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Nonlinear Stability of Rotating Two Superposed Magnetized Fluids With The Homotopy Perturbation Technique

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ABSTRACT

In the present work, the Rayleigh-Taylor instability of two rotating superposed magnetized fluids within the presence of a vertical or a horizontal magnetic flux has been investigated. The nonlinear theory is applied, by solving the equation of motion and uses the acceptable nonlinear boundary conditions. However, the nonlinear characteristic equation within the elevation parameter is obtained. This equation features a transcendental integro-Duffing kind. The homotopy perturbation technique has been applied by exploitation the parameter growth technique that results in constructing the nonlinear frequency. Stability conditions are derived from the frequency equation. It's illustrated that the rotation parameter plays a helpful result. It's shown that the stability behavior within the extremely uniform rotating fluids equivalents to the system while not rotation. A periodic solution for the elevation function has been performed. Numerical calculations area unit created for linear analysis furthermore the nonlinear scope. Moreover, the elevation function has been premeditated versus the time parameter. The strategy adopted here is vital and powerful for solving nonlinear generator systems with a really high nonlinearity arising in nonlinear science and engineering.

1. Introduction

The dynamics of wave motion of nice importance in physical investigations, as wave motion, constitutes one in each of the principal modes of transmission of energy. The energy received from the sun is transmitted by waves within the ether and therefore the energy of sound by airwaves. A wave suggests that the continual transference of a specific state or from one facet of a medium to a different. This will imply the transference of the medium itself from one space to a different however simply the propagation through it's of a specific kind, state or condition. The waves because of little oscillating motion that crop up at and close to the surface of a vast of a vast sheet of fluids are known as surface waves [1].

The properties of the linear stability for many flows are mentioned clearly and given by designer [2] and Chandrasekhar [3]. Bauer [4] has performed a close analysis of the natural frequencies of a rapidly rotating viscous and periodical fluid column in an exceeding type of geometries. El-Dib [5] investigated the instability of the flow ensuing from the oscillations of a rapidly periodic rotating cylindrical fluid column constant flow instability downside beneath the result of the constant vertical magnetic field of force was mentioned by Moatimid and El-Dib [6]. Hatay et al [7] think about the stability with linear analysis for the compressible of viscous rotate Couette flow having high-speed in 3 dimensions of the disturbance. Sarma et al [8] examined the linear hydrodynamic stability of a thin-liquid layer flowing on the within wall of a vertical tube rotating regarding its axis within the flowing of a core-gas beneath the constraints that the density and viscosity ratios of gas to liquid square

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measure little and also the relative the vertical element of the pressure within the gas is reminiscent of constant order as gravity. Mohamed et al [9] investigate the linear stability of a two rotating liquid jet beneath a periodic axial field of force. Leblanc and Cambon [10] investigated linear stability of an inviscid plane flows in an incompressible homogeneous rotating fluid. The vector of the angular rate $\Omega = \Omega \underline{e}_z$ of the rotating frame is taken into account vertical to the plane of the fundamental system. El-Dib [11] investigated the instability of a rotate a fluid column beneath the influence of a periodic magnetic field. At Prandtl variety, the stability of flow in an exceedingly horizontal fluid layer subjected to uniformly rotation was investigated by Schwarz [12].

The possibility of erratic stability of the interface between two fluids by suggests that of fluids by means of an external body field, apart from gravity, the encouragement from phenomena in stars and planetary interiors were a number of the needs for the extent of the RTI analysis to incorporate the result of rotation and magnetic flux. Coriolis and centrifugal forces are more common in these interiors and play a very important role in determining many phenomena including RTI. The result of rotation at associate in periodic angle with gravity was initially investigated by Hide [13], who the first one gave an exhaustive analysis for the rotation parallel to gravity. Bjerknes et al. [14] were thought-about the primary to research the influence of vertical rotation in an exceedingly two-fluid layer system. Davalos-Orozco and Aguilar-Rosas [15] studied the rotating for magnetic fluid in an exceedingly general rotation field. They found that, for sufficiently high values of the wavenumber, a bifurcation happens within the plane of the angle of most rates against the wavenumber for a vertical component of rotation component in magnitude with the tangential component. Davalos-Orozco[16] investigated the RTI of two stratified fluid below a horizontal rotation field yet because of the action of horizontal magnetic fields. Chakraborty and Chandra [17] studied the RTI of two uniform fluids separated by a layer of transition of finite thickness within which the density will increase exponentially within the vertical direction and therefore the full system rotates uniformly around a columnar. Chakraborty [18] studied a similar drawback subjected to a tangential magnetic flux. Davalos-Orozco [19] investigated the RTI of two superposed fluid layer system below a general rotation field, the conditions that gravity was perpendicular to the two horizontal layers associate in the periodic rotation had a capricious angle with relation to the vertical. Two stratified plasmas of the RTI-type, consisting of interacting ions and a neutral, in an exceedingly tangential magnetic flux was investigated by Sharma and Chhajlani [20]. Hemamalini and Anjali Hindu deity [21] take into account the rotating RTI subject to a time-dependent horizontal magnetic flux and relevant the solutions by exploitation the multiple scales methodology.

Nonlinear interfacial Rayleigh–Taylor instability of two-layers flow with an ac electric field was analyzed by Moatimid and El-Bassiouny [22]. Anjalidevi and Hemamalini [23] considered the effect of parallel rotation and a standard flux. RTI in three dimensions was studied by Stone and Gardiner[24]. Yu and Livescu [25] mentioned RTI in an exceedingly cylindrical pure mathematics with compressible fluids. RTI in nonconductor fluids was examined by Joshi et al [26]. On the very long time simulation of the RTI was analyzed by Lee et al [27]. Wang et al [28] thought-about density gradient effects in nonlinear ablative RTI.Tao et al [29] analyzed the nonlinear RTI of rotating inviscid fluids with the axis of rotation traditional to the acceleration of the interface between two uniform inviscid fluids. Rayleigh-Taylor stability boundary at solid-liquid interfaces was mentioned by Priz et al. [30].

Murakami [31], El-Dib [32, 33], Hemamalini and Anjali Devi [21] investigate the interaction of finite amplitude waves by victimization the strategy of multiple scales they introducing the scales

$$T_n = \varepsilon^n t \text{ and } X_n = \varepsilon^n x;\ n = 01,2 .$$

They assume a small parameter ε expressing the steepness ratio of the wave. They didn't get the formula of the approximate solution in terms of the amplitude wave. The goal of this work is to review the nonlinear stability of two superposed rotating magnetic fluids subjected to a vertical or a horizontal magnetic flux while not employing a small parameter.

A generalization of the nonlinear RTI under the result of rigid-body rotation and therefore the magnetic flux is that the goal of the study. The homotopy perturbation technique [34-37] is applied. This technique doesn't use the small parameter as mentioned before within the previous studies. The applying of the homotopy technique results in formulating the associate approximate periodical answer to the nonlinear generator with a very high nonlinearity, whereas the classical perturbation ways square measure invalid for determination such high nonlinear oscillators. The technology of the parameter growth [38, 39] is utilized to get the associate approximate nonlinear frequency that permits explanation the stability criteria of the nonlinear downside adopted here.

