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Nonlinear free vibration of viscoelastic nanoplates based on modified couple stress theory

Masoud Ajri[1], Mir Masoud Seyyed Fakhrabadi [1,]*

[1] *School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran*

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ABSTRACT

In this paper, a new viscoelastic size-depended model developed based on a modified couple stress theory and the for nonlinear viscoelastic material in order to vibration analysis of a viscoelastic nanoplate. The material of the nanoplate is assumed to obey the Leaderman nonlinear constitutive relation and the von Kármán plate theory is employed to model the system. The viscous parts of the classical and nonclassical stress tensors are obtained based on the Leaderman integral and the corresponding work terms are calculated. The viscous work equations are balanced by the terms of size-dependent potential energy, kinetic energy. Then the equations of motion are derived from Hamilton's principle. The governing nonlinear integro-differential equations with coupled terms are solved by using the fourth-order Runge-Kutta method and Galerkin approach. The results are validated by carrying out the comparison with existing results in the literature when our model is reduced into an elastic case. In order to explore the vibrational characteristics, the influences of the thickness ratio, relaxation coefficient, and aspect ratio on the frequency and damping ratio were also examined. The results revealed that the frequency, vibration amplitude and damping ratio of viscoelastic nanoplate were significantly influenced by the relaxation coefficient of nanoplate material, and length scale parameter. Also, it was found that with increasing (h/l) the vibration frequency decreases and its amplitude and damping ratio increase.

1. Introduction

Nanoplates are a subgroup of nanostructures with two-dimensional geometries. They are used as thin-film elements [1], nano-sheet resonators [2], paddle-like resonators [3], gas sensors [4,5] and mass sensors [6] due to their exceptional mechanical, thermal and electrical properties. Researchers studied various mechanical behavior of nanostructures based on the experimental methods, molecular dynamics simulations [7-8] and classical continuum theories but their higher accuracy demands the need for developing more accurate models to analyze.

At nanostructure, experimental and atomistic simulation results have shown a significant size effect [7, 9-10] on the mechanical properties. Computational methods such as molecular dynamic (MD) simulation is reasonable in the analysis of nanostructures [11], however this method is time consuming for nanostructure with large numbers of atom. The classical elasticity theories governing the mechanical behavior of the macro-materials often fail to predict the properties of the nanostructures. This is the reason why researchers have tried to develop some size-depended theories to compensate the shortages of the classical techniques [12-16].Based on these theories various mechanical behavior of micro/nanoplates have been studied [17-24].In view of the difficulties in determining the material length scale parameters, the modified couple stress theory first proposed by Yang et al. [13] that is used in this paper. This theory takes an advantage over the aforementioned size-dependent continuum theories due to involving only one material length scale parameter. This theory is used to determine the mechanical behavior of the micro-nanostructures including their vibration [25-27] buckling [28, 29] and electromechanical properties [30-32].

As a result of nanoplate's application as resonators and sensors, it is important to understand their vibration characteristics. Therefore, the vibration and dynamic analysis of the nanoplates has become a subject of primary interest in recent studies [33-35].Kiani[36] studied vibrations of an embedded nanoplate subjected to biaxially applied loads and a moving

[*] Corresponding author. +98 21 61114033;
e-mail: mfakhrabadi@ut.ac.ir ; msfakhrabadi@gmail.com

nanoparticle. At this study the nanoplate is modeled via nonlocal Kirchhoff plate. The vibration of elastic thin nanoplates traversed by a moving nanoparticle involving Coulomb friction is investigated using the nonlocal continuum theory of Eringen[37]. Murmu and Pradhan [38] studied small-scale effect on free in-plane vibration of the nanoplates by employing nonlocal continuum mechanics. They also investigated the vibration response of the nanoplates under uniaxially pre-stressed conditions [39]. Size-dependent natural frequencies and mode shapes of circular nanoplates under the effect of surface properties has been investigated by Assadi and Farshi [40]. In another work, a three dimensional vibration analysis of anisotropic layered composite nanoplates has been studied based on modified couple-stress theory [41]. Buckling and vibration of nanoplates such as single layered graphene sheets have been considered using nonlocal elasticity theory [42]. In that case, Navier type solution was used for simply supported sheets and Levy type method was used for nanoplates with two opposite edge simply supported and remaining ones arbitrary. Malekzadeh et al. [43] investigated the free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates. Recently, free vibration of a circular FG Microplates in thermal environments at different boundary condition have been investigated [44].

It should be noted that a large amount of previous researcha activities have been concentrated on the calculation of frequencies of the nanoplates with elastic structure whereas the nanostucture similar to many materials can also reveal viscoelastic structural damping. Recently some experiments have revealed that the viscoelastic phenomena widely exist in NEMS and MEMS materials such as silicon [45], polysilicon [46] and metals [47]. Su et al. recently reported the viscoelastic properties of graphene oxide nanoplate proved by their experimental investigations [48]. The tensile tests on this nanoplate show clear hysteresis loops, indicating the viscoelastic property of the graphene oxide nanoplate. In addition, Liu and Zhang [49] discussed the vibration of a double-viscoelastic FGM nanoplate, and Pouresmaeeli et al [50] investigated the vibration analysis of viscoelastic orthotropic nanoplates resting on viscoelastic medium. Jamalpoor et al [51] studied free transverse vibration analysis of orthotropic multi-viscoelastic microplate system via modified strain gradient. The vibration and instability of axially moving viscoelastic micro-plate is investigated. The viscoelastic structural properties of micro-plate are taken into consideration based on Kelvin's model [52]. In all above mentioned work, the Boltzmann superposition principle was incorporated, enabling the modeling of linear viscoelastic materials. However, it is well known that many viscoelastic materials are not linear hence they should be modeled nonlinearily in order to give an adequate description of a viscoelastic structure behavior. The comparison research [53] has shown that the Leaderman model [54] is one of useful representations, when prediction and simplicity are concerned. This model have been used in this paper.

