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Free vibration analysis of BNNT with different cross-Sections via nonlocal FEM

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ABSTRACT

In the present study, free vibration behaviors of carbon nanotube (CNT) and boron nitride nanotube (BNNT) have been investigated via Eringen's nonlocal continuum theory. Size effect has been considered via nonlocal continuum theory. Nanotubes have become popular in the world of science thanks to their characteristic properties. In this study, free vibrations of Boron Nitride Nanotube (BNNT) and Carbon Nanotube (CNT) are calculated using the Nonlocal Elasticity Theory. Frequency values are found via both analytical and finite element method (FEM). Galerkin weighted residual method is used to obtain the finite element equations. BNNT and CNT are modeled as Euler - Bernoulli Beam and solutions are gained by using four different cross-section geometries with three boundary conditions. Selected geometries are circle, rectangle, triangle, and square. Frequency values are given in tables and graphs. The effect of cross-section, boundary conditions and length scale parameter on frequencies has been investigated in detail for BNNT.

1. Introduction

With the developmen[1]ts in technology, nanotechnology emerged and has become a focus of interest in recent years. In nanotechnology, nanotubes attract attention as one of the highly studied subjects. Nanotube is, the most general definition, tube-shaped 1-D nano-scale structure. In 1959, physicist Richard Feynman gave a speech called "There's plenty of room at the Bottom" [1]. He talked about the possibility of producing more powerful devices at smaller size and the advantages of miniaturization in his speech. With this speech, the idea of nanotechnology was first introduced by Feynman. Nanotechnology is an interdisciplinary field with great potential. Nanotechnology is studied in many disciplines such as Applied Physics, Materials Science, Device Physics, Chemistry, Chemical Engineering, Electrical Engineering, Health, Civil Engineering, Aerospace Engineering. One of the most important subjects of nanotechnology is nanotubes. Nanotubes are 1-D, tube-shaped nanostructures and have remarkable properties. Thanks to their remarkable properties, the interest on nanotubes has increased.

When nanotubes are classified in terms of the atoms forming themselves, they can be divided into two groups: organic and inorganic [2]. Organic nanotubes are Carbon Nanotubes (CNT). CNT, one of the most famous nano structures, was discovered by Sumio Iijima in 1991 [3] CNT is a C allotropic only composed of C atoms [4]. Hexagons obtained by adding the C atoms to the corners combine with each other to form a long, cylindrical structure, which causes the CNT structure. The mechanical, electrical, thermal, physical, chemical properties of CNTs are unconventional. CNTs have high Young modulus and tensile strength, low density, large length/diameter ratio [2]. The Boron nitride Nanotube (BNNT), which is an inorganic nanotube (see Figure 1), was theoretically estimated in 1994 [5,6]. Following this prediction, the first synthesis was produced in 1995 [7]. BNNT and the CNT are very similar structurally. CNT is formed by rolling the graphene layer consisting only of C atoms, while BNNT is formed by rolling the BN layer [8].

According to the classical physics theories, equilibrium equations can be applied to every point of the object. However, this applies to macro-dimensional structures, and as the dimensions get smaller, the internal structure of the material and the interactions at the other points must also be considered [9]. Since these interactions are not handled in classical theories, calculations are not exactly accurate. In other words, taking the size effect into accounts is important in order to

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obtain more accurate results. Therefore, Nonlocal Theory of Elasticity presented by Eringen is used [10]. This theory based on the fact that the stress of a point must depend not only on that point but also the function of the shape changes of all other points.

Many researchers have analyzed nano/micro structures using various methods. Reddy [11] reformulated various beam theories, such as the Euler–Bernoulli, Timoshenko, Reddy, and Levinson beam theories, for bending, buckling and vibration using the nonlocal differential constitutive relations of Eringen. Kong et al. [12] solved the dynamic problems of Bernoulli–Euler beams based on modified couple stress theory. Civalek and Demir [13] developed elastic beam model using nonlocal elasticity theory for the bending analysis of microtubules (MTs) based on the Euler–Bernoulli beam theory. Vibration analysis of the Euler-Bernoulli beam was reported using the finite element method by Eltaher et al [14]. They used Eringen's Nonlocal constitutive equation. Khan and Hashemi [15] modeled double-walled CNTs as local and nonlocal Euler Bernoulli beams. They found the natural frequencies of double-walled CNTs with various boundary conditions by using finite element formulation. Dinçkal [16] analyzed vibration of CNTs by using finite element method. CNTs were modeled according to Euler-Bernoulli and Timoshenko beam theory. She presented the results obtained with tables and graphs. Norouzzadeh and Ansari [17] investigated static bending of Timoshenko nanobeams by using finite element analysis.

