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Variation of Parameters Method for Thermal Analysis of Straight Convective- Radiative Fins with Temperature Dependent Thermal Conductivity

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ABSTRACT

In this study, thermal performance across straight convective-radiative fin with temperature dependent thermal conductivity is considered. The variation of parameters method (VPM) is adopted to analyze the nonlinear higher order differential equations arising due to thermal conductivity and heat transfer coefficient on temperature distribution. Where pertinent parameters such as thermo geometric and radiation parameters effect on temperature profile are investigated. Result obtained depicts quantitative increase of thermo geometric parameter causes significant increase in temperature distribution due to increasing ratio of convective/conduction heat transfer which influence is significant toward fin base. While increasing radiation parameter leads to decrease in temperature distribution due to increasing heat transfer from fins surface to ambient environment. Comparative analysis of result obtained in study against literature proves to be in satisfactory agreement. Therefore study provides useful insight to fins operational performance in devices such as radiators, boilers, refrigerator, oil pipelines amongst other heat transfer applications.

1. Introduction

Thermal performance analysis of fins as been a motivating subject of interest amongst reseachers overtime [1-16]. Owing to its vast range of applications not limited to refrigeration and airconditioning, heat exchangers, steam and power plant amongst others.In the bid to determine the performance of fins Aziz and Enamul-Huq[1] used perturbation method to analyse convecting fins. Shortly after Aziz [2] extended his research on convecting fins but considered internal heat generation. Mosayebidorcheh et al.[3] provided approximate analytical solutions to temperature dependent thermal conductivity with power law while Ganji and Dogonchi [4] investigated convective heat transfer in longitudinal fins utilizing approximate analytical methods. Least square method of solution was adapted by Aziz and Bouaziz [5] for longitudinal fins with temperature dependent internal heat generation. Hosseini et al. [6] analysed fins with temperature dependent thermal conductivity and heat generation.Analysis for thermal performance on convective fins was investigated by Ghasemi et al. [7] considering thermal conductivity and heat generation. Hatami et al. [8-12] studied fins for practical applications such as refrigeration, internal combustion engines and exhaust waste recovery system. In the bid to improve operational performance.Atay and Coskun [13] compared finite element method of solution against variational iteration method for power law fin type problems. Homotopy perturbation method was applied by Chowdhury et al. [14] to heat transfer equations.Moitheki et al. [15] provided exact solution of fin problems with power law temperature dependent thermal conductivity.As khan et al. [16] analyzed nonlinear fin problems with temperature dependent thermal conductivity and heat transfer coefficient.

Therefore the use of numerical and analytical approximate solutions was applied by researchers [17-26]. Methods of solutions utilised include the pertubation method (PM),homotopy analysis method (HAM), homotopy pertubation method (HPM), differential transform method (DTM),variational iteration method (VIM) , garlerkin method of weighted residuals and adomian decomposition method (ADM). Methods such as PM are limited owing to the problems of linear restrictive assumptions. The need to find initial condition or auxilliary parameter to satisfy the boundary condition makes methods such as HPM,VIM,DTM,HAM require computational tools in handling solution of large parameters resulting to large computational cost and time . The garlerkin method of weighted residual scheme, no doubt a powerful approximate analytical method requires the weighted residuals to satisfy weighting functions which may be arbitrary . The method of solution by decomposing nonlinear coupled equations into linear and nonlinear terms as the case of ADM makes it necessary to determine lagragian polynomial which makes this method cumbersome and labourious for yet simple problems. In the search for convenient and relatively simple method of solution, the variation of parameters method (VPM) is considered. Since it has the capacity to solve weakly and strongly dependent nonlinear equations. It as a rapid convergent rate without taking the highest order term into consideration as compared with VIM.

Solid structured systems made of nanometer sized molecular components plays crucial role in determining fin type material. This requires the manipulation of various material matter and modelling to satisfy performance . Therefore the nanotechnology, a field relevant for engineering advancement in the nanorealm applies nanometer control for material fabrication integrated into functional working device. The application of this science as provided materials functionally efficient for various heat and mass transfer applications . Significant progress as been made in the application of nanoelectromechanical systems to determine and analyze mechanical properties and behaviour of solid structures. As controlled experiments are highly expensive. Thus, the renaissance amongst researchers to develop continuum models to study static and dynamic behaviour of nanosized solid structured systems [27-40].