To the best of our knowledge, no investigation has been performed on the vibration of nonlinear viscoelastic nanoplates by modified continuum models. The aim of this study is to present a new size-depended model based modified couple stress for nonlinear viscoelastic material in order to vibration analysis of the nanoplates. The material of the nanoplate is assumed to obey the Leaderman nonlinear constitutive relation and the von Kármán plate theory is employed to model the system. The dynamic governing equations together with initial conditions and boundary conditions are obtained by a combination of the basic equations of modified couple stress theory [13] and Hamilton's principle. Then, the nonlinear size-dependent viscoelastically coupled integro-differential equation are solved by incorporating the expansion theory and Galerkin method with the fourth-order Runge–Kutta technique, and some useful results are obtained.

2. Governing equations of viscoelastic nanoplates

Consider a rectangular plate of length a, width b, and thickness h with viscoelastic material behavior. To obtain the governing equation of this viscoelastic nanoplate the generalized is applied

$$\delta\int_{t_1}^{t_2}\left[T-U+W\right]dt=0 \tag{1}$$

where δT indicates the variation of the kinetic energy, δU represents the variation of the elastic strain energy or elastic potential energy, and δW denotes the virtual work done by non-conseravtive forces such as external and viscous dissipation.

Here, it is convenient to divide the virtual work δW into two parts. One is the virtual work δW_{ext} performed by all external forces and the other is the virtual work δW_{vis} performed by the viscous dissipative forces. Therefore, we get

$$\delta W=\delta W_{ext}+\delta W_{vis} \tag{2}$$

Substituting Eq. (2) into Eq. (1), the generalized Hamilton's principle can be rewritten as

$$\int_{t_1}^{t_2}\left[\delta T-\delta U+\delta W_{ext}+\delta W_{vis}\right]dt=0 \tag{3}$$

In the following analysis, the formulation is limited to small strains, and moderate rotations and displacements, so that there is no geometric update of the domain, and consequently, there is no difference between the Cauchy stress tensor and the second Piola–Kirchhoff stress tensor. In the modified couple stress theory proposed by Yang et al. [13] the strain tensor ε and the symmetric part of the curvature tensor χ can be given as

$$\varepsilon=\frac{1}{2}\left[\nabla u+\left(\nabla u\right)^T\right]\quad \chi=\frac{1}{2}\left[\nabla\omega+\left(\nabla\omega\right)^T\right] \tag{4}$$

where u is the displacement vector and ω is the rotation vector defined as

$$\omega=\frac{1}{2}curl\,(u) \tag{5}$$

According to the Leaderman constitutive relation [54], for a viscoelastic structure the stress tensor σ and deviatoric part of the couple stress tensor m are given as

$$\sigma=\lambda\otimes tr(\varepsilon)I+2\mu\otimes\varepsilon \tag{6}$$

$$m=2l^2\mu\otimes\chi$$

where λ and μ are time-dependent Lame's constants, l is the material length-scale parameter, and the Stieltjes's convolution operation symbol '$\otimes$' is defined as

$$g(t)\otimes k(t)=g(0^+)k(t)+\int_{0^+}^{t}\frac{\partial g(t-\tau)}{\partial(t-\tau)}k(\tau)d\tau \tag{7}$$

Based on the definition in Eq. (7), the Eq. (6) can be rewritten as

$$\sigma=\left[\lambda_0 tr(\varepsilon(t))I+2\mu_0\varepsilon(t)\right]+$$

$$\int_0^t\left[\dot\lambda(t-\tau)tr(\varepsilon(\tau))I+2\dot\mu(t-\tau)\varepsilon(\tau)\right]d\tau=\sigma_e+\sigma_v \tag{8}$$

$$m = 2l^2\mu_0\chi(t) + \int_0^t \left[2l^2\dot{\mu}(t-\tau)\chi(\tau)\right]d\tau = m_e + m_v \qquad (9)$$

where λ_0 and μ_0 are the initial Lame's constants; namely, their values at t=0 and $\mu(t) = G(t) = E(t)/2(1+\upsilon)$. $E(t)$ is the relaxation function and $G(t)$ is the shear modulus related to the relaxation function $E(t)$ and the time-independent Poisson's ratio ν The over dot (·) denotes differentiation with respect to the time. Therefore, the total stress tensor and deviatoric part of the total couple stress tensor could be both decomposed into two parts: current stress tensor σ_e and past history stress tensor $\boldsymbol{\sigma_v}$; current couple stress tensor me and past history couple stress tensor m_v . Considering the Cartesian system (x, y, z) shown in Fig.1, where the xy-plane is coincident with the geometrical mid-plane of the undeformed nanoplate, the displacement field according to the Kirchhoff's plate theory [55] can be expressed as

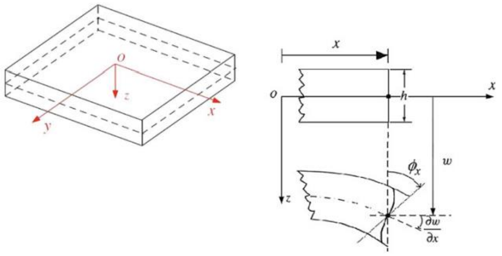


Fig 1. Configuration and coordinate system of the nanoplate [55]

$$u_x = u(x,y,t) - z\frac{\partial w}{\partial x}, v_y = v(x,y,t) - z\frac{\partial w}{\partial y}, w_z = w(x,y,t) \qquad (10)$$

This approximation is applicable for the considered aspect ratios in this paper. However, some limitations related to this plate theory is also applicable here. For example, very large aspect ratios are better to considered as the beam model. In addition, small length to thickness ratios are better to be treated as the Mindlin or higher-ordered plate theories. The time-dependent variables u, v and w are the middle surface displacement components in the x, y and z directions, respectively. considering the von-Karman non-linearity, the strain components of any point in the nanoplate are expressed as

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)
\end{aligned} \qquad (11)$$

Substituting Eq. (10) in Eq. (5), we obtain

$$\omega_1 = \frac{\partial w}{\partial y} \quad \omega_2 = -\frac{\partial w}{\partial x} \quad \omega_3 = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (12)$$