Demir and Civalek [18] presented nonlocal finite element formulation for vibration. Some important studeies have also been made by researchers for nano modeling [19-30]. Recently, stress analytsis of nano structures have been investigated by Hosseini et al. [31,32]. Shishesaz et al. [33] gives detailed review for size-dependent elasticity for nanostructures. Analysis of functionally graded nanodisks under thermoelastic loading based on the strain gradient theory has been given by Shishesaz et al. [34]. Vibrations of three-dimensionally graded nanobeams and buckling of FGM Euler-Bernoulli nano-beams were discussed by Hadi et al. [35,37]. Adeli et al. [36] proposed torsional vibration of nano-cone based on nonlocal strain gradient elasticity theory. Buckling analysis of arbitrary two-directional functionally graded Euler-Bernoulli nano-beams based on nonlocal elasticity theory has been made by Nejad et al. [۳۸]. Some other effects such as thermal, magnetic and piezoelectricity on mechanical modeling of nanostructures have been detailed discussed [39-48].

By this time carbon nanotubes have been detailed investigated via some higher-order continuum theories. In this study, however, the authors analyzed the boron nitride nanotube via size-dependent continuum theory. Different cross-sections have been considered and finite element formulation has been applied. Also, nonlocal matrix and their elements have been listed in detailed via beam and size-dependent parameters. Galerkin weighted residual method is used to obtain the finite element parameters.

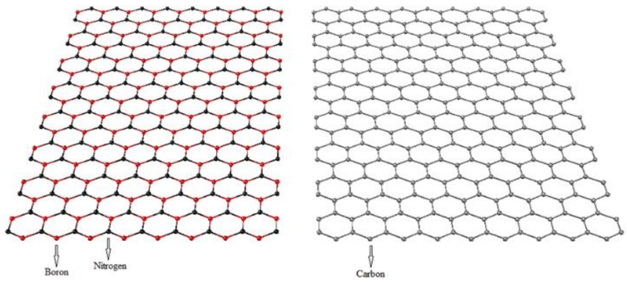

Figure 1. Demonstration of BN layer and graphene

2. The Nonlocal Euler-Bernoulli Beam Theory

The nonlocal stress tensor at point x is expressed as [10,19]

$$\sigma = \int K(|x' - x|, \tau) t(x') dx' \tag{1}$$

where t(x)=C(x): ε(x) is the classical, macroscopic stress tensor at point x, $K(|x'-x|,\tau)$ is kernel function, $|x'-x|$ is the distance in Euclidean form and τ is a material constant that depends on internal and external characteristic lengths, C is the fourth-order elasticity tensor. The nonlocal constitutive formulation is

$$\left[1-(e_0 a)^2 \nabla^2\right]\sigma = t \tag{2}$$

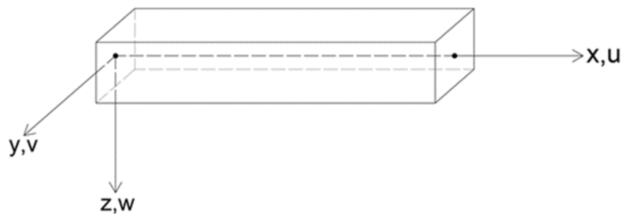

Figure 2. Illustration of coordinates of beam

According to the coordinates via Figure 2 selected above, x, y, z indicate the length, width and height of the beam. u, v, w are the displacements in the x, y, z directions, respectively. The displacements for a Bernoulli–Euler beam can be written as [20]

$$u(x,z,t) = -z\frac{\partial w(x,t)}{\partial x} \tag{3}$$

$$v(x,z,t) = 0 \tag{4}$$

$$w(x,z,t) = w(x,t) \tag{5}$$

ε is the strain tensor, expressed as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{6}$$

