80	64.90	56.65	8.17	60.70	56.27	1.17	
55	2.7	1170	83	61.82	56.13	12.40	57.09	55.68	3.81	

 Table 2. Oil temperature effect on original coupling model

Catalog data				Fi	xed oil tempera	ature	Variable oil temperature		
Input power [hp]	slip(%)	speed [RPM]	fill angle[°]	power [hp]	oil temp.[°C]	error [%]	power [hp]	oil temp.[°C]	error [%]
100	3.8	1170	64	99.99	60	-0.01	99.99	69.78	-0.01
87.5	3.3	1170	68	88.80	60	1.49	88.70	64.17	1.37
75	3	1170	73	78.70	60	4.93	78.49	60.51	4.65
67.5	2.9	1170	77	72.00	60	6.67	71.98	58.81	6.64
60	2.7	1170	80	65.30	60	8.83	64.90	56.65	8.17
55	2.7	1170	83	62.50	60	13.64	61.82	56.13	12.40

Table 3. Oil temperature effect on modified coupling model

	Catalo	og data		Fi	xed oil tempera	ature	Variable oil temperature		
Input power [hp]	slip(%)	speed [RPM]	fill angle[°]	power [hp]	oil temp.[°C]	error [%]	power [hp]	oil temp.[°C]	error [%]
100	3.8	1170	64	99.99	60	-0.01	99.99	70.71	-0.01
87.5	3.3	1170	68	86.76	60	-0.85	87.39	64.59	-0.13
75	3	1170	73	75.98	60	1.31	75.56	60.50	0.75
67.5	2.9	1170	77	69.11	60	2.39	68.22	58.60	1.06
60	2.7	1170	80	61.85	60	3.09	60.70	56.27	1.17
55	2.7	1170	83	58.64	60	6.61	57.09	55.68	3.81

Table 4. Modified model versus original model										
	Catalo	og data		Model			Modified model			
Input power [hp]	slip(%)	speed [RPM]	fill angle[°]	power [hp]	oil temp.[°C]	error [%]	power [hp]	oil temp.[°C]	error [%]	
100	3.8	1170	64	99.99	60	-0.01	99.99	70.71	-0.01	
87.5	3.3	1170	68	88.06	60	0.64	87.39	64.59	-0.13	
75	3	1170	73	78.26	60	4.34	75.56	60.50	0.75	
67.5	2.9	1170	77	71.94	60	6.58	68.22	58.60	1.06	
60	2.7	1170	80	65.11	60	8.51	60.70	56.27	1.17	

62.08

60

12.87

57.09

55.68

3.81



Figure 5. fill angle definition and its effect on the coupling oil volume

Different simulation has been done to evaluate the effect of oil temperature and modified circulation loss formulation. In first simulation, the effect of variable circulation loss coefficient has been investigated. For this purpose, the performance of original model and the modified model has been evaluated in different operating points and different fill angles. The results of these simulations are given in Table 1. The results show that the new estimation method for circulation loss coefficient can improve the model accuracy. This improvement is greater for larger fill angles where fluid vortex diameter is reduced and circulation losses are increased. The is clear because, in the given diagram for the loss coefficient for modified model, the coefficient of circulation losses increased by reduction of the fluid vortex diameter.

Another simulation is done to study the effect of oil temperature on both models. The results of these simulations are shown for original model and modified model in Table 2 and Table 3 respectively. As it is clear, the calculation of the oil temperature for each operating point can improve the model accuracy in different operating points.

In the last simulation both modified circulation loss and the variable oil temperature are considered at the same time. The results of this simulation are summarized in Table 4. As the results show, the model accuracy for variable fill coupling can be improved very well by using new estimation method for circulation loss coefficient and taking into account the oil temperature variations.

The model error increases as the power decreases for both models. This is because, the model has few parameters such as loss coefficient that should be tuned for each coupling. These parameters are assumed to be fixed but as we know they are not. Usually these parameters are tuned so the model have the best performance in the design point (full power). When the model deviates from the design point the model accuracy will be degraded because of the assumption of fixed parameters. This issue needs more investigation in future works.