Similarly, from Eqs. (4) and (10) it follows that

$$\begin{aligned}
&\chi_{11} = \frac{\partial^2 w}{\partial x\partial y} \quad \chi_{22} = -\frac{\partial^2 w}{\partial x\partial y} \quad \chi_{33} = 0 \\
&\chi_{12} = \frac{1}{2}\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right) \quad \chi_{13} = \frac{1}{4}\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x\partial y}\right) \\
&\chi_{23} = \frac{1}{4}\left(\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2}\right)
\end{aligned} \qquad (13)$$

substituting Eq. (11) into Eq. (8), the following equations can be obtained

$$\begin{aligned}
\sigma_{exx} &= \frac{E_0}{(1-\upsilon^2)}\left[\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + \upsilon\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)\right] \\
\sigma_{vxx} &= \int_0^t \frac{\dot{E}(t-\tau)}{(1-\upsilon^2)}\left[\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right) + \upsilon\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)\right]d\tau \\
\sigma_{eyy} &= \frac{E_0}{(1-\upsilon^2)}\left(\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + \upsilon\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)\right) \\
\sigma_{vyy} &= \int_0^t \frac{\dot{E}(t-\tau)}{(1-\upsilon^2)}\left[\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right) + \upsilon\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)\right]d\tau \\
\sigma_{exy} &= G_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) \\
\sigma_{vxy} &= \int_0^t \dot{G}(t-\tau)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)d\tau
\end{aligned} \qquad (14)$$

Similarly, after substituting Eq. (13) into Eq. (9), we can obtain

$$\begin{aligned}
&m_{exx} = 2l^2 G_0\left(\frac{\partial^2 w}{\partial x\partial y}\right), \quad m_{vxx} = 2l^2\int_o^t \dot{G}(t-\tau)\left(\frac{\partial^2 w}{\partial x\partial y}\right)d\tau \\
&m_{eyy} = -2l^2 G_0\left(\frac{\partial^2 w}{\partial x\partial y}\right), \quad m_{vyy} = -2l^2\int_0^t \dot{G}(t-\tau)\left(\frac{\partial^2 w}{\partial x\partial y}\right)d\tau \\
&m_{exy} = l^2 G_0\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right), \quad m_{vxy} = l^2\int_o^t \dot{G}(t-\tau)\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right)d\tau \\
&m_{exz} = \frac{1}{2}l^2 G_0\left(\frac{\partial^2 v}{\partial^2 x} - \frac{\partial^2 u}{\partial x\partial y}\right), \quad m_{vxz} = \frac{1}{2}l^2\int_0^t \dot{G}(t-\tau)\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x\partial y}\right)d\tau \\
&m_{eyz} = \frac{1}{2}l^2 G_0\left(\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2}\right), m_{vyz} = \frac{1}{2}l^2\int_0^t G(t-\tau)\left(\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2}\right)d\tau
\end{aligned} \qquad (15)$$

According to the modified couple stress theory of Yang et al. [13], we know that the variation of the elastic potential energy is in the following form

$$\delta U = \int(\sigma_e.\delta\varepsilon + m_e.\delta\chi)dv \qquad (16)$$

Integration on volume for nanoplate can be written as

$$\int_V F\,dv = \int_A \int_{-\frac{h}{2}}^{\frac{h}{2}} F\,dz\,dA \qquad (17)$$

After substituting Eq. (17) into Eq. (16) and integrating by parts and collecting the coefficients of (δu , δv , δw), the following result can be obtained

$$\begin{aligned}
\delta U = \int_A [&\left(-\frac{\partial N_{exx}}{\partial x} - \frac{\partial N_{exy}}{\partial y} - \frac{1}{2}\frac{\partial^2 R_{exz}}{\partial x\partial y} - \frac{1}{2}\frac{\partial^2 R_{eyz}}{\partial y^2}\right)\delta u \\
&+\left(-\frac{\partial N_{eyy}}{\partial y} - \frac{\partial N_{exy}}{\partial x} + \frac{1}{2}\frac{\partial^2 R_{exz}}{\partial x^2} + \frac{1}{2}\frac{\partial^2 R_{eyz}}{\partial x\partial y}\right)\delta v - \\
&\left(\begin{aligned}&\frac{\partial^2 M_{exx}}{\partial x^2} + \frac{\partial^2 M_{eyy}}{\partial y^2} + 2\frac{\partial^2 M_{exy}}{\partial x\partial y} - \frac{\partial^2 R_{exx}}{\partial x\partial y} \\ &-\frac{\partial^2 R_{exy}}{\partial y^2} + \frac{\partial^2 R_{exy}}{\partial x^2} + \frac{\partial^2 R_{eyy}}{\partial x\partial y} + P_e(w)\end{aligned}\right)\delta w\,]\,dA
\end{aligned} \qquad (18)$$

where stress and couple resultants can be defined by integrating corresponding stresses along the thickness as follows

$$N_{eij}=\int_{-h/2}^{h/2}\sigma_{eij}dz \quad R_{eij}=\int_{-h/2}^{h/2}m_{eij}dz \tag{19}$$

and

$$P_e(w)=\frac{\partial}{\partial x}\left(N_{exx}\frac{\partial w}{\partial x}+N_{exy}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_{exy}\frac{\partial w}{\partial x}+N_{eyy}\frac{\partial w}{\partial y}\right) \tag{20}$$

Similarly, the virtual work performed by the viscous dissipative forces can be givens as

$$\delta W_{vis}=\delta U_{vis}=\int(\sigma_v .\delta\varepsilon+m_v .\delta\chi)dv \tag{21}$$