The analysis of linear theory as comes in Chandrasekhar's book [3] depends on removing the nonlinear terms from the equation of motion further as from the boundary conditions then yielding a dispersion relation while not nonlinear terms. The thought of the frail nonlinear elaboration may be a very little departure from the linear viewpoint. At this stage, the nonlinear drawback can embrace the linear elaboration with some further terms that perform a correction for the most solution. The analysis of the nonlinear elaboration given here depends on dropping the nonlinear terms from the equation of motion and applying the acceptable boundary conditions while not neglecting the nonlinear terms. At this stage, the dispersion equation ought to be extended to incorporate nonlinear terms. This method ends up to extending the well-known Chandrasekhar dispersion relation [3] to gift nonlinear terms within the surface elevation. Then, the nonlinear dispersion equation springs, so it depends not solely on the national frequency and also for the wave number however on all the physical parameters of the matter. This conclusion of the nonlinear dispersion equation is incredibly sophisticated and contains high nonlinear terms within the elevation parameter with its derivatives.

2. Formulation of the problem

For the base flow in the RTI problem, two incompressible, immiscible fluids having constant properties occupy the half-spaces. The lighter fluid with density $\rho^{(1)}$ and hydrostatic pressure $-\rho^{(1)}gz$, occupies the upper half-space for $z>0$, while the heavier fluid with density $\rho^{(2)}<\rho^{(1)}$ and pressure $-\rho^{(2)}gz$, occupies the lower half-space for $z<0$, where g is the gravitational acceleration acts in the negative z-direction. The interface between the fluids is assumed to be well defined and initially flat and has infinite extension having the horizontal surface in the x-y plane. It is acted upon by a vertical magnetic field $\underline{H}(0,0,H)$ and uniform rotation with an angular velocity $\underline{\Omega}(0,0,\Omega)$ and a gravitational field $\underline{g}(0,0,-g)$.

As the underlying condition of the framework, we expect that both liquid stages are immiscible and have a typical level interface at $z=0$. The appropriation balance of the interface between both fluid stages has been built up. We are worried about the interfacial reaction of the two stages after an inconvenience for the balance setup. The surface diversion is communicated by [3]

$$z=\eta(x,y,t) \tag{1}$$

If the surface is determined as the locus of points satisfying the relation

$$S(x,y,z,t)=z-\eta(x,y,t)=0 \tag{2}$$

Then the unit normal vector to the interface is given by

$$\underline{n}=\frac{\nabla S}{|\nabla S|}=\left(-\eta_x\underline{e}_x-\eta_y\underline{e}_y+\underline{e}_z\right)\left(1+\eta_x^2+\eta_y^2\right)^{-1/2} \tag{3}$$

where $\underline{e}_x$, $\underline{e}_y$ and $\underline{e}_z$ are the unit vectors in the x-, y- and z- directions.

The elements of the issue are qualified by the synchronous arrangement of three field conditions: Maxwell's conditions, the condition of movement and the progression condition. In planning Maxwell's conditions for the issue, we guess that the magneto-semi static estimation is substantial for the issue [40, 41].

The topical in the attractive fluid is intrigued with marvels in which attractive vitality extraordinarily surpasses electric vitality stockpiling and where the engendering times of electromagnetic waves are finished in moderately brief occasions contrasted with those important to us. The fluids are subjected to vertical magnetic fields $H^{(1)}$ and $H^{(2)}$ acting in the negative z-direction. As needs are, Maxwell's conditions lessen to

$$\nabla.\left(\mu^{(j)}\underline{H}^{(j)}\right)=0, \quad \text{and} \tag{4}$$

$$\nabla\times\underline{H}^{(j)}=\underline{0};\ j=1,2, \tag{5}$$

where μ is the magnetic permeability for the fluid phase and the superscript j refers to the fluid phase. The superscript (1) and (2) refer to the upper fluid and lower fluid respectively. In conformity with the legality of the quasi-static approximation, a potential function $\phi(x,y,z,t)$ can be introduced such that

$$\underline{H}^{(j)}=-H^{(j)}-\nabla\phi^{(j)}(x,y,z,t). \tag{6}$$

Clearly, the function $\phi(x,y,z,t)$ satisfies Laplace's equation

$$\nabla^2\phi^{(j)}(x,y,z,t)=0. \tag{7}$$

Smooth movement is chosen by an arrangement of nonlinear incomplete differential conditions communicating protection of mass, force, and vitality. We consider media that are at first uniform so movement is of homogenous fluids in a homogeneous medium. The key conditions overseeing the movement, for the main part of attractive liquid stages, written in the $\underline{\Omega}$ rotating casing of reference Weidman et. al. [42] as

$$\rho\left(\frac{\partial\underline{V}}{\partial t}+(\underline{V}.\nabla)\underline{V}+2(\underline{\Omega}\times\underline{V})-\tfrac{1}{2}\nabla\left(\left|\underline{\Omega}\times\underline{r}\right|^2\right)\right)=-\nabla p-\rho g\underline{e}_z, \tag{8}$$

related to the continuity equation

$$\nabla.\underline{V} = 0, \tag{9}$$

where p is the hydrodynamic pressure, ρ is the fluid density and $\underline{V} = (u, v, w)$ represents the velocity of the fluid. The term $2(\underline{\Omega} \times \underline{V})$ in equation (8) represents the Coriolis acceleration and the term $\frac{1}{2}\nabla\left(\left|\underline{\Omega} \times \underline{r}\right|^2\right)$ platforms for the centrifugal force where $\underline{r}$ the position vector of any point of the fluid is. The Coriolis term is usefully presented in the equation of motion in the rotating frame of reference when the angular velocity Ω is uniform. The total pressure is defined as

$$\pi = p - \tfrac{1}{2}\rho\Omega^2 r^2 - \tfrac{1}{2}\mu H^2. \tag{10}$$

The pure equilibrium configuration gives,

$$\pi_0^{(j)}(x, y, z, t) = -\rho^{(j)} g z + \tfrac{1}{2}\rho^{(j)}\Omega^{(j)2} r^2 + \lambda_0^{(j)}, \tag{11}$$

where $\lambda_0^{(j)}$ is due to the integration. The balance of the normal stress tensor at the interface leads to

$$\lambda_0^{(2)} - \lambda_0^{(1)} = -\tfrac{1}{2}\left(\rho^{(2)}\Omega^{(2)2} - \rho^{(1)}\Omega^{(1)2}\right) r^2 + \tfrac{1}{2}\left(\mu^{(2)} H^{(2)2} - \mu^{(1)} H^{(1)2}\right). \tag{12}$$

2.1 Boundary Conditions

It is convenient to enclose that $\phi(x, y, z, t)$ is a finite function presented due to the perturbed interface and far from the interface, its influence vanishes. Therefore both the partial derivative for $\phi(x, y, z, t)$ with respect to x, y and z must vanish as $z \to \pm\infty$. At the dividing surface, the following boundary conditions must be satisfied:

(i) The continuity of the vertical component of the magnetic displacement at the surface of separation is

$$\underline{n}.\left(\mu^{(1)}\underline{H}^{(1)} - \mu^{(2)}\underline{H}^{(2)}\right) = 0, \quad z = \eta. \tag{13}$$

This leads to

$$\eta_x\left(\mu^{(1)}\phi_x^{(1)} - \mu^{(2)}\phi_x^{(2)}\right) + \eta_y\left(\mu^{(1)}\phi_y^{(1)} - \mu^{(2)}\phi_y^{(2)}\right) - \left(\mu^{(1)}\phi_z^{(1)} - \mu^{(2)}\phi_z^{(2)}\right) = 0, \quad z = \eta. \tag{14}$$