Therefore VPM been free from discretization ,linearization or determination of lagragian polymian is the favoured scheme adopted to study thermal performance in the nanostructured material. Hence thermal analysis of convective radiative fins with themperature dependent thermal conductivity is investigated.

2. Model Development and Problem Formulation

A straight fin undergoing convective and radiative heat transfer having length L, temperature dependent thermal conductivity k(T) and thickness δ , is exposed to the convective environment with both faces at a temperature T_a and convective heat transfer coefficient h as depicted in the Fig. 1. Heat transfer in the fin is assumed constant with time and surrounding medium of the fin with fin base temperature are at uniform temperature. Also fin base joining prime surface as no contact resistance. Fin thickness compared with width and length is small . Therefore heat transfer from fin edges and temperature gradient across fin may be neglected .The co-ordinate length as its origin from the fin's tip with a positive orientation from the tip to the base of the fin. With respect to the above assumptions ,the problem governing differential equation is presented as:

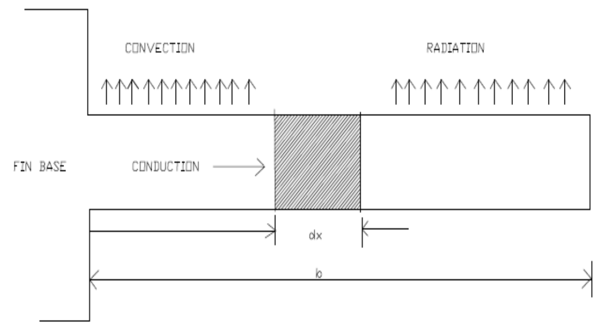

Figure 1. Physical model of problem.

$$\frac{d}{dx}\left(k_a\left[1+\lambda\left(T-T_a\right)\right]A_{cr}\frac{dT}{dx}+\frac{4\sigma A_{cr}}{3\beta_R}\frac{dT^4}{dx}\right)= hP(T-T_\infty)+\sigma\varepsilon P(T^4-T_\infty^4)dx \tag{1}$$

$$\frac{d}{dx}\left(\left[1+\lambda\left(T-T_\infty\right)\right]\frac{dT}{dx}\right)+\frac{4\sigma}{3\beta_R k_a}\frac{d}{dx}\left(\frac{dT^4}{dx}\right) -\frac{h}{k_a t}(T-T_\infty)+\frac{\sigma\varepsilon}{k_a t}(T^4-T_\infty^4)=0 \tag{2}$$

$$x=0,\frac{dT}{dx}=0 \tag{3}$$

$$x=b,T=T_b \tag{4}$$

But

$$\frac{J_c x J_c}{\sigma}=\sigma B_o^2 u^2 \tag{5}$$

Here small temperature difference exists during heat flow within material. The difference is necessitated using thermal fin properties and temperature invariant physical models. However in such situation, T^4 may be expressed as linear function of temperature. This is expressed as:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{6}$$

Substituting the Eq. (6) into Eq. (2) can be expressed as

$$\frac{d}{dx}\left(\left[1+\lambda\left(T-T_{\infty}\right)\right]\frac{dT}{dx}\right)+\frac{16\sigma}{3\beta_R k_a}\frac{d^2T}{dx^2}$$
$$-\frac{h}{k_a t}(T-T_{\infty})+\frac{4\sigma\varepsilon P}{k_a t}(T-T_{\infty})=0 \tag{7}$$

Where non-dimensional parameters are introduced as:

$$X=\frac{x}{b},\theta=\frac{T-T_{\infty}}{T_b-T_{\infty}},\beta=\lambda(T_b-T_{\infty}),M^2$$
$$=\frac{pbh}{A_b k_a},R_d=\frac{4\sigma_{st}T_{\infty}^3}{3\beta_R k_a},N_r=\frac{4\sigma_{st}\varepsilon bT_{\infty}^3}{k_a} \tag{8}$$

With the aid of the dimensionless parameters introduced in Eq. (8), the governing equation can be expressed as:

$$\left(1+4R_d\right)\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dx}\right)^2$$
$$-M^2\theta-N_r\theta=0 \tag{9}$$

This is further expressed as

$$\frac{d^2\theta}{dX^2}+\beta^*\theta\frac{d^2\theta}{dX^2}+\beta^*\left(\frac{d\theta}{dx}\right)^2-\left(M^*\right)^2\theta-N^*{}_r\theta=0 \tag{10}$$