By integration

$$\begin{aligned}\delta W_{visco}=&\int_A[\left(-\frac{\partial N_{vxx}}{\partial x}-\frac{\partial N_{vxy}}{\partial y}-\frac{1}{2}\frac{\partial^2 R_{vxz}}{\partial x\partial y}-\frac{1}{2}\frac{\partial^2 R_{vyz}}{\partial y^2}\right)\delta u\\&+\left(-\frac{\partial N_{vyy}}{\partial y}-\frac{\partial N_{vxy}}{\partial x}+\frac{1}{2}\frac{\partial^2 R_{vxz}}{\partial x^2}+\frac{1}{2}\frac{\partial^2 R_{vyz}}{\partial x\partial y}\right)\delta v-\\&\left(\begin{array}{l}\frac{\partial^2 M_{vxx}}{\partial x^2}+\frac{\partial^2 M_{vyy}}{\partial y^2}+2\frac{\partial^2 M_{vxy}}{\partial x\partial y}-\frac{\partial^2 R_{vxx}}{\partial x\partial y}\\-\frac{\partial^2 R_{vxy}}{\partial y^2}+\frac{\partial^2 R_{vxy}}{\partial x^2}+\frac{\partial^2 R_{vyy}}{\partial x\partial y}+N_\upsilon(w)\end{array}\right)\delta w\,]dA\end{aligned} \tag{22}$$

Where stress and couple resultants can be defined by integrating corresponding stresses along the thickness as follows

$$N_{\upsilon ij}=\int_{-h/2}^{h/2}\sigma_{\upsilon ij}dz \quad R_{\upsilon ij}=\int_{-h/2}^{h/2}m_{\upsilon ij}dz \tag{23}$$

and

$$P_\upsilon(w)=\frac{\partial}{\partial x}\left(N_{\upsilon xx}\frac{\partial w}{\partial x}+N_{\upsilon xy}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(N_{\upsilon xy}\frac{\partial w}{\partial x}+N_{\upsilon yy}\frac{\partial w}{\partial y}\right) \tag{24}$$

The virtual work done by external forces consists of three parts: (1) virtual work done by the body forces in $V=\Omega\times(-h/2,h/2)$, (2) virtual work done by surface tractions acting on the top and bottom surfaces Ω, and (3) virtual work done by surface tractions acting on the lateral surface $S=\Gamma\times(-h/2,h/2)$, where Ω denotes the middle surface of the plate and Γ is the boundary of the middle surface [56]. Let (f_x ,f_y ,f_z) be the body forces, (c_x ,c_y ,c_z) be the body couples, (q_x ,q_y ,q_z) be the tractions acting on Γ, and (t_x ,t_y ,t_z) and (s_x ,s_y ,s_z)be the Cauchy traction and surface couple, respectively, acting on *S*. Then, the virtual work done by external forces is [57].

$$\begin{aligned}\delta W_{ext}=&-\int_\Gamma(t_x\delta u+t_y\delta v+t_z\delta w+s_x\delta\omega_1+s_y\delta\omega_2+s_z\delta\omega_3)d\Gamma\\&-\int_\Omega(f_x\delta u+f_y\delta v+f_z\delta w+q_x\delta u+q_y\delta v+q_z\delta w+c_x\delta\omega_1+c_y\delta\omega_2+c_z\delta\omega_3)dA\end{aligned} \tag{25}$$

The first variation of kinetic energy of structure is given as

$$\delta K=\int\rho\left[\dot u_x\delta\dot u_x+\dot v_y\delta\dot v_y+\dot w_z\delta\dot w_z\right]dV \tag{26}$$

where ρ is mass density of the nanoplate. In the Eq. (26) and throughout this paper, the overhead "·" and"··" denote, respectively, the first and second time derivatives.

Derivation Eq. (10) respect to time and substituting in Eq. (26) gives

$$\begin{aligned}\delta K=\int_A\Big\{&I_0(\dot u\delta\dot u+\dot v\delta\dot v+\dot w\delta\dot w)+I_2(\frac{\partial\dot w}{\partial x}.\frac{\partial\delta\dot w}{\partial x}+\frac{\partial\dot w}{\partial y}.\frac{\partial\delta\dot w}{\partial y})\\&-I_1(\dot u\frac{\partial\delta\dot w}{\partial x}+\delta\dot u\frac{\partial\dot w}{\partial x}+\dot v\frac{\partial\delta\dot w}{\partial y}+\delta\dot v\frac{\partial\dot w}{\partial y})\Big\}dA\end{aligned} \tag{27}$$

Where

$$\int_{-h/2}^{h/2}\left(1,z,z^2\right)\rho dz=\left(I_0,I_1,I_2\right) \tag{28}$$

Substituting the expressions for δU , δW_{vis} , δW_{ext} , and δK from the Eqs. (18), (22), (25) and (27) into Eq. (3) and integrating by parts, and collecting the coefficients of (δu , δv , δw), the dynamic governing equation of the viscoelastic nanoplate can be obtained as

$$\begin{aligned}&\delta u:\ \frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}+\frac{1}{2}\left(\frac{\partial^2 R_{xz}}{\partial x\partial y}+\frac{\partial^2 R_{yz}}{\partial y^2}\right)+f_x\\&+q_x+\frac{1}{2}\frac{\partial c_z}{\partial y}=I_0\ddot u-I_1\frac{\partial\ddot w}{\partial x}\\&\delta v:\ \frac{\partial N_{yy}}{\partial y}+\frac{\partial N_{xy}}{\partial x}-\frac{1}{2}\left(\frac{\partial^2 R_{xz}}{\partial x^2}+\frac{\partial^2 R_{yz}}{\partial x\partial y}\right)+f_y\\&+q_y-\frac{1}{2}\frac{\partial c_z}{\partial x}=I_0\ddot v-I_1\frac{\partial\ddot w}{\partial y}\\&\delta w:\ \frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}\\&+\frac{\partial^2(R_{yy}-R_{xx})}{\partial x\partial y}+\frac{\partial^2 R_{xy}}{\partial x^2}-\frac{\partial^2 R_{xy}}{\partial y^2}+P(w)\\&+f_z+q_z-\frac{\partial c_x}{\partial y}+\frac{\partial c_y}{\partial x}=\\&I_0\ddot w-I_2(\frac{\partial^2\ddot w}{\partial x^2}+\frac{\partial^2\ddot w}{\partial y^2})+I_1(\frac{\partial\ddot u}{\partial x}+\frac{\partial\ddot v}{\partial y})\end{aligned} \tag{29}$$

where, $N_{ij}=N_{eij}+N_{\upsilon ij}$, $P(w)=P_e+P_\upsilon$ and $M_{ij}=M_{eij}+M_{\upsilon ij}$

The Equation (29) is the system of integral-differential equations for a viscoelastic nanoplate based on the modified couple stress theory, which contains an additional internal material length scale parameter. Moreover, this additional material constant not only affects the current situation, but also affects the past history situation. Furthermore, when the past history term is neglected the Eq. (29) could be reduced into the size-depended model for elastic nanoplate, and when the size effect is neglected, i.e. $l=0$, it could be reduced into the classical viscoelastic plate model.

