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Internal heat source in a temperature dependent thermoelastic half space with microtemperatures

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ABSTRACT

A two dimensional deformation due to internal heat source in a thermoelastic solid with micro-temperatures under the effect of dependence of reference temperature on all elastic and thermal parameters have been studied. A mechanical force is applied at the free surface of thermoelastic half space. The normal modes technique has been applied to obtain the exact expressions for the components of normal displacement, micro temperature, normal force stress, temperature distribution, heat flux moment tensor and tangential couple stress for thermoelastic solid with micro-temperatures. The effect of internal heat source, reference temperature and micro-rotation on the derived components have been depicted graphically.

1. Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronatics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. Eringen and Suhubi [1, 2] developed a nonlinear theory of micro-elastic solids. Later Eringen [3-5] developed a theory for the special class of micro-elastic materials and called it the "linear theory of micropolar elasticity". Under this theory, solids can undergo macro-deformations and micro-rotations.

The concept of micro-temperatures was derived by assuming the fact that in a thermoelastic body there exit microelements which have distinct temperatures and which further depends homogeneously on microcoordinates of these microelements, that are based on the concept of the microstructure of the continuum. This micro-temperatures theory is widely used in nano materials which are of great importance in the current research area. Many researchers have contributed to the problems on micro-temperatures in the past. Grot [6] investigated thermodynamics theory of elastic solids with microstructure whose microelements exhibit micro-temperatures and obtained the heat conduction equation for micro-temperatures. In this theory, the inverse of the temperature of microelements is assumed as a linear function of microcoordinates of the microelements. Riha [7] investigated the heat transportation in materials possessing micro-temperatures and the constitutive coefficients are then specified as a function of the volume concentration of inner structure and thermal characteristics of the materials. Iesan and Quintanilla [8] studied a theory of thermoelasticity with micro-temperatures. Iesan [9] proposed the theory of micromorphic elastic solids with micro-temperatures. Ezzat et. al. [10] discussed the dependence of modulus of elasticity on the reference temperature in generalized thermoelasticty. Exponential stability in thermoelasticity with micro-temperatures was studied by Casas and Quintanilla [11]. Scalia and Svandze [12] gave the solutions of the theory of thermoelasticity with micro-temperatures. Iesan [13] discussed thermoelasticity of bodies with microstructure and micro-temperatures. Aouadi [14] discussed some theorems in the isotropic theory of micro-stretch thermoelasticity with micro-temperatures. Iesan and Quintanilla [15] discussed thermoelastic bodies with inner structure and micro-temperatures. Scalia et al. [16] studied basic theorems in the equilibrium theory of thermoelasticity with micro-temperatures. Quintanilla [17] discussed the growth and continuos dependence in thermoelasticity with micro-temperatures. Steeb et al. [18] studied time harmonic waves in thermoelastic material with micro-temperatures. Chirita et. al. [19] studied the theory of thermoelasticity with micro-temperatures. Kumar et. al. [20] studied the Reflection and refraction of plane waves at the interface of an elastic solid and micro-stretch thermoelastic solid with micro-temperatures.

The thermal and mechanical properties of the materials vary with temperature, so the temperature dependent of the material properties must be taken into consideration in the

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thermal stress analysis. When the temperature variation from the initial stress is not strongly varying, the properties of materials remain unchanged. In the refractory industries, the structural components are exposed to a high temperature change. In that case, the materials characteristics such as modulus of elasticity, thermal conductivity and the coefficient of linear thermal expansion are no longer constants. In the industries involving very high temperature, we cannot neglect the effect of temperature on the physical quantities. The effect of temperature on the parameters, hence become an important part of study in many practical applications. Noda [21] studied he thermal stress in a material with temperature-dependence properties. Youssef [22] discussed the dependence of the modulus of elasticity and the thermal conductivity on a reference temperature in the generalized thermoelasticity for an infinite material with a spherical cavity. Othman et al. [23] studied the two-dimensional problem of generalized magneto thermoelasticity with temperature dependent elastic moduli for different theories. The effect of gravity on elastic surface waves is discussed by Biot [24]. Kumar and Devi [25] studied thermomechanical interactions in porous generalized thermoelastic material permeated with heat source. Lotfy [26] have studied the transient disturbance in a half-space under generalized magneto-thermoelasticity with a stable internal heat source. Lotfy [27] discussed the transient thermo-elastic disturbances in a visco-elastic semi-space due to moving internal heat source. Othman [28] studied the generalized thermoelastic problem with temperature-dependent elastic moduli and internal heat sources. Kumar and Devi [29] discussed deformation in porous thermoelastic material with temperature dependent properties. Ailawalia and Budhiraja [30] studied the Internal heat source in temperature rate dependent thermoelasticity using dual-phase lag model. Othman [31] discussed the effect of gravitational field and temperature dependent properties on two-temperature thermoelastic medium with voids under G-N Theory. Kumar et. al. [32] investigated a problem in a micro-stretch thermoelastic solid which have micro-temperatures. Ailawalia et. al. [33] investigated the two dimensional deformation in micro-stretch thermoelastic half space with micro-temperatures and internal heat source. Ailawalia et. al. [34] investigated a two dimensional problem in a rotating micro-stretch thermoelastic solid with micro-temperatures.