Where

$$\beta^*=\frac{\beta}{(1+4R_d)},\left(M^*\right)^2=\frac{M^2}{(1+4R_d)},N_r^*=\frac{N_r}{(1+4R_d)} \tag{11}$$

With appropriate boundary conditions stated as

$$X=0,\frac{d\theta}{dX}=0$$
$$X=1,\theta=1 \tag{12}$$

2.1. Principles of Variation of Parameters Method (VPM)

The procedural concept or technique of the variation of parameters method (VPM) for analysis of differential equation is expressed as follows. Nonlinear form of differential equation is in the operator form

$$Lf(\eta)+Rf(\eta)+Nf(\eta)=g \tag{13}$$

Given

L is easily convertible and the highest order derivative

R is the linear operator remainder and is less compared with L

G is the source term or system input

u is the system output

Nu is the nonlinear equation terms

Decomposing Eq. (13) above into L+R .Therefore the VPM can be defined as follows

$$f_{n+1}(\eta)=f_0(\eta)+\int_0^{\eta}\lambda(\eta\xi)(-\mathrm{R}f_n(\xi)-Nf_n(\xi)-g(\xi)d\xi \tag{14}$$

Where initial approximation $f_0(\eta)$ is given by

$$f_0(\eta)=\sum_{i=0}^{m}\frac{k_i f^i(0)}{i!} \tag{15}$$

Where

m is the order of the given differential equation

k_i is an unknown constant which could be obtained using initial/boundary conditions

$\lambda(\eta,\xi)$ is a multiplier which reduces the equation order of integration, which is determined adopting the Wronskian technique stated as Sobamowo et al. [25]

$$\lambda(\eta,\xi)=\sum_{i=0}^{m}\frac{(-1)^{i-1}\xi^{i-1}\eta^{m-1}}{(i-1)!(m-i)!}=\frac{(\eta-\xi)^{m-1}}{(m-1)!} \tag{16}$$

2.2 Application of the Variation of Parameters Method

Applying the standard procedure of the VPM the Eq. (10) is presented as

$$\theta_{n+1}(x)=k_1+k_2\xi-\int_0^x\left(x-\xi\right)\left[\begin{array}{l}\beta^*\theta\dfrac{d^2\theta}{d\xi^2}+\beta\\ {}^*\left(\dfrac{d\theta}{d\xi}\right)^2-\left(M^*\right)^2\theta-N^*{}_r\theta\end{array}\right]d\xi \tag{17}$$

Here k_1 and k_2 are constant. They are derived by taking the highest order in the linear term Eq. (10) which is integrated twice, to generate the scheme final form. Applying the boundary condition Eq. (12). The above equation can be written as

$$\theta_{n+1}(x)=k_2\xi-\int_0^x\left(x-\xi\right)\left[\begin{array}{l}\beta^*\theta\dfrac{d^2\theta}{d\xi^2}+\\ \beta^*\left(\dfrac{d\theta}{d\xi}\right)^2-\left(M^*\right)^2\theta-N^*{}_r\theta\end{array}\right]d\xi \tag{18}$$

Following the iterative scheme, it can be easily shown that

$$\theta_0=1 \tag{19}$$

(13)

$$\theta_1=-\frac{M^{*2}}{6}\left(1-x^3\right)+\frac{N_r^*}{6}\left(1-x^3\right)+\frac{M^{*2}}{2}\left(1-x^3\right)-\frac{N_r^2}{2}\left(1-x^3\right)-1 \tag{20}$$