The equations of motion can be expressed in terms of displacements (*w*) as:

$$\begin{aligned}&\delta w:-\left(D+A_n\right)\left(E_0\nabla^4 w+\int_0^t\dot E(t-\tau)\nabla^4 w(\tau)d\tau\right)\\&+B_n\left(E_0 P(w)+\int_0^t\dot E(t-\tau)P(w(\tau))d\tau\right)\\&+f_z+q_z-\frac{\partial c_x}{\partial y}+\frac{\partial c_y}{\partial x}=I_0\ddot w)-I_2(\frac{\partial^2\ddot w}{\partial x^2}+\frac{\partial^2\ddot w}{\partial y^2})+I_1(\frac{\partial\ddot u}{\partial x}+\frac{\partial\ddot v}{\partial y})\\&D=\frac{h^3}{12(1-\upsilon^2)}\quad A_n=\frac{hl^2}{2(1+\upsilon)}\quad B_n=\frac{h}{1-\upsilon^2}\end{aligned} \tag{30}$$

where

$$P(w)=\frac{h}{1-\upsilon^2}\left[\begin{array}{l}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial x}+\upsilon\frac{\partial v}{\partial y}+\frac{\upsilon}{2}\frac{\partial w}{\partial y}\frac{\partial w}{\partial y}\right)\\+\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial v}{\partial y}+\frac{1}{2}\frac{\partial w}{\partial y}\frac{\partial w}{\partial y}+\upsilon\frac{\partial u}{\partial x}+\frac{\upsilon}{2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial x}\right)\\+(1-\upsilon)\frac{\partial^2 w}{\partial x\partial y}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\end{array}\right] \tag{31}$$

For homogenous rectangle plate I_1 become zero. For convenience, the following nondimensional variables and parameters are introduced

$$\bar{x}=\frac{x}{a},\bar{y}=\frac{y}{b},\bar{w}=\frac{w}{h},\bar{t}=\frac{t}{T},\zeta=\frac{a}{b},l_0=\frac{l}{h},T=\sqrt{\frac{I_0 a^4}{E_0 h^3}} \tag{32}$$

In this paper, the standard anelastic linear solid model which is suggested by most experiment results [58, 59] is introduced for the material viscoelasticity, and the relaxation function is given as

$$E(t)=A+Be^{-\lambda t} \tag{33}$$

where λ is the element relaxation coefficient of the nanoplate material.The Eq.(33) implies the initial Young's modulus $E_0 = E(0) = A + B$ .Then the nondimensional form of relaxation function is introduced as

$$n(t)=\frac{E(t)}{E_0}=\bar{A}+\bar{B}e^{-\bar{\lambda}t} \tag{34}$$

where $\bar{A}=\frac{A}{A+B}$, $\bar{B}=\frac{A}{A+B}$ and $\bar{\lambda}=\lambda T$

The present study considers the nanoplate with all edges simply supported. The solutions are assumed as [60]

$$u(\bar{x},\bar{y})=\frac{1}{16}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\alpha\frac{h}{a}\Phi^2_{mn}(\bar{t})\sin 2\alpha\bar{x}(\cos 2\beta\bar{y}-1+\upsilon\zeta^2\frac{\beta^2}{\alpha^2})$$

$$v(\bar{x},\bar{y})=\frac{1}{16}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta\frac{h}{b}\Phi^2_{mn}(\bar{t})\sin 2\beta\bar{y}(\cos 2\alpha\bar{x}-1+\upsilon\frac{1}{\zeta^2}\frac{\alpha^2}{\beta^2})$$

$$w(\bar{x},\bar{y})=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Phi_{mn}(\bar{t})\sin\alpha\bar{x}\sin\beta\bar{y} \tag{35}$$

where, α=mπ and β=nπ. Clearly, this choice of displacements (u,v,w) satisfies the simply supported boundary conditions of the plate. Furthermore, the expressions of u and v also satisfy the first two equations of the Eq. (30) automatically. To solve the last equation of Eq. (30) for $\Phi_{mn}(t)$,we use the Bubnov–Galerkin approach, and compute the integral. (With dropping the asterisk notation for brevity)

$$\int_0^1\int_0^1 \Lambda\sin\alpha x\sin\beta y\,dx\,dy=0 \tag{36}$$

where Λ is the left-hand side of the Eq. (30 c). Substituting the expressions of (u,v,w) into Eq. (30c), the following expression of Λ is obtained

$$\Lambda=-\left\{\frac{1}{16(1-\upsilon^2)}\left(4\upsilon\zeta^2\alpha^2\beta^2+2(\alpha^4+\zeta^4\beta^4)+2(\upsilon^2-1)(\alpha^4\cos 2\beta y+\zeta^4\beta^4\cos 2\alpha x)\right)\right.$$
$$\times\left(\Phi^3(t)-B\lambda\int_0^t e^{-\lambda(t-\tau)}\Phi^3(\tau)d\tau\right)-$$
$$\left.\left(\frac{1}{12(1-\upsilon^2)}+\frac{1}{2(1+\upsilon)}l_0^2\right)(\alpha^2+\zeta^2\beta^2)^2\left(\Phi(t)-B\lambda\int_0^t e^{-\lambda(t-\tau)}\Phi(\tau)d\tau\right)-\ddot{\Phi}_{mn}\right\}\sin\alpha x\sin\beta y \tag{37}$$