(ii) The continuity of the tangential component of the magnetic field is assumed at the interface $z = \eta$. Thus,

$$\underline{n} \times \left(H^{(1)} - H^{(2)}\right) = 0, \qquad z = \eta. \tag{15}$$

It yields that

$$\left(\eta_y - \eta_x\right)\left[\left(H^{(1)} - H^{(2)}\right) + \left(\phi_z^{(1)} - \phi_z^{(2)}\right)\right] + \left(\phi_y^{(1)} - \phi_y^{(2)}\right) - \left(\phi_x^{(1)} - \phi_x^{(2)}\right) = 0, \ z = \eta. \tag{16}$$

(iii) At the dividing surface, the velocity field u, v and w are subject to the following boundary condition:

$$w^{(j)} = \eta_t + u^{(j)}\eta_x + v^{(j)}\eta_y, \ z = \eta. \tag{17}$$

This equation comes across the assumed material character of the dividing surface. Far from the interface, the fluid velocity vanishes. Thus,

$$\underline{V}^{(j)}(x, y, \pm\infty, t) = 0. \tag{18}$$

(iv) At the interface between fluids, the fluids and the magnetic stresses must be balanced. The components of these consist of the hydrodynamic pressure, surface tension effects and magnetic stresses. The magnetic stresses result from the magnetization forces [40] and [41]. Thus, the normal component of the stress tensor σ_{ij} is discontinuous at the interface by the surface tension, i.e.

$$\underline{n}.\left(\underline{F}^{(1)} - \underline{F}^{(2)}\right) = \sigma_T \nabla.\underline{n}, \ \ z = \eta, \tag{19}$$

where $\underline{F}$ is the force vector acting on the interface, the surface tension coefficient is denoted by the parameter σ_T and the tensor σ_{ij} given by

$$\sigma_{ij} = -\pi\delta_{ij} + \mu H_i H_j - \tfrac{1}{2}\mu H^2 \delta_{ij}, \tag{20}$$

where π is the total pressure.

(v) The boundary conditions performed here are prescribed at the interface $z = \eta(x, y, t)$. As the interface is disfigured, all variables are slightly perturbed from their equilibrium values. Because the interfacial displacement is small, then the boundary conditions on perturbation with interfacial variables need to be estimated at the equilibrium position rather than at the interface. Therefore, it is needful to accurate all the physical quantities involved in terms of the Maclaurin series about $z = 0$.

3. Solutions of the equation of motion

In assessing them in the light of straight annoyance, the second request, and in addition the higher-order terms containing the rise parameter η, is disregarded. For nonlinear extension, these terms of higher-requests of η won't be dropped. To treat the issue under thought, three-dimensional limited unsettling influences are brought into the condition of movement and congruity condition and in addition limit conditions. As it is common in hydrodynamic stability examination [3], where all amounts have exponential intermittent spatial reliance and obscure time reliance. In the conclusion of a check Fourier disintegration, we may expect that the mass arrangements are of the shape

$$\underline{V}(x,y,z,t)=\underline{\hat{V}}(z,t)e^{ik\zeta}, \tag{21}$$

$$\pi(x,y,z,t)=\hat{\pi}(z,t)e^{ik\zeta}, \tag{22}$$

$$\phi(x,y,z,t)=\hat{\phi}(z,t)e^{ik\zeta}. \tag{23}$$

We suppose there is a uniform monochromatic wave train propagating along the interface such that

$$\eta(x,y,t)=\xi(t)e^{ik\zeta}+c.c, \tag{24}$$

where *c.c.* represents complex conjugate for preceding terms, the spatial variable ζ is defined as

$$\zeta=\frac{1}{k}\left(k_x x+k_y y\right);\quad k=\sqrt{k_x^2+k_y^2}, \tag{25}$$

which is assumed to be real and positive. The amplitude $\xi(t)$ has the initial conditions $\xi(0)=A$ and $\xi_t(0)=0$, where A is a constant.

Employing (21) and (22) into equations of motion (8) and (9) yields the following system:

$$\hat{\pi}(z,t)=-\frac{\rho}{k^2}\left(1+\frac{4\Omega^2}{D^2}\right)\frac{\partial^2\hat{w}(z,t)}{\partial z\partial t}, \tag{26}$$

$$\frac{\partial^2\hat{w}(z,t)}{\partial z^2}-q^2\hat{w}(z,t)=0, \tag{27}$$

where the integral operator q is given by

$$q^2=k^2\left(1+\frac{4\Omega^2}{D^2}\right)^{-1};\ D=\frac{\partial}{\partial t} \tag{28}$$

The development of the administrator q with the indispensable impact needs to accept that the angular velocity is little. In the event that the prerequisite is to grow q as an arrangement in forces of the differential administrator D^2 , then it very well may be looked for in the shape as

$$q=\frac{kD}{2\Omega}\left(1+\frac{D^2}{4\Omega^2}\right)^{-1/2}. \tag{29}$$

The above formal of the administrator q needs the impact of the highly rotating fluids, with the end goal to extend it. Condition (27) is a direct halfway differential condition where its correct arrangement is performed through the kinematic limit condition (17), the conveyance of the vertical speed segment gives

$$\begin{aligned} w^{(1)}(x,y,z,t)&=\left(1-\xi q^{(1)}\right)^{-1}\frac{d\xi}{dt}e^{ik\zeta-q^{(1)}z};\quad z>0,\\ w^{(2)}(x,y,z,t)&=\left(1+\xi q^{(2)}\right)^{-1}\frac{d\xi}{dt}e^{ik\zeta+q^{(2)}z};\ z<0. \end{aligned} \tag{30}$$

Employing the distribution of (30) into (25) yields the pressure distribution in the two fluids in the form

$$\begin{aligned} \pi^{(1)}(x,y,z,t)&=\frac{\rho^{(1)}}{q^{(1)}}\frac{d}{dt}\left(1-\xi q^{(1)}\right)^{-1}\frac{d\xi}{dt}e^{ik\zeta-q^{(1)}z};\ \ z>0,\\ \pi^{(2)}(x,y,z,t)&=-\frac{\rho^{(2)}}{q^{(2)}}\frac{d}{dt}\left(1+\xi q^{(2)}\right)^{-1}\frac{d\xi}{dt}e^{ik\zeta+q^{(2)}z};z<0. \end{aligned} \tag{31}$$

To derive the solution for the magnetic function $\phi(x,y,z,t)$, we insert (23) into the Laplace equation (7), for using both conditions (14) and (16), the resulting solution is

$$\begin{aligned} \phi^{(1)}(x,y,z,t)&=H^{(1)}\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\frac{\xi}{(1-k\xi)}e^{ik\zeta-kz};\ z>0,\\ \phi^{(2)}(x,y,z,t)&=-H^{(2)}\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\frac{\xi}{(1+k\xi)}e^{ik\zeta+kz};z<0. \end{aligned} \tag{32}$$

As the nonlinear terms are disregarded, the linear profile emerges and it is identical to those acquired already by El-Dib [43] for interface supporting free electric surface streams and by Chandrasekhar [3] for unadulterated fluids.