$$\begin{aligned}
\theta_2 = {} & \frac{\beta M^{*2}}{12}\left(1-x^4\right)-\frac{\beta N_r^{*}}{12}\left(1-x^4\right) \\
& -\frac{\beta M^{*2}}{6}\left(1-x^4\right)+\frac{\beta N_r^{*}}{6}\left(1-x^4\right) \\
& -\frac{M^{*4}}{36}\left(x^3-x^6\right)+\frac{M^{*4}N_r^{*}}{36}\left(x^3-x^6\right) \\
& +\frac{M^{*4}}{12}\left(x^3-x^6\right)-\frac{M^{*2}N_r^{*}}{36}\left(x^3-x^6\right) \\
& -\frac{M^{*2}}{6}\left(1-x^3\right)-\frac{N_r^{*}M^{*4}}{36}\left(x^3-x^6\right) \\
& +\frac{N_r^{*}}{36}\left(x^3-x^6\right)+\frac{N_r^{*}M^{*2}}{12}\left(x^3-x^6\right) \\
& -\frac{N_r^{*}}{12}\left(x^3-x^6\right)-\frac{N_r^{*}}{6}\left(1-x^3\right) \\
& -\frac{\beta M^{*2}}{6}\left(1-x^4\right)+\frac{\beta N_r^{*}}{6}\left(1-x^4\right)+ \\
& \frac{\beta M^{*2}}{2}\left(1-x^4\right)-\frac{\beta N_r^{*}}{2}\left(1-x^4\right)+ \\
& +\frac{M^{*4}}{12}\left(x^3-x^6\right)-\frac{N_r^{*}M^{*2}}{12}\left(x^3-x^6\right)- \\
& \frac{M^{*4}}{4}\left(x^3-x^6\right)+\frac{N_r^{*}M^{*2}}{4}\left(x^3-x^6\right) \\
& +\frac{M^{*2}}{2}\left(1-x^3\right)-\frac{N_r^{*}M^{*2}}{12}\left(x^3-x^6\right) \\
& \frac{N_r^{*}}{12}\left(x^3-x^6\right)+\frac{N_r^{*}M^{*2}}{4}\left(x^3-x^6\right) \\
& -\frac{N_r^{*2}}{4}\left(x^3-x^6\right)-\frac{N_r^{*2}}{2}\left(1-x^3\right)+1
\end{aligned} \tag{21}$$

The Newton law of cooling is applied in determining the fins heat transfer. Therefore ratio of actual heat transfer from fin surface to heat transfer from the surface of the entire fin is at the same temperature as the base. This is regarded as efficiency of the fin, derived as:

$$\eta=\frac{Q}{Q_{ideal}}=\frac{\int_0^b \mathrm{P}(\mathrm{T}-\mathrm{T}_\infty)dx}{Pb(T_b-T_\infty)}=\int_{\xi=0}^{1}\theta(x)dx \tag{22}$$

Therefore fins efficiency can be obtained upon simplifying the Eq. (22) which can be easily shown as

$$\begin{aligned}
\eta = {} & \frac{M^{*2}}{2}-\frac{N_r^{*}}{2}-\frac{3M^{*2}}{2}+\frac{3N_r^{*}}{2}-\frac{\beta M^{*2}}{3}+\frac{\beta N_r^{*}}{3} \\
& +\frac{2\beta M^{*2}}{3}-\frac{2\beta N_r^{*}}{3}+\frac{M^{*4}}{12}-\frac{N_r^{*}M^{*2}}{12}-\frac{2M^{*4}}{4} \\
& +\frac{N_r^{*}M^{*2}}{4}+\frac{M^{*2}}{2}+\frac{N_r^{*}M^{*2}}{12}-\frac{N_r^{*}}{12}-\frac{N_r^{*}M^{*2}}{4} \\
& +\frac{N_r^{*}}{4}+\frac{N_r^{*}}{2}+\frac{2\beta M^{*2}}{3}-2\beta M^{*2}+2\beta N_r^{*}+ \\
& \frac{N_r^{*}M^{*2}}{4}+\frac{3M^{*4}}{4}-\frac{3N_r^{*}M^{*2}}{4}-\frac{3M^{*2}}{2}+ \\
& \frac{N_r^{*}M^{*2}}{4}-\frac{N_r^{*2}}{4}-\frac{3N_r^{*}M^{*2}}{4}+\frac{3N_r^{*2}}{4}+\frac{3N_r^{*}}{2}
\end{aligned} \tag{23}$$

3. Results and Discussion

The validation of result of present study against numerical solutions (NM) and the Chebychev spectral collocation method (CSCM) is illustrated in Table 1.This proves the accuracy of the variation of parameter method (VPM) in providing solutions to strongly dependent nonlinear solution through yet a simple and convenient method of solution. The effect of thermal conductivity or nonlinear parameter (β) on heat transfer is illustrated in Figs. 2 and 3. As depicted from the plots increasing numerical values of β shows increasing temperature distribution across the fin length which is due to heat transfer increase across fins surface to ambient environment. Owing to rapid heat conduction from fins prime surface to base of fin.

Table 1. Comparison of various values of x for dimensionless temperature.