Substituting Eq. (37) into Eq. (36), the solution of $\Phi_{mn}(t)$ can be obtained from

$$\frac{1}{16(1-\upsilon^2)}\left(4\upsilon\zeta^2\alpha^2\beta^2+(3-\upsilon^2)(\alpha^4+\zeta^4\beta^4)\right)\left(\Phi^3(t)-B\lambda\int_0^t e^{-\lambda(t-\tau)}\Phi^3(\tau)d\tau\right)+$$
$$\left(\frac{1}{12(1-\upsilon^2)}+\frac{1}{2(1+\upsilon)}l_0^2\right)(\alpha^2+\zeta^2\beta^2)^2\left(\Phi(t)-B\lambda\int_0^t e^{-\lambda(t-\tau)}\Phi(\tau)d\tau\right)+\ddot{\Phi}_{mn}(t)=0 \tag{38}$$

It is worth mentioning that, Eq. (38) is the resulting nonlinear integro-differential equation of the viscoelastic nanoplate based on the modified couple stress theory. The fourth-order Runge-Kutta method can be used to solve this equation after some algebraic processes [61].

3. Numerical results and discussion

For verification the accuracy of the present results, a comparison has been carried out with available data in the literature [62, 63] for an elastic rectangular plate ($\lambda=0$). The first linear dimensionless natural frequency of the out-of-plane motion $\omega(a/h)^2\sqrt{\rho/E}$ are listed in table-1 for several thickness ratios (h/l). It can be found that the present results are in good agreement with the results given by Ref. [62, 63]. For illustrative examples, the nanoplate is assumed to be made of epoxy with the following material properties:

$\rho=1220\ Kg/m^3$ $E=1.44\ GPa$ $\upsilon=0.38$

Table1. Comparison of first linear dimensionless natural frequency for elastic nano plate (a/b=1).

h/l	Non Classical plate		classical plate (l=0)	
	Ref [63]	Present	Ref [62]	Present
1	13.381	13.383	6.16	6.16
2	8.552	8.558	6.16	6.16
3	7.320	7.323	6.16	6.16
4	6.833	6.839	6.16	6.16
5	6.584	6.602	6.16	6.16
6	6.461	6.470	6.16	6.16

At above table two significant effects due to the length scale parameter can be observed. First, it can be seen that the natural frequencies predicted by the new model are always higher than those predicted by the classical Kirchhoff plate theory. This is due to the increased stiffness predicted by the present model. Also, it can be found that the differences in natural frequencies for classical and present model are larger when the thickness ratio is small (e.g. $h/l<3$), while they are decreasing or even diminishing with increase in thickness ratio. This reveals that the size effect is significant only when the plate thickness is as small as the material length scale parameter.

To study the effect of variation of the length scale parameter on the vibration behavior of the nanoplate, the first three dimensionless natural frequencies are tabulated in Table 2 for linear and nonlinear model (von-karman strain), various Length scale thickness ratios(l/h) and some aspect ratios a/b.

It is apparent that the dimensionless natural frequency increases for increase in values of the length scale parameter for all modes of vibration. This is because of the fact that the plate becomes stiffer by increasing the length scale parameter. It can be seen that the first frequency of a nonoplate with $a/b=2$ equals to the second frequency of a nanoplate with a$/b=1$.This is rated to the constants of the equating (38) and the same effect can be seen at Ref.[63].

Table 2. First three dimensionless natural frequencies with various length scale parameters

	(l/h)	a/b=1 ω_1	a/b=1 ω_2	a/b=1 ω_3	a/b=2 ω_1	a/b=2 ω_2	a/b=2 ω_3
Lm[a]	0.1	6.27	15.68	31.36	15.68	53.32	116.06
	0.5	8.55	21.39	42.79	21.39	72.74	158.32
	1	13.38	33.45	66.91	33.45	113.76	247.6
	5	59.72	149.31	298.63	149.31	507.67	1105
	10	118.97	297.44	594.96	297.44	1011	2202
Nm[b]	0.1	7.59	16.53	31.88	16.53	53.65	116.23
	0.5	9.22	21.75	43	21.75	72.87	158.39
	1	13.57	33.55	66.97	33.55	113.79	247.62
	5	59.72	149.31	298.63	149.31	507.67	1105
	10	118.97	297.44	594.96	297.44	1011	2202

[a]: Linear model [b]: Nonlinear model

Table 3. First dimensionless damped frequencies with different dimensionless relaxation coefficient and various length scale parameters.

λ	l/h	a/b=1 Lm	a/b=1 Nm	a/b=2 Lm	a/b=2 Nm
1	0.1	6.23	10.94	15.66	20.38
	0.5	8.52	11.79	21.38	23.95
	1	13.36	14.76	33.45	34.26
	5	59.72	59.74	149.32	149.33
	10	118.97	118.97	297.44	297.44
5	0.1	5.78	10.07	15.35	19.72
	0.5	8.10	11.07	21.13	23.51
	1	13.02	14.30	32.27	34.03
	5	59.61	59.63	149.27	149.28
	10	118.92	118.92	297.44	297.44
15	0.1	5.35	9.21	14.23	18.24
	0.5	7.40	10.12	19.96	22.19
	1	11.97	13.17	32.25	32.96
	5	58.90	58.91	148.93	148.94
	10	118.50	118.50	297.26	297.26

The first dimensionless damped frequencies are tabulated in Table 3 for various Length scale thickness ratios (l/h), dimensionless relaxation coefficient (λ) and some aspect ratios *a/b*. it can be seen that dimensionless damped frequencies increases for increase in values of the length scale parameter like natural frequencies. As expected the frequencies decrease with increasing dimensionless relaxation coefficient λ . Also nonlinear model frequencies are larger than linear model for all aspect ratio and length scale parameters. In order to highlight the difference between the linear frequencies and the corresponding nonlinear ones, the variation of these frequency are depicted at different relaxation times versus initial excitation in Fig. 2. It is observed that the nonlinear vibration frequency is higher than its linear counterpart under the action of the same initial excitation condition. This phenomenon are attributed to the intrinsic stiffening effect of the nanoplate brought by geometric nonlinearity Moreover, one can find that the nonlinear frequency gets larger with the increase of the excitation velocity. This is also due to the intrinsic stiffening effect.