Most of the earlier investigations were performed on the pretext of temperature-independent material properties, which restrict the applicability of the solution obtained in the certain ranges of temperature. In case of high temperature, the characteristics such as modulus of elasticity, thermal conductivity and the coefficient of linear thermal expansion are no longer constants. Various authors have discussed the problems in thermoelastic medium with micro-temperatures, but not many problems have been discussed in thermoelastic medium with micro-temperatures based on the temperature dependence. The study of temperature dependence on the deformation of body led to the study of the present problem.

2. Formulation Of Problem

The constitutive equation of motion for a homogeneous, isotropic thermoelastic solid with micro-temperatures without body forces, body couples, heat sources, and first heat source moment following Iesan and Quintanilla [8] are,

$$(\mu+K)u_{i,jj}+(\lambda+\mu)u_{i,ij}+K\varepsilon_{ijr}\phi_{r}-\nu T_{,i}=\rho\ddot{u}_{i}, \tag{1}$$

$$\gamma\phi_{i,jj}+K\varepsilon_{ijr}u_{r}-2K\phi_{i}-\mu_{1}\varepsilon_{ijr}w_{r}=\rho J\ddot{\phi}_{i}, \tag{2}$$

$$K^{*}T_{,ii}-\rho c^{*}\dot{T}-\nu T_{0}u_{i,i}+k_{1}w_{i,i}=Q_{1}, \tag{3}$$

$$k_{6}w_{i,jj}+(k_{4}+k_{5})w_{i,ij}+\mu_{1}\varepsilon_{ijr}\dot{\phi}_{r}-b\dot{w}_{i}-k_{2}w_{i}-k_{3}T_{,i}=0, \tag{4}$$

The constitutive relations are,

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\phi_{r})-\nu T\delta_{ij}, \tag{5}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}, \tag{6}$$

$$q_{ij}=-k_{4}w_{r,r}\delta_{ij}-k_{5}w_{i,j}-k_{6}w_{j,i}, i,j,m=1,2,3 \tag{7}$$

where $\nu=(3\lambda+2\mu+K)\alpha_{t}$ and $\beta=(3\lambda+2\mu)\alpha_{t}$, α_{t} is coefficents of linear thermal expansion, λ and μ are Lame's constants, K,α,β,γ are the micropolar constants of the solid, $\mu_{1},k_{1},k_{2},k_{3},k_{4},k_{5},k_{6}$ are the constitutive coefficients. t_{ij} is the component of stress tensor, m_{ij} is the couple stress tensor, q_{ij} is the first heat flux moment tensor, $\vec{u}=(u_{i})$ is the displacement vector, $\vec{\phi}=(\phi_{i})$ is the micro-rotation vector, $\vec{w}=(w_{i})$ is the micro-temperature vector, ρ is the density, J is the micro-inertia, c^{*} is the specific heat at constant strain, Q_{1} is the internal heat source, K^{*} is the thermal conductivity and T is the thermodynamic temperature above reference temperature T_{0}.

Within the context of linear theory, the C-D inequality provides the following restrictions on the elastic moduli are given by Grot [6],

$$3k_{4}+k_{5}+k_{6}\geq 0,\ k_{6}\pm k_{5}\geq 0,\ k_{2}\geq 0,\ K^{*}\geq 0,$$

$$\left(k_{1}+T_{0}k_{3}\right)^{2}\leq 4T_{0}K^{*}k_{2}.$$

We also assume that all functions are continuous and differentiable upto required order in the domain of definition.

We have restricted our analysis to the plane strain parallel to xz -plane having origin on the surface $z=0$ and z- axis pointing vertically downward into the thermoelastic medium with micro-temperatures. We consider a normal force of magnitude F_1 acting at the free surface of thermoelastic medium with micro-temperatures occupying the region $0\leq z<\infty$ as shown in figure-1.

We have restricted our analysis to the plane strain parallel to xz plane with displacement vector $\vec{u}=(u_{1},0,u_{3})$, micro-temperature vector $\vec{w}=(w_{1},0,w_{3})$, and micro-rotation vector $\vec{\phi}_{i}=(0,\phi_{2},0)$.

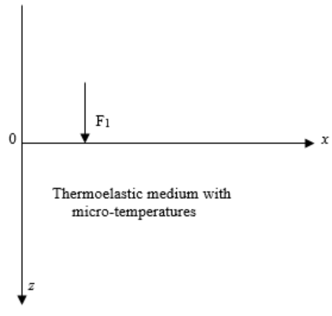

Figure 1. Geometry of the problem

Our aim is to investigate the effect of dependence of reference temperature on all elastic and thermal parameters. Therefore we may assume [10],