4. Transcendental nonlinear characteristic equation

At the limit between the fluids, the fluid and the magnetic stresses must be adjusted. The segments of these burdens rely upon hydrodynamic weight, surface pressure stresses, and attractive anxieties. In what pursues, we will determine the nonlinear condition overseeing the interfacial relocation. Utilizing the vertical part of the speed circulation (30), the weight dissemination (31), and the attractive potential dispersion (32) to the typical pressure tensor (19), the subsequent is the accompanying nonlinear trademark condition as far as the displacement $\xi(t)$:

$$\begin{aligned} &\left(\frac{\rho^{(1)}}{q^{(1)}}\frac{d}{dt}\left(1-\xi\,q^{(1)}\right)^{-1}+\frac{\rho^{(2)}}{q^{(2)}}\frac{d}{dt}\left(1+\xi\,q^{(2)}\right)^{-1}\right)\frac{d\xi}{dt}\\ &-\left(\rho^{(1)}-\rho^{(2)}\right)g\xi+k^2\sigma_T\xi\left(1-k^2\xi^2\right)^{-3/2}\\ &-k\sigma_{H\perp}\xi\left(1-k^2\xi^2\right)^{-1}\left[1+Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right]=0, \end{aligned} \tag{33}$$

where the notion $\sigma_{H\perp}$ is referred to the performance of the normal magnetic field which given by

$$\sigma_{H\perp}=H^{(1)}H^{(2)}\frac{\left(\mu^{(1)}-\mu^{(2)}\right)^2}{\mu^{(1)}+\mu^{(2)}}. \tag{34}$$

On the other hand, if the system is stressed by a tangential magnetic field such that $H=H_0\underline{e}_z$ then the same characteristic equation will be obtained except that the term $\sigma_{H\perp}$ should be replaced by

$$\sigma_{H\|} = -H_0^2 \frac{\left(\mu^{(1)} - \mu^{(2)}\right)^2}{\mu^{(1)} + \mu^{(2)}}, \tag{35}$$

where the notion $H\|$ denotes the horizontal magnetic term. The parameter $J = 1$ in the stage of the vertical field, while $J = -1$ refers to the application of a horizontal magnetic field.

Condition (33) is a cubic supernatural integro-Duffing condition which administers the wave engendering along the interface between two rotating liquids. This alters the trademark condition that got in the linear discussion given by Chandrasekhar [3] for rotating fluids. It speaks to an augmentation of the scattering connection for Chakraborty and Chandra [17] and Davalos-Orozco [16, 19] by including some higher-arrange terms of the height parameter ξ. In the non- rotated fluids, it diminishes for magnetic fluids in permeable media by El-Dib and Ghaly [44].

Due to the very complicated of equation (33), a simplification of equal rotation can be considered such that the angular velocities $\Omega^{(1)} = \Omega^{(2)} = \Omega$, which leads to $q^{(1)} = q^{(2)} = q$, finial, the wavy surface of the two rotating fluids with the same angular velocity is described by

$$\begin{aligned} &\frac{1}{q}\frac{d}{dt}\left(\rho^{(1)}(1-\xi q)^{-1} + \rho^{(2)}(1+\xi q)^{-1}\right)\frac{d\xi}{dt} \\ &-\left(\rho^{(1)} - \rho^{(2)}\right)g\xi + k^2\sigma_T\xi\left(1-k^2\xi^2\right)^{-3/2} \\ &-k\sigma_{H\perp}\xi\left(1-k^2\xi^2\right)^{-1}\left[1 + Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right] = 0. \end{aligned} \tag{36}$$

5. The case of finite rotating fluids and the implication of the homotopy perturbation method

In this section, we deal with the examination of the influence of finite angular velocity on the stability behavior. In this case, the formal of the operator q is as defined by (28). Keeping with the integral form for the operator q, and operating on both sides by q^{-1}, then equation (36) transforms to

$$\begin{aligned} &\left(1+\frac{4\Omega^2}{D^2}\right)\frac{d}{dt}\left(\rho^{(1)}(1-\xi q)^{-1} + \rho^{(2)}(1+\xi q)^{-1}\right)\frac{d\xi}{dt} \\ &+k\left(1+\frac{4\Omega^2}{D^2}\right)^{1/2}\left[k^2\sigma_T\xi\left(1-k^2\xi^2\right)^{-3/2} - \left(\rho^{(1)}-\rho^{(2)}\right)g\xi\right] \\ &= k^2\sigma_{H\perp}\xi\left(1+\frac{4\Omega^2}{D^2}\right)^{1/2}\left(1-k^2\xi^2\right)^{-1}\left[1+Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right]. \end{aligned} \tag{37}$$

To solve the above nonlinear system (37), we may use the expansion procedure obtained formally by the homotopy perturbation [34-39]. By introducing the homotopy parameter $\delta \in [0,1]$, the homotopy equation corresponding to equation (37) can be constructed in the form

$$\begin{aligned} &\left(1+\frac{4\Omega^2}{D^2}\right)\frac{d}{dt}\left(\rho^{(1)}(1-\delta\xi q)^{-1} + \rho^{(2)}(1+\delta\xi q)^{-1}\right)\frac{d\xi}{dt} \\ &+k\left(1+\delta\frac{4\Omega^2}{D^2}\right)^{1/2}\left[k^2\sigma_T\xi\left(1-\delta k^2\xi^2\right)^{-3/2} - \left(\rho^{(1)}-\rho^{(2)}\right)g\xi\right] \\ &= k^2\sigma_{H\perp}\xi\left(1+\delta\frac{4\Omega^2}{D^2}\right)^{1/2}\left(1-\delta k^2\xi^2\right)^{-1}\left[1+\delta Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right]. \end{aligned} \tag{38}$$

Clearly as $\delta \to 1$ the initial equation (37) is obtained, whereas $\delta \to 0$ the linear part of the equation (38) arises.

5.1 *The estimation of the linear influence*

In this subsection, the linear form of the equation (38) will be discussed. The linearity of the equation (38) can be obtained as $\delta \to 0$, and $\xi(t)$ becomes $\xi_0(t)$ which is given by

$$\ddot{\xi}_0 + \left\{4\Omega^2 - \frac{k}{\left(\rho^{(1)}+\rho^{(2)}\right)}\left[\left(\rho^{(1)}-\rho^{(2)}\right)g - k^2\sigma_T + k\sigma_{H\perp}\right]\right\}\xi_0 = 0. \tag{39}$$

This is the second-harmonic equation its solution, satisfying the initial conditions, has the form

$$\xi_0(t) = A\cos\omega_0 t, \tag{40}$$

where the argument ω_0 is given by

$$\omega_0^2 = 4\Omega^2 + \frac{k}{\rho^{(1)}+\rho^{(2)}}\left[k^2\sigma_T - \left(\rho^{(1)}-\rho^{(2)}\right)g - k\sigma_{H\perp}\right]. \tag{41}$$

In the above equation, ω_0 appears as a square term only, while the right-hand side is real. Thus, the values of ω_0 are either real or imaginary. When ω_0 is imaginary, an instability is expressed through the dependence of ω_0^2 on the wavenumber *k*. However, stability occurs when the angular velocity satisfies the following relation:

$$\Omega^2 > \frac{k}{4\left(\rho^{(1)}+\rho^{(2)}\right)}\left[k\sigma_{H\perp} + \left(\rho^{(1)}-\rho^{(2)}\right)g - k^2\sigma_T\right]. \tag{42}$$

In the absence of rotation i.e. in the limiting case as $\Omega \to 0$, the stability occurs when the magnetic field satisfies the following relation:

$$\sigma_{H\perp} < \frac{1}{k}\left[k^2\sigma_T - \left(\rho^{(1)}-\rho^{(2)}\right)g\right]. \tag{43}$$

It is useful to investigate the numerical assess for linear stability of the wave propagating on the interface. In order to present this examination, numerical calculations for stability condition (42) are made for both vertical and tangential magnetic fields influence. The results for the calculations are displayed in Figures. 1 to 4.