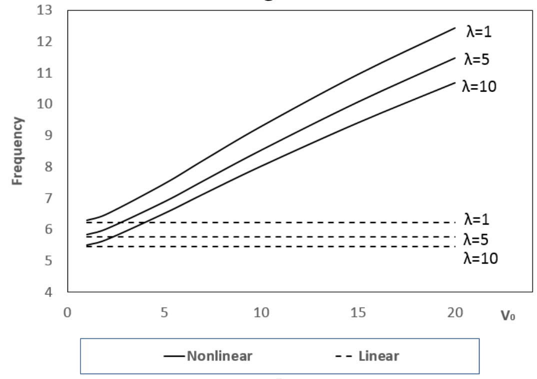
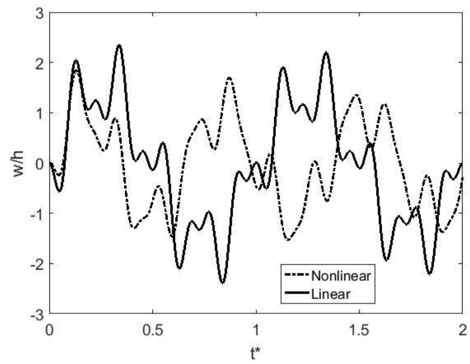
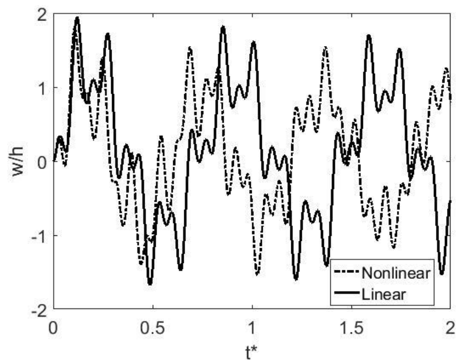


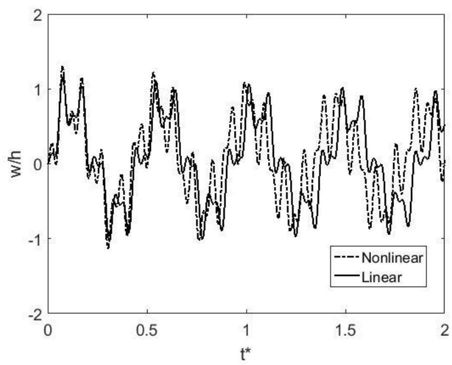
Fig 2. Variation of Linear and nonlinear damped frequency at different relaxation time versus excitation velocity (a/b=1 and l/h=0.1)



a: (l/h=0.1, λ=0.5,V$_0$=10)

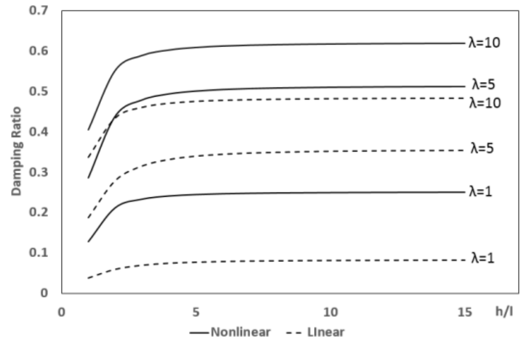


b: (l/h=0.5, λ=0.5,V$_0$=10)

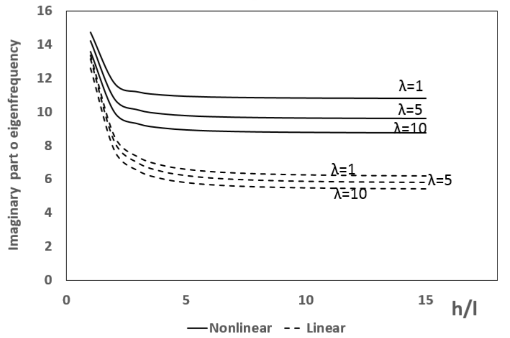


c: (l/h=1, λ=0.5,V$_0$=10)

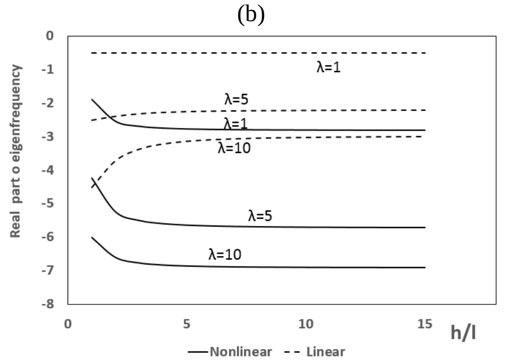
Fig 3. Vibration response curves of the center deflection vs. time for the viscoelastic nanoplate.