$$\lambda=\lambda_0(1-\alpha^*T_0),\ \mu=\mu_0(1-\alpha^*T_0),\ K=K_0(1-\alpha^*T_0),$$
$$\nu=\nu_0(1-\alpha^*T_0),\ \alpha=\alpha_0(1-\alpha^*T_0),\ \beta=\beta_0(1-\alpha^*T_0),$$
$$\gamma=\gamma_0(1-\alpha^*T_0),\ K^*=K_0^*(1-\alpha^*T_0),\ c^*=c_0^*(1-\alpha^*T_0),$$
$$J=J_0(1-\alpha^*T_0),\ b=b_0(1-\alpha^*T_0),\ \mu_1=\mu_{10}(1-\alpha^*T_0),$$
$$k_1=k_{10}(1-\alpha^*T_0),$$
$$k_2=k_{20}(1-\alpha^*T_0),\ k_3=k_{30}(1-\alpha^*T_0),\ k_4=k_{40}(1-\alpha^*T_0),$$
$$k_5=k_{50}(1-\alpha^*T_0),\ k_6=k_{60}(1-\alpha^*T_0).$$

where λ_0, μ_0, K_0, ν_0, α_0, β_0, γ_0, K_0^*, c_0^*, J_0, b_0, μ_{10}, k_{10}, k_{20}, k_{30}, k_{40}, k_{50} and k_{60} are considered constants, α^* is called empirical material constant, in case of the system independent of reference temperature, $\alpha^*=0$.

For convenience and to simplify the numerical calculations, the following non-dimensional variables are used:

$$x'=\frac{1}{L}x,\quad z'=\frac{1}{L}z,\quad u_1'=\frac{1}{L}u_1,\quad u_3'=\frac{1}{L}u_3,$$
$$w_1'=Lw_1, w_3'=Lw_3,\quad t'=\frac{c_1}{L}t,\ t_{ij}'=\frac{t_{ij}}{\nu T_0},\quad \phi_2'=\phi_2,$$
$$m_{ij}'=\frac{m_{ij}}{L\nu T_0}, q_{ij}'=\frac{q_{ij}}{Lc_1\nu T_0},\ T'=\frac{T}{T_0},\ F_1'=\frac{F_1}{\nu T_0},\ Q_1'=\frac{Q_1}{Q_0}.$$

where

$$L=\left(\frac{b_0}{\rho c_0^*T_0}\right)^{\frac{1}{2}},\ c_1^2=\frac{\lambda_0+2\mu_0+K_0}{\rho}.$$

We assume the scalar potential functions $\psi_1(x,z,t)$, $\psi_2(x,z,t)$, $\psi_3(x,z,t)$ and $\psi_4(x,z,t)$ defined by the relation in non dimensional form as,

$$u_1=\frac{\partial\psi_1}{\partial x}-\frac{\partial\psi_2}{\partial z};\ u_3=\frac{\partial\psi_1}{\partial z}+\frac{\partial\psi_2}{\partial x};$$
$$w_1=\frac{\partial\psi_3}{\partial x}-\frac{\partial\psi_4}{\partial z};\ w_3=\frac{\partial\psi_3}{\partial z}+\frac{\partial\psi_4}{\partial x}. \tag{8}$$

Using above non dimensional variables and scalar potentials given by equation (8), the equations (1)-(4) reduces to (after dropping superscripts),

$$\left\{(A_1+1)\nabla^2-A_2\frac{\partial^2}{\partial t^2}\right\}\psi_1-A_3T=0, \tag{9}$$

$$\left(\nabla^2-A_2\frac{\partial^2}{\partial t^2}\right)\psi_2+A_4\phi_2=0, \tag{10}$$

$$\left(\nabla^2-2A_5-A_6\frac{\partial^2}{\partial t^2}\right)\phi_2-A_5\nabla^2\psi_2+A_7\nabla^2\psi_4=0, \tag{11}$$

$$\left(\nabla^2-A_8\frac{\partial}{\partial t}\right)T-A_9\nabla^2\psi_1+A_{10}\nabla^2\psi_3=YQ_1, \tag{12}$$

$$\left(\nabla^2(1+A_{11})-A_{12}-A_{13}\frac{\partial}{\partial t}\right)\psi_3-A_{14}T=0, \tag{13}$$

$$\left(\nabla^2-A_{12}-A_{13}\frac{\partial}{\partial t}\right)\psi_4+A_{15}\frac{\partial\phi_2}{\partial t}=0. \tag{14}$$

where

$$A_1=\frac{\lambda_0+\mu_0}{\mu_0+K_0}, A_2=\frac{\rho A^*c_1^2}{\mu_0+K_0}, A_3=\frac{\nu_0T_0}{\mu_0+K_0},\ A_4=\frac{K_0}{\mu_0+K_0},$$
$$A_5=\frac{K_0L^2}{\gamma_0}, A_6=\frac{\rho J_0c_1^2}{\gamma_0},\ A_7=\frac{\mu_{10}}{\gamma_0}, A_8=\frac{\rho c_0^*c_1L}{K_0^*},$$
$$A_9=\frac{\nu_0c_1L}{K_0^*},\ A_{10}=\frac{k_{10}}{K_0^*T_0}, A_{11}=\frac{k_{40}+k_{50}}{k_{60}}, A_{12}=\frac{k_{20}L^2}{k_{60}},$$
$$A_{13}=\frac{b_0c_1L}{k_{60}}, A_{14}=\frac{k_{30}T_0L^2}{k_{60}}, A_{15}=\frac{\mu_{10}c_1L}{k_{60}}, Y=\frac{L^2}{K_0^*}A^*Q_0,$$
$$A^*=\frac{1}{(1-\alpha^*T_0)}.$$