$u_{i,j}$ means

$$u_{i,j} = \frac{\partial u_i}{\partial j} \tag{7}$$

We obtain from Eq. (6) the strains of the Euler-Bernoulli beam as follows

$$\varepsilon_{xx}=\frac{1}{2}(\frac{\partial u(x,z,t)}{\partial x}+\frac{\partial u(x,z,t)}{\partial x})=-z\frac{\partial^2 w(x,t)}{\partial x^2} \quad (8)$$

$$\varepsilon_{xx}=\frac{1}{2}(\frac{\partial u(x,z,t)}{\partial x}+\frac{\partial u(x,z,t)}{\partial x})=-z\frac{\partial^2 w(x,t)}{\partial x^2} \quad (9)$$

$$\varepsilon_{xz}=\varepsilon_{zx}=\frac{1}{2}(\frac{\partial u(x,z,t)}{\partial z}+\frac{\partial w(x,z,t)}{\partial x})= \frac{1}{2}(-\frac{\partial w(x,t)}{\partial x}+\frac{\partial w(x,t)}{\partial x})=0 \quad (10)$$

$$\varepsilon_{yy}=0 \quad (11)$$

$$\varepsilon_{yz}=\varepsilon_{zy}=0 \quad (12)$$

$$\varepsilon_{zz}=0 \quad (13)$$

Only ε_{xx} has a non-zero value. E elasticity modulus and σ stress, the strain for the linear elastic materials is expressed as follows

$$\sigma=E\varepsilon \quad (14)$$

σ_{xx} is obtained if ε_{xx} is written in Eq. (14) as we obtained in Eq. (8)

$$\sigma_{xx}=E\varepsilon_{xx}=-Ez\frac{\partial^2 w(x,t)}{\partial x^2} \quad (15)$$

Moment (M) and the moment of inertia (I) are given by

$$M=\int_A z\,\sigma_{xx}dA \quad (16)$$

$$I=\int_A z^2 dA \quad (17)$$

Here, A is the cross-sectional area.

For one dimensional case, the nonlocal constitutive relations can be written as below [10,21,22]

$$\sigma_{xx}-(e_0a)^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\varepsilon_{xx} \quad (18)$$

Where e_0 is the constant which is determined experimentally, a is the internal characteristic length. Multiplying z on both sides of Eq. (18) and integrating over the cross-sectional area (A) of the beam, we obtain

$$\int_A z\sigma dA-(e_0a)^2\int_A z\frac{\partial^2\sigma}{\partial x^2}dA=\int_A zE\varepsilon dA=0 \quad (19)$$

Substituting Eq. (8), (16) and (17) into (18), we get

$$M(x,t)-(e_0a)^2\frac{\partial^2 M(x,t)}{\partial x^2}=-EI\frac{\partial^2 w(x,t)}{\partial x^2} \quad (20)$$

For the transverse vibration of Euler-Bernoulli beam, the equilibrium conditions are

$$V(x,t)=\frac{\partial M(x,t)}{\partial x} \quad (21)$$

$$\frac{\partial V(x,t)}{\partial x}=\rho A\frac{\partial^2 w(x,t)}{\partial t^2} \quad (22)$$

$$\frac{\partial^2 M(x,t)}{\partial x^2}=\rho A\frac{\partial^2 w(x,t)}{\partial t^2} \quad (23)$$

Where ρ is the mass density. By differentiating equation (20) twice with respect to the variable x and substituting Eq. (23) into Eq. (20), we get the equation of free vibration of Euler-Bernoulli nanobeams

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}-(e_0a)^2\rho A\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2}+\rho A\frac{\partial w^2(x,t)}{\partial t^2}=0 \quad (24)$$

3. Galerkin Weighted Residual Method

For finite element formulation, we considred the Fig. 3 as below. As seen in Fig. 3, beam element has two end nodes and four degrees of freedom.