To depict the stability picture it is useful to present the transition curve of inequality (42) in the non- dimensional form as defined by

- The characteristic length $L=(\sigma_T/g\rho^{(2)})^{1/2}$,
- The characteristic time $t=(L/g)^{1/2}$ and
- Other dimensionless quantities are given by

$k=k^*/L,\ \omega=\omega^*/t,\ \ \Omega=\Omega^*/t,\ \sigma_H=\sigma_H^* Lg\rho^{(2)}$

and $\rho=\rho^{(1)}/\rho^{(2)}$.

Note that the superposed asterisk refers to the dimensionless quantity, which will be omitted later for simplicity.

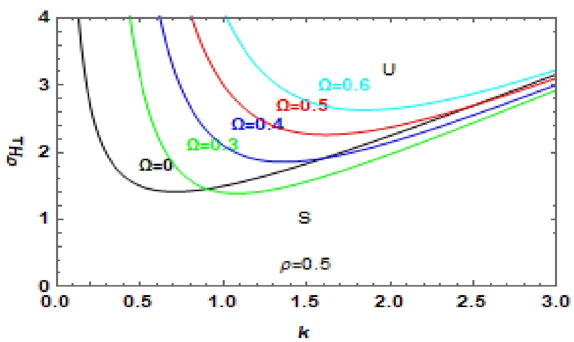

Figure 1: The graph is constructed for $\sigma_{H\perp}$ versus the wavenumber k of condition (42). Influence of the angular velocity on the linear stability criteria for variation of Ω. The system has $\rho=0.5$.

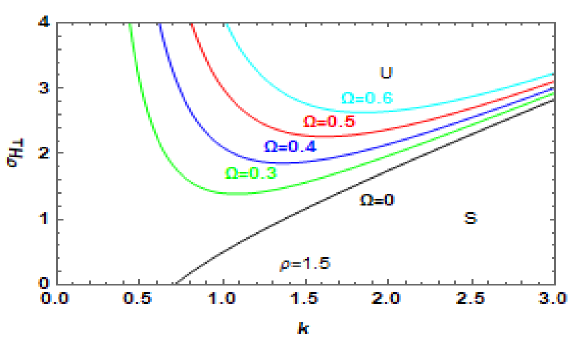

Figure 2: The graph is constructed for the same system. as in Figure 1 except that the system has $\rho=1.5$. The system has $\rho=0.5$.

In Figure 1, the stability image has been displayed within the plane $(\sigma_{H\perp}-k)$ for variation within the angular rate $\Omega=0,0.3,0.4,0.5$ and 0.6 .

The black curve indicates the transition curve for un-rotated fluids that separates the stable region from the unstable region. The image S refers to the stable region and also the image U indicate the unstable region. The stability behavior for variation of the angular rate has been illustrated during this figure for the case that refers to the stable system and within the presence of a vertical magnetic field of force. The graph shows that within the non-rotate system the plane has divided into stable region lies beneath the transition curve and unstable region settled on the opposite facet of the curve. This instability is because of the appliance of the vertical field of force to the statically stable system. The implication for the rotation of the system has been indicated by the colors curves.

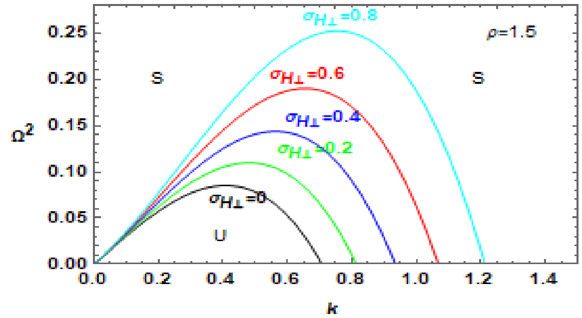

Figure 3: Influence of the variation of the vertical magnetic field on the stability diagram for the system of Fig (2). The graph is constructed for Ω^2 Versus k.

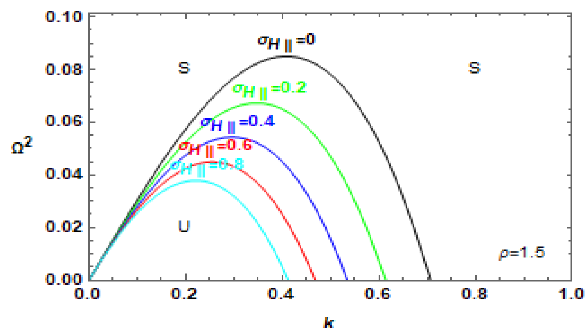

Figure 4: As in Fig (3), except that the variation of the vertical magnetic field is replaced by the variation of the tangential field

It seems that the rise within the angular speed will increase the stable region and also the unstable region decreases in size as k will increase. However, this shows the helpful influence of angular speed, particularly at little wavenumber values. Similar conclusions are discovered, for the influence of the rotating fluids even within the statically unstable system, as shown in Figure 2. The influence of each vertical and tangential magnetic field has been displayed in Figures 3 and 4, wherever the calculations for transition curves Ω^2 are performed as a function of the magnetic field. In these graphs, the black curve refers to un-magnetic fluids. The plane (Ω^2-k) showing in Figure 3 refers to the vertical field of force whereas the graph in Figure 4 refers to the horizontal field of force. The plane $(\Omega^2 - k)$ is split by the transition curves into two regions. The higher one is that the stable region and labeled by the letter. The lower one is that the unstable region and labeled by the letter. The introduction of the vertical field plays a destabilizing role whereas the tangential field plays a helpful role that is associate degree earlier vital development, studied by Melcher [41] associate degreed by different many researchers for an inviscid flow through the linear stability theory. Melcher [41] demonstrated that in the linear stability theory, the tangential field has a stabilizing effect as it increases the surface tension effect, while the vertical field has a destabilizing influence as it decreases the surface tension effect. Even if the fluids have a viscosity, the vertical field still plays a destabilizing role, as demonstrated by El-Dib [44]. He studied the surface wave propagation in the interface between two magnetic fluids in porous media.

5.2 *The examination of the nonlinear influence on the rotating system*

To develop the nonlinear effects for the amplitude modulation for the progressive waves, we'd like to travel to the total nonlinear equation (38) with $\delta > 0$ and considering the subsequent homotopy expansion:

$$\xi(t) = \xi_0(t) + \delta\xi_1(t) + \delta^2\xi_2(t) + \dots \tag{44}$$

where $\xi_0(t)$ is as given by (40) and therefore the unknowns $\xi_n(t)$ are determined in turns. To analyze the stability behavior, through a nonlinear approach, we are going to proceed with the nonlinear frequency analysis [38, 39]. Therefore, a nonlinear frequency ω^2 is also prompt within the following form:

$$\omega^2 = \omega_0^2 + \delta\omega_1 + \delta^2\omega_2 + \dots \tag{45}$$

where $\omega_n, n = 1,2,\dots$ are arbitrary parameters to be determined and ω_0^2 as outlined in (41).