3. Method of Solution

Here, we use normal mode analysis technique to find the solution of the considered physical variables in the following form,

$$(\psi_i,T,\phi_2,t_{ij},q_{ij},m_{ij},Q_1)(x,z,t)=(\bar{\psi}_i,\bar{T},\bar{\phi}_2,\bar{t}_{ij},\bar{q}_{ij},\bar{m}_{ij},\bar{Q}_1)(z)e^{\omega t+iax}. \tag{*}$$

where ω is complex frequency, a is wave number in x direction and $\bar{\psi}_i(z),\bar{T}(z),\bar{\phi}_2(z)$, $\bar{t}_{ij}(z),\bar{q}_{ij}(z)$, $\bar{m}_{ij}(z),\bar{Q}_1(z)$ are the amplitudes of field quantities.

Using (*) in the equations (9)-(14), we get,

$$(D^2-B_6)\bar{\psi}_1-B_2\bar{T}=0, \tag{15}$$

$$(D^2-B_7)\bar{\psi}_2+A_4\bar{\phi}_2=0, \tag{16}$$

$$(D^2-B_8)\bar{\phi}_2-A_5(D^2-a^2)\bar{\psi}_2+A_7(D^2-a^2)\bar{\psi}_4=0, \tag{17}$$

$$(D^2-B_9)\bar{T}-A_9(D^2-a^2)\bar{\psi}_1+A_{10}(D^2-a^2)\bar{\psi}_3=Y\bar{Q}_1, \tag{18}$$

$$(D^2-B_{10})\bar{\psi}_3-B_5\bar{T}=0, \tag{19}$$

$$(D^2-B_{11})\bar{\psi}_4+B_{12}\bar{\phi}_2=0. \tag{20}$$

where

$$D\equiv\frac{d}{dz},\ B_1=\frac{A_2}{A_1+1},\ B_2=\frac{A_3}{A_1+1},\ B_3=\frac{A_{12}}{A_{11}+1},$$
$$B_4=\frac{A_{13}}{A_{11}+1},\ B_5=\frac{A_{14}}{A_{11}+1}, B_6=a^2+B_1\omega^2, B_7=a^2+A_2\omega^2,$$
$$B_8=a^2+2A_5+A_6\omega^2, B_9=a^2+A_8\omega, B_{10}=a^2+B_3+B_4\omega,$$
$$B_{11}=a^2+A_{12}+A_{13}\omega,\ B_{12}=A_{15}\omega.$$

The constitutive equations (5)-(7) takes the form,

$$\bar{t}_{xx}=\left(A_{17}D^2-a^2A_{16}\right)\bar{\psi}_1+ia\left(A_{17}-A_{16}\right)D\bar{\psi}_2-\bar{T}, \tag{21}$$

$$\bar{t}_{zx}=ia\left(A_{18}+A_{19}\right)D\bar{\psi}_1-\left(A_{18}D^2+a^2A_{19}\right)\bar{\psi}_2-(A_{19}-A_{18})\bar{\phi}_2, \tag{22}$$

$$\bar{t}_{zz}=\left(A_{16}D^2-a^2A_{17}\right)\bar{\psi}_1+ia\left(A_{16}-A_{17}\right)D\bar{\psi}_2-\bar{T}, \tag{23}$$

$$\bar{q}_{xx}=\left(A_{20}a^2-A_{21}D^2\right)\bar{\psi}_3+ia\left(A_{20}-A_{21}\right)D\bar{\psi}_4, \tag{24}$$

$$\bar{q}_{zx} = -ia\left(A_{22}+A_{23}\right)D\bar{\psi}_3 + \left(A_{22}D^2+a^2A_{23}\right)\bar{\psi}_4, \tag{25}$$

$$\bar{q}_{zz} = \left(A_{21}a^2 - A_{20}D^2\right)\bar{\psi}_3 + ia\left(A_{21}-A_{20}\right)D\bar{\psi}_4, \tag{26}$$

$$\bar{m}_{zy} = A_{24}D\bar{\phi}_2. \tag{27}$$

where

$$A_{16} = \frac{\lambda_0+2\mu_0+K_0}{\nu_0T_0},\ A_{17} = \frac{\lambda_0}{\nu_0T_0},\ A_{18} = \frac{\mu_0}{\nu_0T_0},$$

$$A_{19} = \frac{\mu_0+K_0}{\nu_0T_0},\ A_{20} = \frac{k_{40}+k_{50}+k_{60}}{L^3c_1\nu_0T_0},\ A_{21} = \frac{k_{40}}{L^3c_1\nu_0T_0},$$

$$A_{22} = \frac{k_{50}}{L^3c_1\nu_0T_0},\ A_{23} = \frac{k_{60}}{L^3c_1\nu_0T_0},\ A_{24} = \frac{\gamma_0}{L^2\nu_0T_0}.$$