5. Conclusion

In this paper, the original model for the partially filled coupling has been modified to have better performance for the variable fill couplings. Since fluid coupling model is based on the loss calculations, coupling loss equations have been examined and a new method has been suggested to evaluate the circulation losses for the variable fill coupling. Moreover, the effect of oil temperature variations on the model performance has been studied and blade thickness has been included in the coupling model. The suggested modification on modeling accuracy has been investigated by couple of simulations and the obtained results are compared with the experimental data from a coupling manufacturer. The simulation results have been shown that the suggested modifications can improve the model accuracy very well.

Nomenclature

- C_{θ} : Tangential component of the fluid velocity
- d : Cup diameter
- *d'* : Vortex diameter
- *D* : Center to center distance of the cups
- *D'* : Center to center distance of the fluid vortex
- D_L : Depth of the lower stream line thickness
- D_u : Depth of the upper stream line thickness
- *f* : Friction factor
- *F* : Defines oil volume fraction
- F_H : Empirical heat transfer coefficient
- *K* : Empirically determined circulation loss coefficient
- P_c : Circulation loss
- P_f : Frictional loss
- P_i : Incidence loss
- *P*_{loss} : Coupling loss
- P_P : Input power
- P_t : Output power

r

R

- : Element radius (Figure. 2)
- : Element radius (Figure. 2)
- R_i : Inner radius of the coupling
- R_m : Mean radius
- R_o : Outer radius of the coupling
- *s* : Slip factor
- *t* : Blade thickness
- T_{air} : Ambient temperature
- *T_{oil}* : Oil temperature
- Z_p : Number of blades in the pump
- Z_t : Number of blades in the turbine
- ω_p : Angular velocity of the pump
- ω_t : Angular velocity of the turbine
- ω_{vor} : Angular velocity of the fluid circulation
- τ : Coupling torque

References

[1] A. Bahreini, A. Sattari, Numerical and Economic Study of Performance of Centrifugal Pump as Turbine, *Journal of Computational Applied Mechanics*, Vol. 48, No. 2, pp. 151-160, 2017.

[2] A. Javanbakht, h. ahmadi danesh ashtian, Impeller and volute design and optimization of the centrifugal pump with low specific speed in order to extract performance curves, *Journal of Computational Applied Mechanics*, pp. -, 2018.

[3] A. Riasi, F. Dianatipoor, Performance Improvements of a Centrifugal Pump with Different Impellers using Polymer Additive, *Journal of Computational Applied Mechanics*, Vol. 48, No. 2, pp. 199-206, 2017.

[4] F. J. Wallace, A. Whitfield, R. Sivalingam, A theoretical model for the performance prediction of fully filled fluid couplings, *International Journal of Mechanical Sciences*, Vol. 20, No. 6, pp. 335-347, 1978/01/01/, 1978.

[5] A. M. Maqableh, Mathematical Modelling of Partially Filled Fluid Coupling Behaviour, *International Journal of Mechanical*, *Aerospace, Industrial, Mechatronic and Manufacturing Engineering*, Vol. 5, pp. 2625-2631, 1/, 2011.

[6] G. H. Rolfe, Research on the Hydraulic Coupling, *Proceedings of the Institution of Mechanical Engineers*, Vol. 183, No. 1, pp. 219-232, 1968.

[7] A. Whitfield, R. Sivalingam, F. J. Wallace, The performance prediction of fluid coupling with the introduction of a baffle plate, *International Journal of Mechanical Sciences*, Vol. 20, No. 10, pp. 729-736, 1978/01/01/, 1978.

[8] F. White, 2015, Fluid Mechanics,

[9] P. Akbarzadeh, S. M. Hashemian, H. Raznahan, Numerical Simulation of Thermal and Hydrodynamic Behavior of Mixtures of Air and Oil in Hydraulic Coupling, *Tabriz Mechanical Engineering*, Vol. 46, No. 3, pp. 11-19, 2016.

[10] O. V. Yaremenko, T. I. Kononenko, Fluid coupling for starting up heavy machinery heat regime calculations and ways to reduce thermal stresses, *Khimicheskoe i Neftyanoe Mashinostroenie*, Vol. 1, pp. 9-11, 1968.