Employing the two expansions (44) and (45) into the homotopy equation (38) and equating like powers of δ on each side yields the zero and the first orders as

$$\delta^0 : \ddot{\xi}_0 + \omega^2\xi_0 = 0, \tag{46}$$

$$\begin{aligned}\delta^1 : \ddot{\xi}_1 + \omega^2\xi_1 &= \omega_1\xi_0 + \frac{2k\Omega^2}{\left(\rho^{(1)} + \rho^{(2)}\right)}\left[\left(\rho^{(1)} - \rho^{(2)}\right)g - k^2\sigma_T + k\sigma_{H\perp}\right]\iint \xi_0 dtdt \\ &- \frac{\left(\rho^{(1)} - \rho^{(2)}\right)}{\left(\rho^{(1)} + \rho^{(2)}\right)}\left(1 + \frac{4\Omega^2}{D^2}\right)\left(\xi_0 q\ddot{\xi}_0 + \dot{\xi}_0 q\dot{\xi}_0\right) \\ &+ \frac{k^3}{\rho^{(1)} + \rho^{(2)}}\left[J\sigma_{H\perp}\left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}}\right)\xi_0^2 + k\left(\sigma_{H\perp} - \tfrac{3}{2}k\sigma_T\right)\xi_0^3\right].\end{aligned} \tag{47}$$

The solution of equation (46) is as given by (40) except that the frequency ω_0 has been replaced by ω. Inserting this solution into equation (47) and removing the secular terms gives

$$\omega_1 = \frac{2k\Omega^2}{\omega^2\left(\rho^{(1)} + \rho^{(2)}\right)}\left[\left(\rho^{(1)} - \rho^{(2)}\right)g - k^2\sigma_T + k\sigma_{H\perp}\right] - \frac{3k^4A^2\left(\sigma_{H\perp} - \frac{3}{2}k\sigma_T\right)}{4\left(\rho^{(1)} + \rho^{(2)}\right)}. \tag{48}$$

Taking into account, the condition (48), the exact solution of equation (47) is arranged in the form

$$\begin{aligned}\xi_1 &= \frac{kA^2}{3}\frac{\left(\rho^{(1)} - \rho^{(2)}\right)}{\left(\rho^{(1)} + \rho^{(2)}\right)}\frac{\left(\omega^2 - \Omega^2\right)}{\left(\omega^2 - 2\Omega^2\right)}\left(\cos\omega t - \cos 2\omega t\right) \\ &+ \frac{k^4A^3}{32\omega^2\left(\rho^{(1)} + \rho^{(2)}\right)}\left(\sigma_{H\perp} - \tfrac{3}{2}k\sigma_T\right)\left(\cos\omega t - \cos 3\omega t\right) \\ &+ \frac{J\sigma_{H\perp}k^3A^2}{6\omega^2\left(\rho^{(1)} + \rho^{(2)}\right)}\left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}}\right)\left(3 - \cos 2\omega t - 2\cos\omega t\right).\end{aligned} \tag{49}$$

In the light of one iteration approaches, the final approximate solution of equation (37) can be performed by employing the uniform solutions of equations (46) and (47) into the expansion (45) and setting $\delta \to 1$. The result is

$$\begin{aligned}\xi(t) &= A\cos\omega t \\ &+ \frac{kA^2}{3}\frac{\left(\rho^{(1)} - \rho^{(2)}\right)}{\left(\rho^{(1)} + \rho^{(2)}\right)}\frac{\left(\omega^2 - \Omega^2\right)}{\left(\omega^2 - 2\Omega^2\right)}\left(\cos\omega t - \cos 2\omega t\right) \\ &+ \frac{k^4A^3}{32\omega^2\left(\rho^{(1)} + \rho^{(2)}\right)}\left(\sigma_{H\perp} - \tfrac{3}{2}k\sigma_T\right)\left(\cos\omega t - \cos 3\omega t\right) \\ &+ \frac{J\sigma_{H\perp}k^3A^2}{6\omega^2\left(\rho^{(1)} + \rho^{(2)}\right)}\left(\frac{\mu^{(1)} - \mu^{(2)}}{\mu^{(1)} + \mu^{(2)}}\right)\left(3 - \cos 2\omega t - 2\cos\omega t\right).\end{aligned} \tag{50}$$

This is the solution for the amplitude of the progressive wave of the surface between the rotating fluids. To complete this solution, it has to construct the formula for the frequency ω .To construct the approximate nonlinear frequency with one iteration technique, we tend to use (48) into (45), using (41) and holding $\delta \to 1$, yields

$$\begin{aligned}&\left(\rho^{(1)} + \rho^{(2)}\right)\omega^4 - \Big[4\Omega^2\left(\rho^{(1)} + \rho^{(2)}\right) + k^3\left(1 + \tfrac{9}{8}k^2A^2\right)\sigma_T \\ &- k\left(\rho^{(1)} - \rho^{(2)}\right)g - k^2\left(1 + \tfrac{3}{4}k^2A^2\right)\sigma_{H\perp}\Big]\omega^2 \\ &+ 2k\Omega^2\left[k^2\sigma_T - k\sigma_{H\perp} - \left(\rho^{(1)} - \rho^{(2)}\right)g\right] = 0.\end{aligned} \tag{51}$$

This is the characteristic equation within the quadratic type in ω^2 and having two roots ω_1^2 and ω_2^2. The stability of this technique needed some conditions on these roots. These conditions are the two roots ω_1^2 an ω_2^2 should be real and positive. These constraints may be obtained from elementary algebra within the type

$$\Omega^4 + 3k^4A^2\frac{(3k\sigma_T - 2\sigma_{H\perp})}{16(\rho^{(1)}+\rho^{(2)})}\Omega^2 + \frac{k^2}{16(\rho^{(1)}+\rho^{(2)})^2}\begin{bmatrix} k^2(1+\frac{9}{8}k^2A^2)\sigma_T - k(1+\frac{3}{4}k^2A^2)\sigma_{H\perp} \\ -(\rho^{(1)}-\rho^{(2)})g \end{bmatrix}^2 > 0, \quad (52)$$

$$\Omega^2 + \frac{k}{4(\rho^{(1)}+\rho^{(2)})}\begin{bmatrix} k^2(1+\frac{9}{8}k^2A^2)\sigma_T - k(1+\frac{3}{4}k^2A^2)\sigma_{H\perp} \\ -(\rho^{(1)}-\rho^{(2)})g \end{bmatrix} > 0, \quad (53)$$

and $\quad k^2\sigma_T - k\sigma_{H\perp} - (\rho^{(1)}-\rho^{(2)})g > 0. \quad (54)$

Satisfying the above conditions will ensure that the solution (50) is periodic.

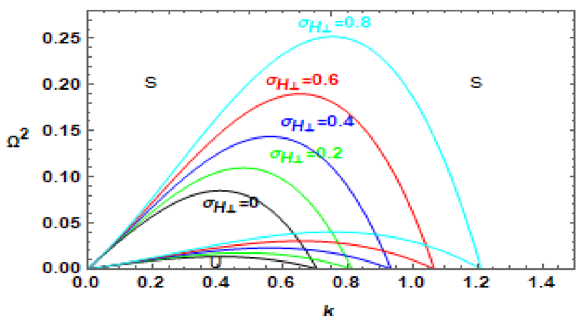

Figure 5: The nonlinear estimation for the same system as given in Fig (3).