Eliminating $\bar{\psi}_3(z)$ and $\bar{T}(z)$ from equations (15), (18)-(19), we get the following sixth order differential equation for $\bar{\psi}_1(z)$ as,

$$(D^6+PD^4+QD^2+R)\bar{\psi}_1(z) = B_{13}\bar{Q}_1. \tag{28}$$

Eliminating $\bar{\psi}_4(z)$ and $\bar{\phi}_2(z)$ from equations (16)-(17) and (20), we get the following sixth order differential equation for $\bar{\psi}_2(z)$ as,

$$(D^6+ED^4+FD^2+G)\bar{\psi}_2(z) = 0. \tag{29}$$

where

$$P = [B_5A_{10} - (B_6 - B_9 - A_9B_2 + B_{10})],$$
$$Q = [B_9B_6 - a^2B_2A_9 + B_{10}(B_6+B_9+B_2A_9) - 2a^2B_5A_{10}],$$
$$R = [a^4B_5A_{10} - B_{10}(B_9B_6 + a^2B_2A_9)],$$
$$E = [A_5A_4 - B_7 - B_8 - B_{11} - A_7B_{12}],$$
$$F = [B_7B_8 - a^2A_4A_5 + B_{11}(B_7+B_8-A_4A_5) + B_{12}A_7(B_7+a^2)],$$
$$G = [B_{11}(a^2A_5A_4 - B_7B_8) - a^2A_7B_7B_{12}],$$
$$B_{13} = -YB_{10}B_2.$$

In a similar manner we can show that $\bar{\psi}_3(z)$, $\bar{T}(z)$ satisfies the equation,

$$(D^6+PD^4+QD^2+R)\,(\bar{\psi}_3(z),\bar{T}(z)) = B_{13}\bar{Q}_1. \tag{30}$$

which can be factorized as,

$$(D^2-r_1^2)(D^2-r_2^2)(D^2-r_3^2)\bar{\psi}_1(z) = B_{13}\bar{Q}_1. \tag{31}$$

where r_n^2; $(n=1,2,3)$ are roots of equation (30).

and $\bar{\psi}_4(z)$ and $\bar{\phi}_2(z)$ satisfies the equation,

$$(D^6+ED^4+FD^2+G)(\bar{\psi}_4(z),\bar{\phi}_2(z)) = 0. \tag{32}$$

which can be factorized as,

$$(D^2-h_1^2)(D^2-h_2^2)(D^2-h_3^2)\bar{\psi}_2(z) = 0. \tag{33}$$

where h_n^2; $(n=1,2,3)$ are roots of equation (32).

The series solution of equation (30) which are bounded as $z\to\infty$ are given by,

$$\bar{\psi}_1(z) = \sum_{n=1}^{3}[M_n(a,\omega)e^{-r_nz}] + N, \tag{34}$$

$$\bar{T}(z) = \sum_{n=1}^{3}[M'_n(a,\omega)e^{-r_nz}] + N_1, \tag{35}$$

$$\bar{\psi}_3(z) = \sum_{n=1}^{3}[M''_n(a,\omega)e^{-r_nz}] + N_2. \tag{36}$$

The series solution of equation (32) which are bounded as $z\to\infty$ given by,

$$\bar{\psi}_2(z) = \sum_{n=1}^{3}[L_n(a,\omega)e^{-h_nz}], \tag{37}$$

$$\bar{\phi}_2(z) = \sum_{n=1}^{3}[L'_n(a,\omega)e^{-h_nz}], \tag{38}$$

$$\bar{\psi}_4(z) = \sum_{n=1}^{3}[L''_n(a,\omega)e^{-h_nz}]. \tag{39}$$

where $M_n(a,\omega), M'_n(a,\omega), M''_n(a,\omega)$ and $L_n(a,\omega), L'_n(a,\omega)$, $L''_n(a,\omega)$ are specific functions depending upon a, ω.

Using (34)-(36) in (15), (18)-(19), we get,

$$M'_n(a,\omega) = H_{1n}M_n(a,\omega), \tag{40}$$

$$M''_n(a,\omega) = H_{2n}M_n(a,\omega). \tag{41}$$

Similarly, using (37)-(39) in (16)-(17) and (20), we get,

$$L'_n(a,\omega) = R_{1n}L_n(a,\omega), \tag{42}$$

$$L''_n(a,\omega) = R_{2n}L_n(a,\omega). \tag{43}$$

Thus we have,

$$\bar{T}(z) = \sum_{n=1}^{3}[H_{1n}M_n(a,\omega)e^{-r_nz}] + N_1, \tag{44}$$

$$\bar{\psi}_3(z) = \sum_{n=1}^{3}[H_{2n}M_n(a,\omega)e^{-r_nz}] + N_2, \tag{45}$$