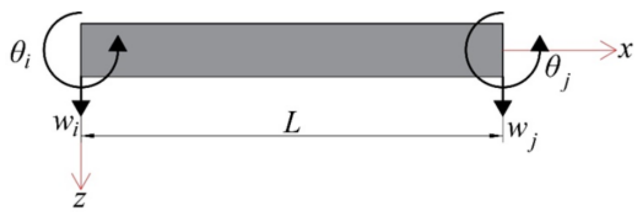

Figure 3. Illustration of a beam element

The degrees of freedom are shown below

w_i: displacement of i , θ_i: rotation of i, w_j: displacement of j , and θ_j: rotation of j. The displacement of the beam element is expressed by four constants due to the degrees of freedom [23-30]

$$w=\alpha_1+\alpha_2x+\alpha_3x^2+\alpha_4x^3=\begin{bmatrix}1 & x & x^2 & x^3\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{bmatrix} \quad (25)$$

Since the rotation is expressed as $\theta=-\frac{dw}{dx}$, it is written as

$$\theta=-(\alpha_2+2\alpha_3x+3\alpha_4x^2) \quad (26)$$

Find the deformations of the beam element at points i (x = 0) and j (x = L) from Eq. (25) and Eq. (26)

i (x=0) $\quad w(0)=\alpha_1 \quad (27)$

$\theta(0)=-\alpha_2 \quad (28)$

j(x=L) $\quad w(L)=\alpha_1+\alpha_2L+\alpha_3L^2+\alpha_4L^3 \quad (29)$

$\theta(L)=-(\alpha_2+2\alpha_3L+3\alpha_4L^2) \quad (30)$

If we write the displacement and rotation expressions in matrix form

$$\begin{Bmatrix}w_i\\ \theta_i\\ w_j\\ \theta_j\end{Bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 1 & L & L^2 & L^3\\ 0 & -1 & -2L & -3L^2\end{bmatrix}\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{Bmatrix} \quad (31)$$

Write the coefficients $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ from Eq. (31)

$$\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{Bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ \frac{-3}{L^2} & \frac{2}{L} & \frac{3}{L^2} & \frac{1}{L}\\ \frac{2}{L^3} & \frac{-1}{L^2} & \frac{-2}{L^3} & \frac{1}{L^2}\end{bmatrix}\begin{Bmatrix}w_i\\ \theta_i\\ w_j\\ \theta_j\end{Bmatrix} \tag{32}$$

Substitution Eq. (32) into Eq. (25), the shape function φ is obtained.

$$\varphi=\begin{Bmatrix}1-3\xi^2+2\xi^3\\ L(-\xi+2\xi^2-\xi^3)\\ 3\xi^2-2\xi^3\\ L(\xi^2-\xi^3)\end{Bmatrix} \tag{33}$$

To obtain the weak form of the governing equation obtained according to the nonlocal Euler-Bernoulli beam theory, the residue can be expressed as

$$I=\left[EI\frac{\partial^4 w}{\partial x^4}-(e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}+\rho A\frac{\partial^2 w}{\partial t^2}\right]=residue \tag{34}$$

Eq. (34) is multiplied by a weighting function (φ) to define the weighted residue. When the weighted residual is integrated over the length

$$\int_0^L \varphi I dx=0 \tag{35}$$

Substituting Eq. (34) into Eq. (35)

$$\int_0^L\left[\varphi EI\frac{\partial^4 w}{\partial x^4}-\varphi(e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}+\varphi\rho A\frac{\partial^2 w}{\partial t^2}\right]dx=0 \tag{36}$$

Eq. (36) is integrated by parts. According to the chain rule, the general form

$$\int_0^L\left[EI\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi^T}{\partial x^2}-(e_0a)^2\rho A\frac{\partial\varphi}{\partial x}\frac{\partial\varphi^T}{\partial x}\ddot{w}+\rho A\varphi\varphi^T\ddot{w}\right]dx=0 \tag{37}$$

By ζ=x/L using the shape functions in Eq. (33) and the dimensionless local coordinate, the stiffness matrix (K) and the mass matrices (M^1, M^2) are obtained

$$K=\frac{EI}{L^3}\begin{bmatrix}12 & -6L & -12 & -6L\\ -6L & 4L^2 & 6L & 2L^2\\ -12 & 6L & 12 & 6L\\ -6L & 2L^2 & 6L & 4L^2\end{bmatrix} \tag{38}$$

$$M^1=\frac{\rho A}{420}\begin{bmatrix}156L & -22L^2 & 54L & 13L^2\\ -22L^2 & 4L^3 & -13L^2 & -3L^3\\ 54L & -13L^2 & 156L & 22L^2\\ 13L^2 & -3L^3 & 22L^2 & 4L^3\end{bmatrix} \tag{39}$$

$$M^2=\frac{(e_0a)^2\rho A}{30L}\begin{bmatrix}36 & -3L & -36 & -3L\\ -3L & 4L^2 & 3L & -L^2\\ -36 & 3L & 36 & 3L\\ -3L & -L^2 & 3L & 4L^2\end{bmatrix} \tag{40}$$

The free vibration of the Euler-Bernoulli beam is found as follows

$$\left|K-\omega^2 M\right|=0 \tag{41}$$

Here M= M^1+M^2 and ω is frequency.