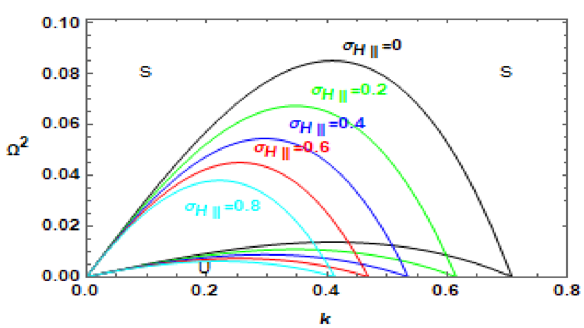

Figure 6: The nonlinear estimation for the same system as given in Fig (4).

Numerical calculations showed that condition (52) has been satisfied automatically. In alternative aspect condition (54) is that the same condition (43) for the non-rotating case within the linear stability. The important condition for the stability theory within the nonlinear estimation is that the condition (53). This condition premeditated for a similar system as given in Figure 3 and picked up with this graph as displayed in Figure 5. It's shown that the term $\sigma_{H\perp}$ continues to be taking part in a similar role within the stability image. A similar conclusion is discovered once the vertical magnetic term has been replaced by the tangential magnetic term $\sigma_{H\|}$ as shown in Figure 6. The main observation in Figures 5 and 6 is that the rotation parameter is helpful within the nonlinear estimation than the influence within the linear examination. The amplitude $\xi(t)$, as given in (50), is illustrated diagrammatically. Many numerical calculations square measure bestowed, for the variation of the rotation parameter Ω, the vertical and tangential magnetic field, together with Figures. 7 to 9. In these figures the function $\xi(t)$ is premeditated versus the variation of the variable quantity t. The vertical axis represents the distribution of the function $\xi(t)$, and therefore the horizontal axis refers to the variations of the time t. In these figures, four totally different values of a particular parameter square measure thought of, whereas the other parameters are considered, while the other parameters are kept fixed.

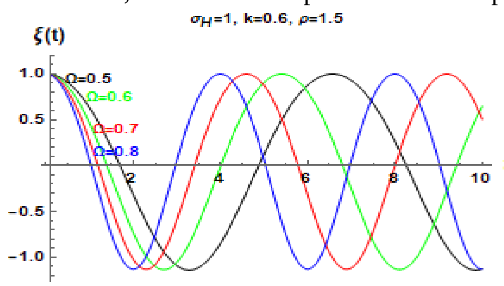

Figure 7: The distribution for the amplitude curve ξ, as a function in the rotation parameter Ω versus t.

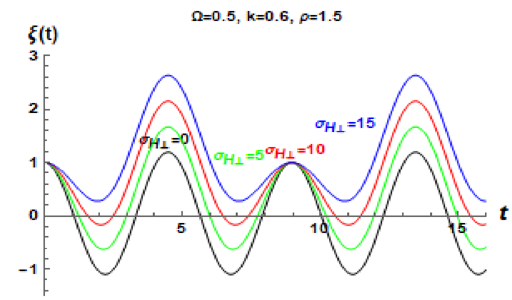

Figure 8: The distribution for the amplitude curve ξ, as a function in the vertical magnetic term $\sigma_{H\perp}$ versus t.

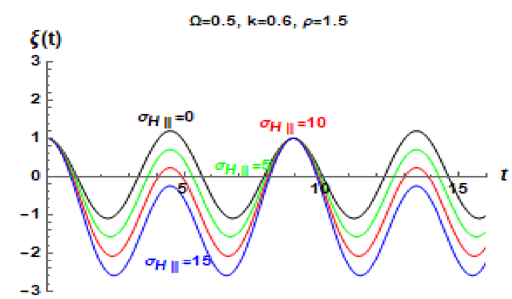

Figure 9: The distribution for the amplitude curve ξ, as a function in the tangential magnetic term $\sigma_{H\|}$ versus t.

The $\xi(t)$-curve, as shown in Figure 7, is premeditated for a system having the fixed parameter $\sigma_{H\perp}=1$, $k=0.6$ and $\rho=1.5$. These four cases square measure started from the identical purpose on the vertical coordinate. Moreover, it's determined that the curves have a periodic behavior of the identical amplitude. Finally, the four curves square measure comparable to the particulars $\Omega=0.5, 0.6, 0.7, 0.8$. It's determined that, because the rotation parameter Ω is enlarged, the $\xi(t)$-curve has shifted within the direction of decreasing the parameter *t*. This mechanism shows that the rise within the rotation parameter Ω plays a helpful role.

As may be seen, Figure 8 illustrates the influence of the variations within the term of the vertical magnetic field $\sigma_{H\perp}=0, 5, 10, 15$ on the amplitude-curve. Related to the mounted of the rotation parameter $\Omega=0.5$, the wavenumber $k=0.6$ and also the density quantitative relation $\rho=1.5$. In these calculations, all the four curves area unit started at the purpose (0, 1) and flinched the first-period case at the purpose (9,1) within the plane ($\xi-t$). It had been seen there are three intervals for the amplitude-curve within the one amount wave. Within the first open interval of $t\in(0,3)$ because the vertical magnetic term $\sigma_{H\perp}$ is multiplied, the magnitude of the amplitude of the wave-curve is shriveled that conform to the stabilizing mechanism. Within the middle open interval of $t\in(3,6)$. It's discovered that because the vertical magnetic term $\sigma_{H\perp}$ is multiplied, the magnitude of the amplitude of the wave-curve is multiplied creating a destabilizing impact. Within the third interval of $t\in(6,9)$, once more the amplitude of the wave has shriveled in its magnitude as $\sigma_{H\perp}$ is multiplied. The twin role within the stability of behavior is discovered. Once the vertical magnetic term is replaced by the tangential magnetic term $\sigma_{H\|}$, the alternative mechanisms is seen within the three intervals for the first-period case as shown in Figure (9).

6. The Allowance for highly rotating fluids

In return back to the initial scheme of equation (36) and reformulated it within the light of the alternative choice style of the operator q as given by (29), within its growth are the differential kind rather than the integral kind. However, after we operative on each side of the equation (36) by the choice style of q yields

$$\left(\rho^{(1)}(1-\xi q)^{-1}+\rho^{(2)}(1+\xi q)^{-1}\right)\frac{d\xi}{dt}= =\frac{k}{2\Omega}\left(1+\frac{D^2}{4\Omega^2}\right)^{-1/2}\left\{\begin{array}{l}\left(\rho^{(1)}-\rho^{(2)}\right)g\xi-k^2\sigma_T\xi\left(1-k^2\xi^2\right)^{-3/2}\\ +k\sigma_{H\perp}\xi\left(1-k^2\xi^2\right)^{-1}\left[1+Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right]\end{array}\right\}. \tag{55}$$