$$\bar{\phi}_2(z) = \sum_{n=1}^{3}[R_{1n}L_n(a,\omega)e^{-h_nz}], \tag{46}$$

$$\bar{\psi}_4(z) = \sum_{n=1}^{3}[R_{2n}L_n(a,\omega)e^{-h_nz}], \tag{47}$$

$$\bar{t}_{xx}(z) = \sum_{n=1}^{3}[H_{3n}M_n(a,\omega)e^{-r_nz}] + \sum_{n=1}^{3}[R_{3n}L_n(a,\omega)e^{-h_nz}] - N_3, \tag{48}$$

$$\bar{t}_{zx}(z) = \sum_{n=1}^{3}[H_{4n}M_n(a,\omega)e^{-r_nz}] + \sum_{n=1}^{3}[R_{4n}L_n(a,\omega)e^{-h_nz}], \tag{49}$$

$$\bar{t}_{zz}(z) = \sum_{n=1}^{3}[H_{5n}M_n(a,\omega)e^{-r_nz}] - \sum_{n=1}^{3}[R_{3n}L_n(a,\omega)e^{-h_nz}] - N_4, \tag{50}$$

$$\bar{q}_{xx}(z) = \sum_{n=1}^{3}[H_{6n}M_n(a,\omega)e^{-r_nz}] + \sum_{n=1}^{3}[R_{5n}L_n(a,\omega)e^{-h_nz}] + N_5, \tag{51}$$

$$\bar{q}_{zx}(z) = \sum_{n=1}^{3}[H_{7n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{6n}L_n(a,\omega)e^{-h_n z}], \tag{52}$$

$$\bar{q}_{zz}(z) = \sum_{n=1}^{3}[H_{8n}M_n(a,\omega)e^{-r_n z}] - \sum_{n=1}^{3}[R_{5n}L_n(a,\omega)e^{-h_n z}] + N_6, \tag{53}$$

$$\bar{m}_{zy}(z) = \sum_{n=1}^{3}[R_{7n}L_n(a,\omega)e^{-h_n z}]. \tag{54}$$

where,

$N = \frac{B_{13}}{R}\bar{Q}_1,\ N_1 = \frac{B_6 N}{B_2},\ N_2 = \frac{-B_5 N_1}{B_{10}},$

$N_3 = (A_{16}a^2 N + N_1),\ N_4 = (A_{17}a^2 N + N_1),\ N_5 = (A_{20}a^2 N_2),$

$N_6 = (A_{21}a^2 N_2),\ H_{1n} = \frac{(r_n^2 - B_6)}{B_2},\ H_{2n} = \frac{(B_5 H_{1n})}{(r_n^2 - B_{10})},$

$H_{3n} = (A_{17}r_n^2 - a^2 A_{16} - H_{1n}),\ H_{4n} = -iar_n(A_{18} + A_{19}),$

$H_{5n} = (A_{16}r_n^2 - a^2 A_{17} - H_{1n}),$

$H_{6n} = (a^2 A_{20} - A_{21}r_n^2)H_{2n},\ H_{7n} = ia(A_{22} + A_{23})r_n H_{2n},$

$H_{8n} = (a^2 A_{21} - A_{20}r_n^2)H_{2n},$

$R_{1n} = \frac{(B_7 - h_n^2)}{(A_4)},\ R_{2n} = -\frac{(B_{12}R_{1n})}{(h_n^2 - B_{11})},\ R_{3n} = ia(A_{16} - A_{17})h_n,$

$R_{4n} = -[A_{18}h_n^2 + a^2 A_{19} + (A_{19} - A_{18})R_{1n}],$

$R_{5n} = ia(A_{21} - A_{20})h_n R_{2n},$

$R_{6n} = (h_n^2 A_{22} + A_{23}a^2)R_{2n},\ R_{7n} = -A_{24}h_n R_{1n}.$

4. Boundary Conditions

To determine the parameters M_n and L_n $,(n = 1,2,3)$ the boundary conditions at the free surface $z = 0$ are given by,

$$t_{zz} = -F_1 e^{\omega t + iax}, t_{zx} = 0, m_{zy} = 0, q_{zz} = 0, q_{zx} = 0, \frac{\partial T}{\partial z} = 0. \tag{55}$$

where F_1 is the magnitude of mechanical force applied at the free surface..

Using the expressions of t_{zz} , t_{zx} , m_{zy} , q_{zz} , q_{zx} , and T into above boundary conditions (55), gives the following non homogeneous equations,

$$\sum_{n=1}^{3}[H_{5n}M_n] - \sum_{n=1}^{3}[R_{3n}L_n] = N_4 - F_1,$$
$$\sum_{n=1}^{3}[H_{4n}M_n] + \sum_{n=1}^{3}[R_{4n}L_n] = 0,$$
$$\sum_{n=1}^{3}[R_{7n}L_n] = 0,$$
$$\sum_{n=1}^{3}[H_{8n}M_n] + \sum_{n=1}^{3}[R_{5n}L_n] = -N_6,$$
$$\sum_{n=1}^{3}[H_{7n}M_n] + \sum_{n=1}^{3}[R_{5n}L_n] = 0,$$
$$\sum_{n=1}^{3}[H_{1n}r_n M_n] = 0.$$

After solving the above system of non homogeneous equations, we get the values of constants $M_1, M_2, M_3, L_1, L_2, L_3$ and hence obtain the components of normal displacement, microtemperature, normal force stress, temperature distribution, heat flux moment tensor and tangential couple stress for thermoelastic half space with micro-temperatures.