X	NM[26]	CSCM[26]	VPM(Present Study)
0.0	0.648054	0.648054 (19)	0.648054
0.1	0.651297	0.651297 (20)	0.651297
0.2	0.661059	0.661059	0.661059
0.3	0.677436	0.677436	0.677436
0.4	0.700594	0.700594	0.700594
0.5	0.730763	0.730763	0.730763
0.6	0.768246	0.768246	0.768246
0.7	0.813418	0.813418	0.813418
0.8	0.866731	0.866731	0.866731
0.9	0.928718	0.928718	0.928718

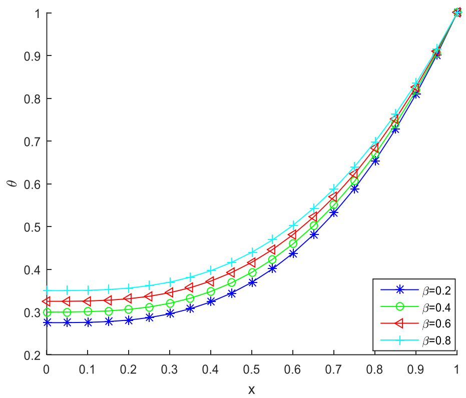

Figure 2. Effect of nonlinear parameter on temperature distribution. Where N_r=1.75, M=1.

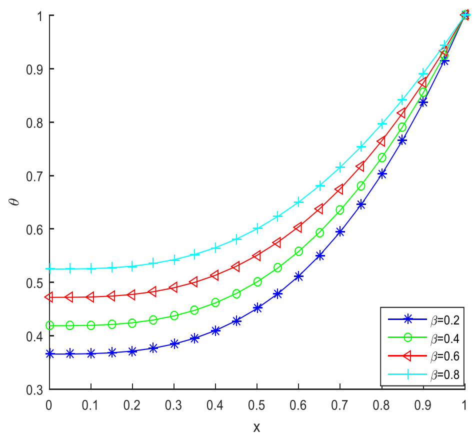

Figure 3. Effect of nonlinear parameter on temperature distribution. Where N_r=2.0, M=1.75.

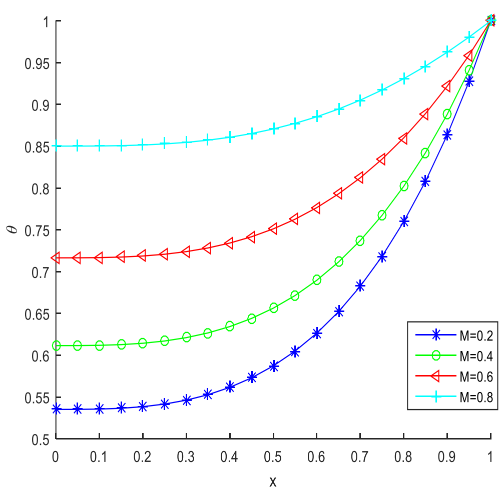

Figure 4. Effect of thermo geometric parameter on temperature distribution. Where N_r=0.5, β=0.1.

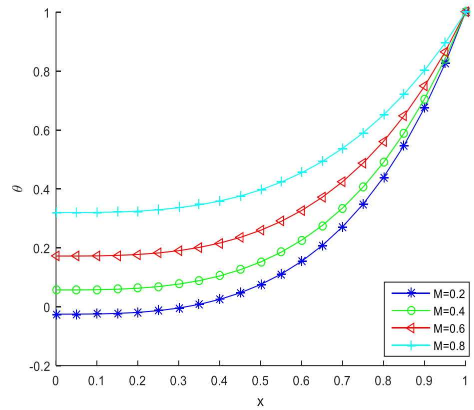

Figure 5. Effect of thermo geometric parameter on temperature distribution. Where N_r,=1.0, β=0.3.

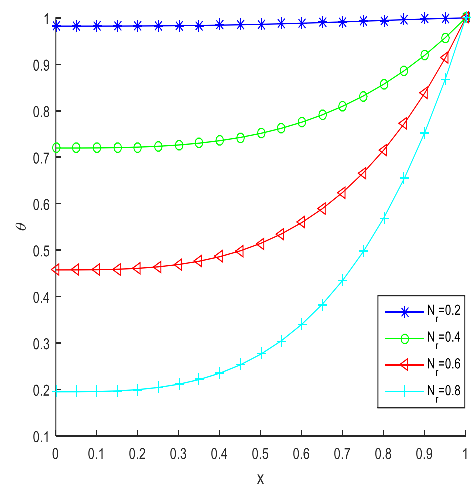

Figure 6. Effect of radiative parameter on temperature distribution. Where M=0.5, β=1.25.