This is the first-order nonlinear equation within the transcendental type. This equation controls the wave propagation on the interface between two extremely rotating fluids. The homotopy equation will be created by introducing the parameter $\delta\in[0,1]$ so once $\delta\to 1$ the initial eq. (55) is found. As $\delta\to 0$ in it, the linear type arises. However, the homotopy equation will be in-built the shape

$$\left(\rho^{(1)}(1-\delta\xi q)^{-1}+\rho^{(2)}(1+\delta\xi q)^{-1}\right)\frac{d\xi}{dt}=\frac{k}{2\Omega}\left(1+\delta\frac{D^2}{4\Omega^2}\right)^{-1/2}\times \times\left\{\begin{array}{l}\left(\rho^{(1)}-\rho^{(2)}\right)g\xi-k^2\sigma_T\xi\left(1-\delta k^2\xi^2\right)^{-3/2}\\ +k\sigma_{H\perp}\xi\left(1-\delta k^2\xi^2\right)^{-1}\left[1+\delta Jk\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi\right]\end{array}\right\}. \tag{56}$$

The two first terms in using the binomial expansion can be rearranged equation (56) in the form

$$\dot{\xi}+\varpi_0\xi=\frac{\delta}{2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)}\left[\frac{1}{8\Omega^2}\varpi_0\left(\rho^{(1)}+\rho^{(2)}\right)\ddot{\xi} +2\Omega\left(\rho^{(1)}-\rho^{(2)}\right)\xi q\dot{\xi}+J\sigma_{H\perp}k^3\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\xi^2 +k^4\left(\sigma_{H\perp}-\tfrac{3}{2}k\sigma_T\right)\xi^3\right]+O\left(\delta^2\right) \tag{57}$$

where ϖ_0 is defined here as

$$\varpi_0=\frac{k}{2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)}\left[k^2\sigma_T-k\sigma_{H\perp}-\left(\rho^{(1)}-\rho^{(2)}\right)g\right]. \tag{58}$$

Clearly, the solution of the linearized style of equation (57) characterized by an exponential in the time and having a negative rate ϖ_0. Thus, the wave propagation on the interface contains a damping nature wherefrom the worth of the right-hand facet of (58) is positive else the infinite solution arises. This stability condition needs that,

$$\sigma_{H\perp}<k\sigma_T-\frac{1}{k}\left(\rho^{(1)}-\rho^{(2)}\right)g \tag{59}$$

At this stage, one can say that the linear stability condition in un-rotating fluids is equivalent to the corresponding condition in highly rotating fluids.

To study the problem within the nonlinear scope, one may apply the technology, of the parameter expansion [38] as

$$\varpi=\varpi_0+\delta\varpi_1+\delta^2\varpi_2+\ldots \tag{60}$$

Employing the expansion (60), using the homotopy expansion (44) and equating the identical powers of δ on both sides of equation (59) yields

$$\xi_0 = Ae^{-\varpi t}. \quad (61)$$

In view of (61), the first-order problem in δ becomes

$$\dot{\xi}_1 + \varpi\xi_1 = \frac{1}{2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)}\Bigg[2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)\left(\varpi_1 + \frac{\varpi^3}{16\Omega^3}\right)Ae^{-\varpi t} + k\varpi^2\left(\rho^{(1)}-\rho^{(2)}\right)\left(1+\frac{\varpi^2}{\Omega^2}\right)^{-1/2}A^2e^{-2\varpi t} + J\sigma_{H\perp}k^3\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)A^2e^{-2\varpi t} + k^4\left(\sigma_{H\perp}-\tfrac{3}{2}k\sigma_T\right)A^3e^{-3\varpi t}\Bigg]. \quad (62)$$

Eliminating the source that producing secular terms from (62) requires that

$$\varpi_1 = -\frac{\varpi^3}{16\Omega^3}. \quad (63)$$

At this end, the exact solution of equation (62) has the form

$$\xi_1 = \frac{k}{2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)}\Bigg[\varpi\left(\rho^{(1)}-\rho^{(2)}\right)\left(1+\frac{\varpi^2}{\Omega^2}\right)^{-1/2} + \frac{J\sigma_{H\perp}k^2}{\varpi}\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\Bigg]\left(1-Ae^{-\varpi t}\right)Ae^{-\varpi t} + \frac{k^4}{4\Omega\varpi\left(\rho^{(1)}+\rho^{(2)}\right)}\left(\sigma_{H\perp}-\tfrac{3}{2}k\sigma_T\right)\left(1-A^2e^{-2\varpi t}\right)Ae^{-\varpi t}. \quad (64)$$

The one iteration technology leads to the following approximate solution:

$$\xi(t) = Ae^{-\varpi t} + \frac{k}{2\Omega\left(\rho^{(1)}+\rho^{(2)}\right)}\Bigg[\varpi\left(\rho^{(1)}-\rho^{(2)}\right)\left(1+\frac{\varpi^2}{\Omega^2}\right)^{-1/2} + \frac{J\sigma_{H\perp}k^2}{\varpi}\left(\frac{\mu^{(1)}-\mu^{(2)}}{\mu^{(1)}+\mu^{(2)}}\right)\Bigg]\left(1-Ae^{-\varpi t}\right)Ae^{-\varpi t} + \frac{k^4}{4\Omega\varpi\left(\rho^{(1)}+\rho^{(2)}\right)}\left(\sigma_{H\perp}-\tfrac{3}{2}k\sigma_T\right)\left(1-A^2e^{-2\varpi t}\right)Ae^{-\varpi t}. \quad (65)$$

Clearly, the above solution has a damping in time within the parameter ϖ has positive values. Employing (63) into (60) and letting $\delta \to 1$, we obtain

$$\varpi^3 + 16\Omega^3\varpi - 16\Omega^3\varpi_0 = 0. \quad (66)$$

From elementary pure mathematics, it is easy; to point out that the on top of frequency equation reads the identical condition for stability as found within the linear analysis, that is

$$\varpi_0 > 0. \quad (67)$$

This means that the stability behavior within the nonlinear scope is adored the stability behavior within the linear scope.

7. Conclusions

The investigation for a surface wave propagating on the free surface between of two superposed ferrofluids subjected to uniform rotation within the presence of the vertical or the tangential flux is taken into account. It's thought-about that the system is subjected to uniform rotating the frame references around the vertical axis. Equations of motion are solved victimization the conventional mods, during which the time-dependence leaves unknown. The appropriate nonlinear boundary conditions are applied that results in a derived nonlinear transcendental integro-differential equation within the elevation parameter. The special case of equal the angular speed of the two fluids is taken into account for simplicity.

Due to the very complicated problem, a perturbation technique is pressing to use so as to get the associate approximate solution. The employment of the homotopy perturbation technique may be a powerful [34-39]. The homotopy parameter δ has been introduced and also the homotopy transcendental equation created. The homotopy perturbation with the technology of the parameter growth has been applied within the conferred study. This technique ends up in a deriving the nonlinear frequency. The configuration of the nonlinear stability derived from the frequency equation for the first-time. A periodic solution for the amplitude $\xi(t)$ performs of the elevation surface was obtained victimization one iteration method. The numerical illustration showed that

- The uniform rotation of the system wills suppers the destabilizing influence of the vertical field for statically stable system or the statically unstable one. This mechanism is determined within the linear stability likewise
as within the nonlinear stability.
- The uniform rotation of the system supports the helpful role within the application of the tangential magnetic field.
- The rotating system behaves because of the
un-rotating system within the case of terribly
high rotation
- The increase in rotation parameter decreases the cyclist of the wave answer.
- The increase in the vertical magnetic term will increase the amplitude of the wave solution, whereas the tangential magnetic term plays the other role.

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