5. Particular Cases

i) If we take $\alpha^* = 0$, we obtain the results in thermoelastic medium with micro-temperatures (TM).

ii) Neglecting micro-rotation effect i.e. $\alpha = \beta = \gamma = b_0 = \mu = K = J = 0$, we obtain the results in thermoelastic medium with micro-temperatures without micro-rotation with temperature dependence (TMWMT).

iii) Letting $\alpha^* \to 0$ in the case (ii), we obtain the results in thermoelastic medium with micro-temperatures without micro-rotation (TMWM).

6. Numerical Results and Discussions

To determine the constants $M_1, M_2, M_3, L_1, L_2, L_3$, we consider the following values of the physical constants:

The values of micropolar constants are given by Eringen [35],

$\lambda = 9.4\times10^{10} N/m^2$, $\mu = 4.0\times10^{10} N/m^2$,

$\rho = 1.74\times10^3 kg/m^3$, $K = 10^{10} Nm^{-2}$,

$\gamma = 7.79\times10^{-10} N$, $J = 0.0000002\times10^{-14} m^2$,

$\beta = 0.32\times10^{10} N/m^2 K$

The values of thermal parameters are given by Dhaliwal and Singh [36]:

$c^* = 0.104\times10^4 Nm/Kg/K$, $T_0 = 298K$,

$K^* = 1.7\times10^2 Ns^{-1}K^{-1}$, $\alpha_t = 0.05K^{-1}$

The values of microtemperature parameters are given by Kumar and Kaur [20]:

$k_1 = 0.0035Ns^{-1}$, $k_2 = 0.045Ns^{-1}$, $k_3 = 0.055NK^{-1}s^{-1}$,

$k_4 = 0.065Ns^{-1}m^2$, $k_5 = 0.076Ns^{-1}m^2$, $k_6 = 0.096Ns^{-1}m^2$,

$\mu_1 = 0.0085N$, $b = 0.15\times10^{-10} N$.

The computations are carried out for the value of non-dimensional time $t = 0.2$ in the range $0 \le x \le 10$ and on the surface $z = 1.0$ for $\alpha^* = 0.051/K$. The numerical values for normal displacement, micro-temperatures, normal force stress, temperature distribution, heat flux moment tensor and tangential couple stress are shown in figures (2)-(7) for $F_1 = 1.0$, $Q_0 = 1$, $\omega = \omega_0 + \iota\xi$, $\omega_0 = -0.3$, $\xi = 0.1$, $Q_1 = 10$ and $a = 0.8$ for

a) Thermoelastic medium with micro-temperatures with temperature dependence (TMT) by solid line with centered symbol ♦.

b) TM by solid line with centered symbol ■

c) TMWMT by dashed line with centered symbol ▲.

d) TMWM by dashed line with centered symbol ×.

7. Discussion

The variations of normal displacement in case of TMWM are more as compared to TMT, TM, and TMWMT, which show the appreciable effect of micro-rotation. These variations are very less for TM as depicted in the range $0 \le x \le 10.0$ in the figure-2.

From figure-3, it is noticed that the variations of microtemperature is more for TMWM as compared to other three mediums. These values are less for TMWMT, which show the influence of temperature dependence.

The variations of normal force stress for TMT and TM are of opposite nature which signifies the influence of temperature dependence on the component of normal force stress. These variations are more in case of TMWMT as depicted in the figure-4.

The variations in temperature distribution is more in case of TMT. These variatrions are of opposite nature for TM and TMWMT. These variations are presented in figure-5.

The variations of heat flux moment tensor follow similar trend for TM and TMWMT with the values differ in magnitude, whereas these variations are of opposite nature for TMT and TMWM as depicted in the figure-6 in the range $0 \le x \le 10.0$.

The variations of tangential couple stress for TMT and TM follow similar pattern in the range $0 \le x \le 3.5$, with the values differ in magnitude, after that the variations show an opposite nature which show the significant effect of temperature dependence as presented in the figure-7.

8. Conclusion

From the graphical representations of the developed analytic solutions of thermoelastic half-space with micro-temperatures, the investigation of the influence of the heat source and temperature dependence in the medium is conducted. The following conclusion have been drawn,

- A significant effect of the heat source is observed in the medium.
- A noticeable effect of micro-rotation is observed.
- Temperature dependenc plays an prominent role in the deformation of the medium.
- The present problem can also be studied in the absence of heat source as well.