4. Numerical Results

In this section, the frequency values of nanotubes are obtained with different lengths, different e0a/L values, different cross-sectional geometries, different boundary conditions and different number of elements (N). Boundary conditions are simply supported at both ends (S–S), clamped-clamped (C–C) and clamped – simply supported (C–S). The results obtained are shown in tables and graphs. It is seen from the Tables 1-6 and Figs. 4-7 that the frequency values of the nanobeam decreases with increasing the nondimensional nonlocal parameters and increasing the mode numbers. The highest frequency value is seen on the triangular cross-section.

Table 1. Frequencies for CNT & BNNT (10^9 rad/sn)

	S – S , Circular , e_0a/L = 0 , L = 40			
Mode Numbers	CNT		BNNT	
	Analytical	FEM (N=100)	Analytical	FEM (N=100)
1	148.3739	148.3739	126.1339	126.1339
2	593.4956	593.4956	504.5356	504.5356
3	1335.3651	1335.3652	1135.2051	1135.2051
4	2373.9824	2373.9828	2018.1424	2018.1427
5	3709.3475	3709.3491	3153.3474	3153.3488

Table 2. Frequencies of different geometries for analytical & FEM (10^9 rad/sn)

C – S , L = 20 nm , BNNT , N=25									
e_0a/L	Mode Numbers	Circular		Rectangular		Quadratic		Triangular	
		Analytical	FEM	Analytical	FEM	Analytical	FEM	Analytical	FEM
0	1	788.1809	788.1812	1046.6324	1046.6328	806.4617	806.4620	1209.6091	1209.6096
	2	2554.2109	2554.2222	3391.7593	3391.7743	2613.4524	2613.4640	3919.9083	3919.9257
	3	5329.1571	5329.2597	7076.635	7076.7711	5452.7599	5452.8648	8178.5758	8178.7332
	4	9113.1741	9113.6851	12101.465	12102.143	9324.542	9325.0649	13985.849	13986.633
	5	13906.262	13908.071	18466.249	18468.650	14228.799	14230.65	21341.728	21344.503
0.1	1	746.3139	746.3142	991.0368	991.0373	763.6237	763.6240	1145.3565	1145.3570
	2	2136.5522	2136.5619	2837.1465	2837.1595	2186.1067	2186.1166	3278.9339	3278.9489
	3	3826.4338	3826.5111	5081.1553	5081.2580	3915.1829	3915.2620	5872.3694	5872.4880
	4	5602.5012	5602.8340	7439.611	7440.0529	5732.4438	5732.7843	8598.0728	8598.5836
	5	7383.1323	7384.1504	9804.1268	9805.4787	7554.3743	7555.4160	11330.780	11332.343
0.2	1	651.5667	651.5670	865.2212	865.2216	666.6789	666.6792	999.9494	999.9499
	2	1550.8298	1550.8377	2059.3607	2059.3711	1586.7993	1586.8073	2380.0348	2380.0468
	3	2448.2587	2448.3126	3251.0643	3251.1360	2505.0428	2505.0979	3757.3051	3757.3878
	4	3318.7311	3318.9417	4406.9724	4407.2520	3395.7046	3395.9201	5093.2058	5093.5290
	5	4169.6422	4170.2464	5536.9049	5537.7072	4266.3516	4266.9697	6399.0861	6400.0133

Table 3. Frequencies for various N (10^9 rad/sn)

C-C , Rectangular , L = 10 nm , e_0a/L = 0.4						
Mode Numbers	CNT					
	Analytical	N=6	N=7	N=8	N=10	N=20
1	4121.8764	4123.9355	4123.0054	4122.5449	4122.1535	4121.8940
2	6752.2336	6771.1086	6762.7933	6758.5661	6754.8972	6752.4061
3	9570.2437	9661.8603	9622.8533	9602.2741	9583.9541	9571.1534
4	12082.4884	12321.4960	12231.1703	12175.6150	12123.4049	12085.2929
5	14740.7602	14837.0893	15081.3687	14968.7276	14844.3726	14748.1165