As observed in Fig. 4 and 5, effect of thermo-geometric parameter (M) influence on convective radiative fin is shown. As depicted M as significant influence on heat transfer, as quantitative increase in M parameter leads to significant increase in temperature distribution. This shows M as high impact on temperature distribution and heat transfer. As this phenomenon can be physically explained due to increase in ratio of convective/conduction heat transfer which influence is significant toward fin base. Radiative parameter (N_r) effect on temperature distribution is observed in Fig. 6. As depicted increasing N_r shows rapid decrease in temperature distribution which is due to increasing heat transfer from fins surface to ambient environment.

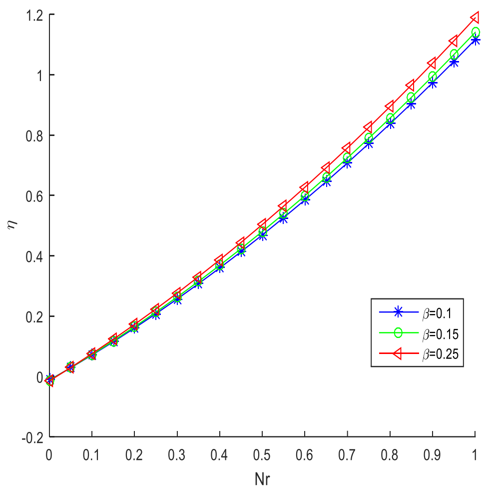

Figure 7. Effect of nonlinear parameter on fins efficiency. Where M=0.1.

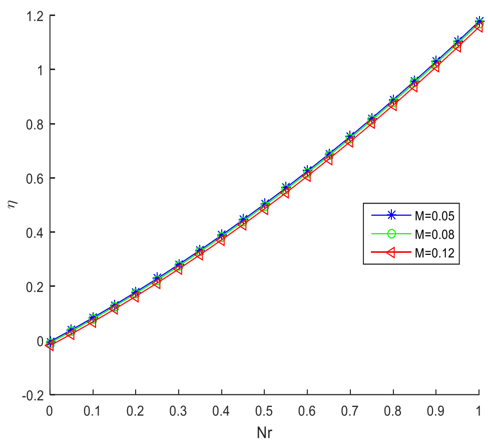

Figure 8. Effect of thermo geometric parameter on fins efficiency. Where β=0.2.

Figure 7 represents the effect of nonlinear parameter (β) on fins efficiency. From the plot it is observed that increase in β causes increase in fins efficiency while effect of thermo-geometric parameter (M) shows increasing M parameter leads to decrease in efficiency. Also it worth noting that increasing radiation improves efficiency of the fin.

4. Conclusion

This paper studies convection- radiation effect on straight fins with temperature dependent thermal conductivity using the variation of parameter method (VPM). The VPM is adopted in generating approximate analytical solutions to strongly nonlinear higher order ordinary equation describing the heat transfer. Solutions obtained are used to investigate pertinent heat transfer parameter including thermo geometric and radiation parameter on heat transfer. Result obtained shows increasing thermo geometric parameter leads to increase in temperature distribution while increase in radiation parameter causes decrease in temperature distribution. Therefore study can be said to provide useful insight to the operational and thermal performance of fins application in heat exchange media such as radiator, gas and steam plants, boiler, refrigeration and air conditioning equipment's and oil pipe lines amongst others.

Nomenclature

a_r	Aspect ratio
b	Fins length
A_c	Cross sectional area fo fins
A_p	Profile area of fins
Bi	Biot number
h	Convective heat transfer coefficient
k	Thermal conductivity of fin material
k_a	Thermal conductivity of fin at ambient temperature
k_b	Thermal conductivity of fin material
K	Dimensionless thermal conductivity of fin material
M	Dimensionless thermo-geometric fin parameter
m^2	Thermo-geometric fin parameter
N_r	Radiative parameter
P	Perimeter of fin
T	Temperature
T_∞	Ambient temperature
T_b	Temperature at fins base
X	Dimensionless length of fin
q	Rate of heat transfer
Q_r	Dimensionless heat transfer

Greek Symbols

β	Nonlinear or Thermal conductivity parameter
δ	Thickness of the fin,m
θ	Dimensionless temperature
θ_b	Dimensionless temperature at base of fin
η	Efficiency of the fin
ε	Effectiveness of the fin

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