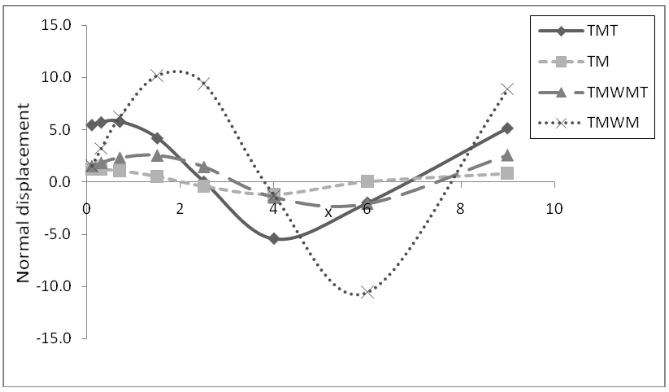

Figure 2. Variation of normal displacement with horizontal distance

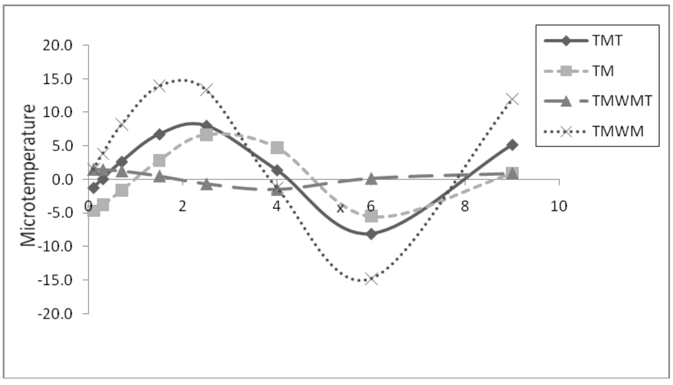

Figure 3. Variation of micro temperature with horizontal distance

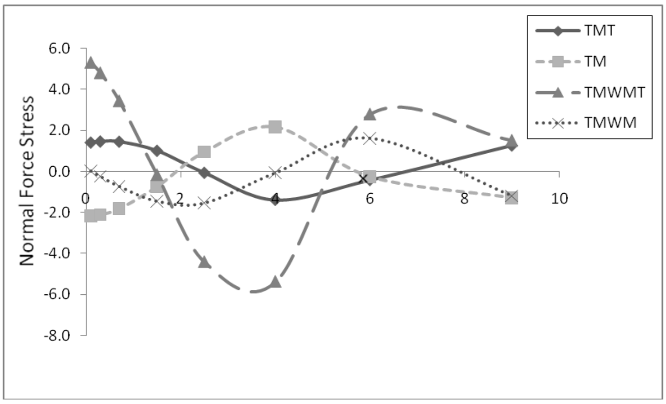

Figure 4. Variation of normal force stress with horizontal distance

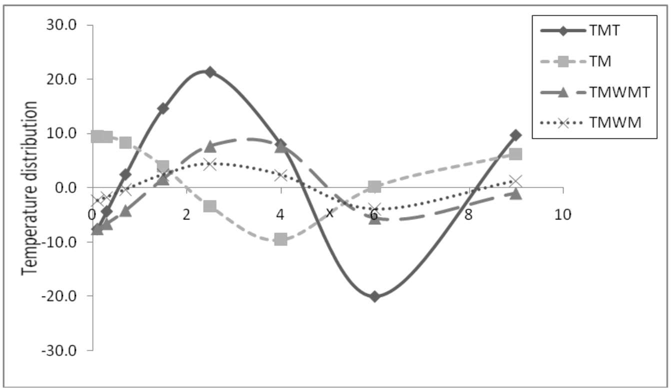

Figure 5. Variation of temperature distribution with horizontal distance

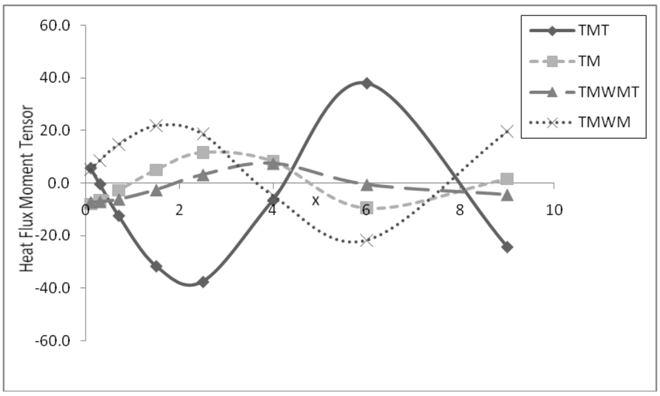

Figure 6. Variation of heat flux moment with horizontal distance

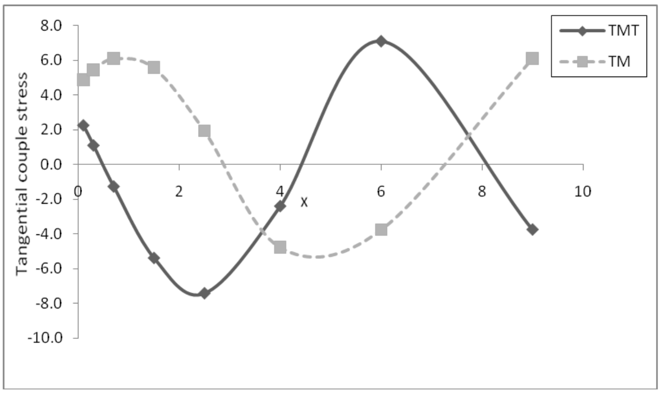

Figure 7. Variation of tangential stress with horizontal distance

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