Table 4. Frequencies of CNT & BNNT for analytical solution (10^9 rad/sn)

C – C , Triangular , L = 80 nm , e_0a/L=0.2		
Mode Numbers	CNT	BNNT
1	105.4913	89.6791
2	210.0892	178.5986
3	314.4882	267.349
4	413.0534	351.1402
5	510.3829	433.8808

Table 5. Frequencies and errors of CNT & BNNT for different N (10^9 rad/sn)

C – C , Triangular , L = 80 nm , e_0a/L=0.2					
	Mode Numbers	CNT		BNNT	
		FEM	Error (%)	FEM	Error (%)
N=15	1	105.4922	0.00085	89.6798	0.00078
	2	210.1034	0.00676	178.6106	0.00672
	3	314.5692	0.02576	267.4179	0.02577
	4	413.3302	0.06701	351.3755	0.06701
	5	511.1137	0.14319	434.5020	0.14317
N=20	1	105.4916	0.00028	89.6793	0.00022
	2	210.0937	0.00214	178.6024	0.00213
	3	314.5142	0.00827	267.3712	0.00830
	4	413.1431	0.02172	351.2164	0.02170
	5	510.6222	0.04689	434.0842	0.04688
N=25	1	105.4915	0.00019	89.6792	0.00011
	2	210.0911	0.00090	178.6002	0.00090
	3	314.4989	0.00340	267.3582	0.00344
	4	413.0906	0.00901	351.1718	0.00900
	5	510.4825	0.01951	433.9654	0.01950
N=30	1	105.4914	0.00009	89.6791	0.00000
	2	210.0901	0.00043	178.5993	0.00039
	3	314.4934	0.00165	267.3535	0.00168
	4	413.0714	0.00436	351.1555	0.00436
	5	510.4314	0.00950	433.9220	0.00950

Table 6. Frequencies for analytical & FEM (10^9 rad/sn)

		C – S , L = 20 nm , N = 100					
e_0a/L	Mode Numbers	Circular					
		CNT			BNNT		
		Analytical	FEM	Error (%)	Analytical	FEM	Error (%)
0	1	927.1534	927.1534	0.00000	788.1809	788.1809	0.00000
	2	3004.5709	3004.5709	0.00000	2554.2109	2554.2109	0.00000
	3	6268.7973	6268.7978	0.00001	5329.1571	5329.1575	0.00001
	4	10720.014	10720.017	0.00003	9113.1741	9113.1761	0.00002
	5	16358.223	16358.231	0.00005	13906.262	13906.269	0.00005
0.1	1	877.9044	877.9044	0.00000	746.3139	746.3139	0.00000
	2	2513.2703	2513.2704	0.00000	2136.5522	2136.5522	0.00000
	3	4501.1129	4501.1133	0.00001	3826.4338	3826.4341	0.00001
	4	6590.3376	6590.3392	0.00002	5602.5012	5602.5025	0.00002
	5	8684.9306	8684.9354	0.00006	7383.1323	7383.1364	0.00006
0.2	1	766.4513	766.4513	0.00000	651.5667	651.5667	0.00000
	2	1824.2731	1824.2731	0.00000	1550.8298	1550.8299	0.00001
	3	2879.9371	2879.9373	0.00001	2448.2587	2448.2589	0.00001
	4	3903.8918	3903.8927	0.00002	3318.7311	3318.7319	0.00002
	5	4904.8361	4904.8389	0.00006	4169.6422	4169.6447	0.00006

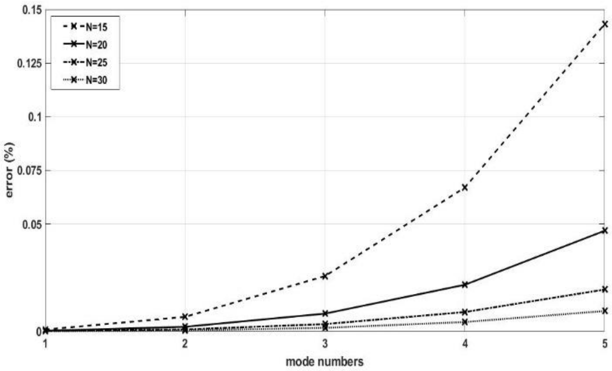

Figure 4. Error percentage between frequencies of analytical and FEM solution with different N