(a)



(b)

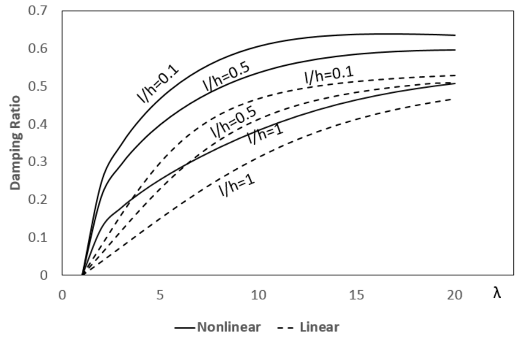


(c)

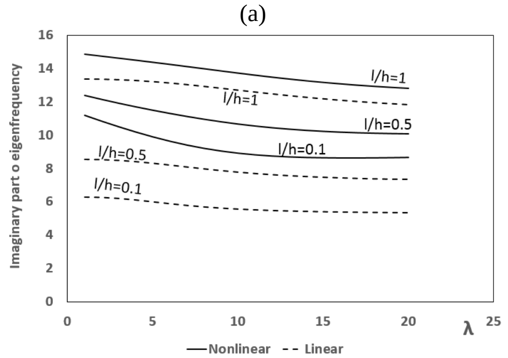
Fig 4. Variation of damping ratio and eigenfrequency with respect to thickness ratio for different values of relaxation time; (a) damping ratio, (b) imaginary part and (c) real part

The Effect of linearity, nonlinearity (von-karman strain) and length scale thickness ratio on vibration response of the centre deflection of the viscoelastic nanoplate are shown in Fig. 3. The aspect ratio is assumed to 1. It is apparent that vibration amplitude decrease with increasing length scale thickness ratio (compare fig.3 a-c). Also difference between linear and nonlinear model is more obvious at smaller thickness ratio.

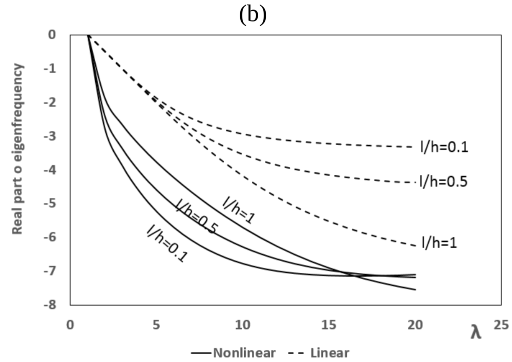
In damped vibration the damping ratio can be obtained from this relation $\zeta = \sqrt{1-(\omega_d/\omega_n)^2}$. Where ω_d , ω_n and ζ is damped frequency, natural frequency and damping ratio respectively. The damping ratio ξ and eigenfrequency of a square nanoplate versus the thickness ratio for some dimensionless relaxation coefficients are depicted in Fig 4. This figure show how the damping ratio of a viscoelastic nanoplate is a function of the length scale parameter. Two significant effects due to the length scale parameter can be observed. First, it can be seen that the imaginary part of frequency decreases by increasing thickness ratio because the length scale parameter tends to have a dampening effect on the vibration frequency [64], also it can be found that the rate of increase of the damping ratio is larger at small thickness ratio (e.g. *h/l<5*) and then decreases to zero at *h/l=15*. Therefore, increasing the thickness ratio beyond this value has no effect on damping ratio. It is observed that the nonlinear vibration damping ratio and imaginary part of frequency is higher than its linear counterparts at the same thickness ratio.



(a)



(b)



(c)

Fig 5. Variation of damping ratio and eigenfrequency with respect to relaxation time for different thickness ratio ;(a) damping ratio, (b) imaginary part and (c) real part

In order to demonstrate the effect of dimensionless relaxation coefficient on free vibration behavior of a viscoelastic nanoplate, the eigenfrequency and damping ratio (ξ) of square nanoplate are depicted in Fig.5. It can be seen the imaginary part of eigenfreuqncy decreases by increasing relaxation time due to the dissipation of system energy. Also, the relation between relaxation coefficient and damping ratio is not linear and its increasing rate decrease and become zero while the relaxation coefficient increase. It is observed that the damping ratio is higher at larger thickness ratio because the length scale parameter tends to have a dampening effect on the vibration frequency. The figure shows that the nonlinear vibration damping ratio and imaginary part of frequency is higher than its linear counterparts at the same relaxation time.

4. Conclusion

In this paper, a new viscoelastic size-depended model developed based on a modified couple stress theory and the for nonlinear viscoelastic material in order to vibration analysis of a viscoelastic nanoplate. The material of the nanoplate is assumed to obey the Leaderman nonlinear constitutive relation and the von Kármán plate theory is employed to model the system. The viscous parts of the classical and nonclassical stress tensors are obtained based on the Leaderman integral and the corresponding work terms are calculated. The viscous work equations are balanced by the terms of size-dependent potential energy, kinetic energy. Then the equations of motion are derived from Hamilton's principle. The governing nonlinear integro-differential equations with coupled terms are solved by using the fourth-order Runge-Kutta method and Galerkin approach. Vibration analysis were performed for a simply supported rectangular viscoelastic nanoplate. In order to explore the vibrational characteristics, the influences of the thickness ratio, relaxation coefficient, and aspect ratio on the frequency and damping ratio were also examined.

- The results were found to be in good agreement with the existing data in the literature.
- It can be found that the differences in natural frequencies for classical and present model are larger when the thickness ratio is small (e.g. $h/l<3$), while they are decreasing or even diminishing with increase in thickness ratio. This reveals that the size effect is significant only when the plate thickness is as small as the material length scale parameter.
- The results revealed that the frequency, vibration amplitude and damping ratio of viscoelastic nanoplate were significantly influenced by the relaxation coefficient, and length scale parameter.
- It is observed that the nonlinear vibration frequency is higher than its linear counterpart under the action of the same initial excitation condition. This phenomena are attributed to the intrinsic stiffening effect of the nanoplate brought by geometric nonlinearity Moreover, one can find that the nonlinear frequency gets larger with the increase of the excitation velocity. This is also due to the intrinsic stiffening effect.
- The imaginary part of frequency decreases by increasing thickness ratio because the length scale parameter tends to have a dampening effect on the vibration frequency
- The Imaginary part of eigenfreuqncy decreases by increasing relaxation time due to the dissipation of system energy.
- It was found that by increasing (l/h) the vibration amplitude and damping ratio decrease.
- It can be seen the relation between relaxation coefficient and damping ratio is not linear and its increasing rate decreases and becomes zero.
- It is observed that the nonlinear vibration damping ratio and imaginary part of frequency is higher than its linear counterparts at the same thickness ratio and relaxation time.

The presented new results for the viscoelastic nanoplates can be used as a benchmark solution for future researches.

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