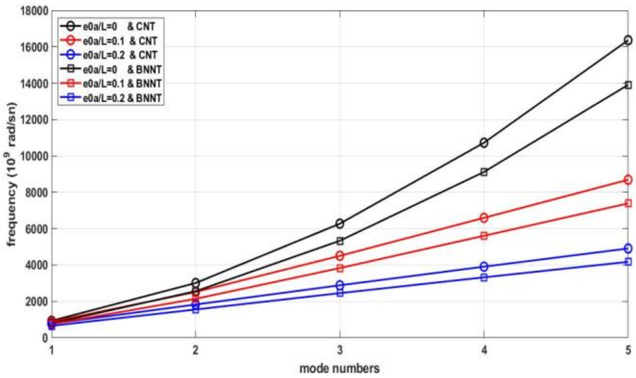

Figure 5. First 5 frequencies of CNT & BNNT with different e_0a/L

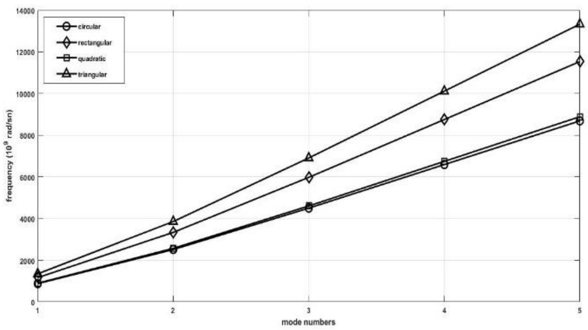

Figure 6a. Frequencies of nanotubes with different cross-sectional geometries with $e_0a/L=0.1$ (CNT)

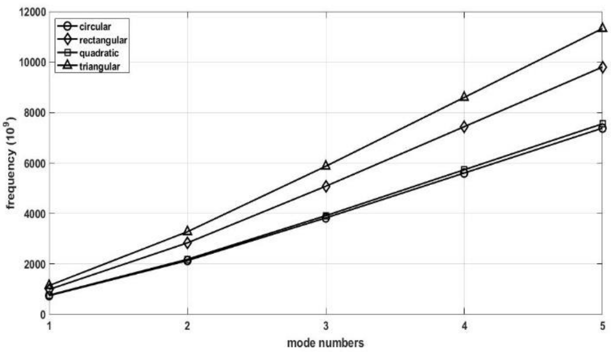

Figure 6b. Frequencies of nanotubes with different cross-sectional geometries with $e_0a/L=0.1$ (BNNT)

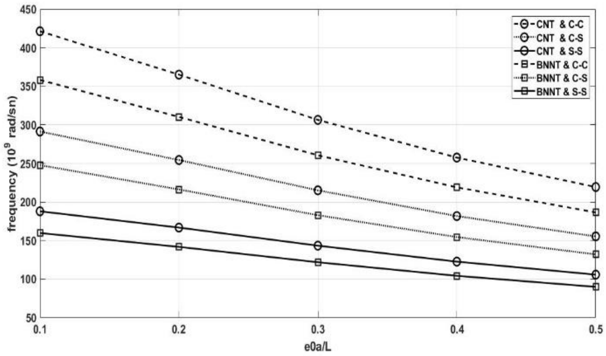

Figure 7. Frequencies of nanotubes with different e_0a/L and boundary conditions

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5. Conclusions

In this paper, free vibration analyzes of BNNT and CNT are investigated based on the Nonlocal Euler-Bernoulli beam theory. Solutions are obtained for S-S, C-S, C-C boundary conditions and four different cross-section geometries such as circular, rectangular, quadratic, and triangular by using analytical and FEM. Results are shown in tables and graphs. The results show that frequency of the nanobeams decreases with increasing the nondimensional nonlocal parameters. The highest frequency value is seen on the triangular cross-section. Circular cross section has the lowest frequency value. Using the same geometric and material parameter, namely under the similar conditions, the frequency values are bigger for CNT than BNNT. For FEM as the number of elements increases, the results approach real value.

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