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Performance, thermal stability and optimum design analyses of rectangular fin with temperature-dependent, thermal properties and Internal heat generation

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ABSTRACT

In this study, we analysed the thermal performance, thermal stability and optimum design analyses of a longitudinal, rectangular fin with temperature-dependent, thermal properties and internal heat generation under multi-boiling heat transfer using Haar wavelet collocation method. The effects of the key and controlling parameters on the thermal performance of the fin are investigated. The thermal stability criteria and optimum design parameter were established. From the investigation, the study reveals that the performance of the fin is enhanced as the boiling condition parameter or the exponent decreases. It is also established that the optimum fin length (at which Q/ζ reaches a maximum value) increases as the non-linear thermal conductivity term β, increases. Furthermore, the study shows that the optimum value of M can be obtained based on the value of the non-linear term. The computational results obtained in this study were compared with established numerical solutions and is found to be in good agreement with the standard numerical solutions.

1. Introduction

In recent times, the study of thermal performance of fin as a passive approach of enhancing heat dissipation from hot primary surfaces is gaining intense attention in the research community. For many engineering applications of fin, the thermal conductivity and the coefficient of heat transfer are temperature-dependent. Therefore, in the analysis of thermal performance of fin, the thermal conductivity is often modelled either by power law or by its linear dependence on temperature, whilst the heat transfer coefficient is often expressed as a power law for which the exponents represent different phenomena as reported in [1, 2]. In either case, such dependence of thermal conductivity and heat transfer coefficient on temperature renders the problem highly non-linear and difficult to solve analytically. The highly non-linear differential equations resulting from thermal and the heat transfer coefficient is solved in the literature by different authors using different methods such as regular perturbation expansion [3, 4], method of successive approximation [5], Adomian decomposition method [6, 7], homotopy perturbation method [8-12], homotopy analysis method [13, 14], variational iteration method [15-17], Galerkin's method of weighted residual [18, 19], and differential transformation method [20-23] to solve the nonlinear fin problem. Most of these approximate analytical methods usually involve a large number of terms which in practice, are not convenient for use by designers and engineers. In addition, other numerical methods such as finite difference method (FDM) finite element methods (FEM) and finite volume method (FVM) have been used to analyze the nonlinear heat transfer problem in fin. Nevertheless, the fact that some of these numerical methods, such as FEM, does not enforce the conservation principle in its original form, they however, come with increased computational time and cost. Therefore, to reduce the computational cost and time in the analysis of nonlinear problems, different wavelet collocation methods such as Legendre, Haar, Chebyshev, Leibnitz-Haar, cubic B-spline, symplectic, multi-symplectic, adaptive, multi-level, interpolating, rational, spectral, ultraspherical, first split-step, sine-cosine and semi-orthogonal B-spline wavelet collocation methods have adopted to solve the nonlinear heat transfer equations [24-27]. The ease of application, simplicity and fast rate of convergence exhibited by these methods has resulted in their increased popularity for nonlinear analysis of most engineering

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systems, including nonlinear heat transfer problems of fins [28-33]. In addition, wavelet collocation method exhibits several desirable characteristics including the ease of handling periodic boundary conditions to solve the nonlinear differential equations directly without simplification, linearization, and perturbation. Furthermore, the need for Taylor's series expansion, mesh independent study, determination of auxiliary parameters, functions, Lagrange multiplier, Adomian polynomials and recursive relations as carried out HAM, VIM, ADM, VIM, DTM, for the analysis of nonlinear problems is not a requisite using wavelet collocation methods. Thus, wavelet collocation methods offer relative simplicity, high accuracy, orthogonal and normalization, possession of close support, the possibility of implementation of standard algorithms. Hence, in this paper, we applied the Haar wavelet collocation method (HWCM) to numerically analyse the thermal performance and thermal stability of longitudinal rectangular fin with temperature-dependent thermal properties and heat generation under multi-boiling heat transfer modes. The numerical solutions established are used to investigate the effects of nonlinear thermal conductivity, convective and porosity parameters on the thermal conductivity of the fin.

2. Model Development

Consider a rectangular longitudinal fin of length *l* that is exposed on both faces to a convective environment at temperature, T_∞ and with temperature-dependent heat transfer coefficient, *h(T)*, thermal conductivity, *K(T)* and internal heat generation, $q_{int.}(T)$ shown in Fig.1. Assume there is no contact resistance where the base of the fin joins the prime surface, and that the fin thickness is negligible compared with its height and length.

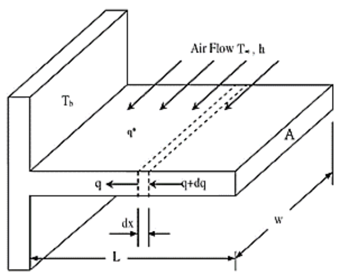


Fig.1 The geometry of straight rectangular fin [30]

The governing differential equation for the fin can therefore be expressed as:

$$\frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right]-\frac{h(T)}{A_{cr}}P(T-T_\infty)+q_{\text{int}}(T)=0 \tag{1}$$

The boundary conditions are

$$x=0,\quad T=T_o$$
$$x=L,\quad \frac{dT}{dx}=0 \tag{2}$$

The temperature-dependent thermal properties and internal heat generation of the fin are given respectively as:

$$k(T)=k_a[1+\lambda(T-T_\infty)] \tag{3}$$

$$h(T)=h_b\left(\frac{T-T_\infty}{T_w-T_\infty}\right)^n \tag{4}$$

$$q_{\text{int}}(T)=q_a[1+\psi(T-T_\infty)] \tag{5}$$

In the literature, the exponential constant *n* (multi-boiling heat transfer mode constant) often varies between -6.6 and 5. However, in most practical applications, *n* lies between -3 and 3 [32, 33]. The value of *n* varies under different conditions. *n* represents laminar film boiling or condensation when *n= -1/4*, laminar natural convection when *n= 1/4*, turbulent natural convection when *n=1/3*, nucleate boiling when *n=2*, radiation when *n=3* and *n=0*, which implies a constant heat transfer coefficient.

substituting Eqs. (3-5) into Eq. (1), we have

$$\frac{d}{dx}\left[k_a[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\frac{h_bP(T-T_\infty)^{n+1}}{A_c(T_w-T_\infty)^n}+q_a[1+\psi(T-T_\infty)]=0 \tag{6}$$

However, to achieve a non-dimensionalise Eq. (6), we introduce the following dimensionless parameters;

$$X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\quad H=\frac{h}{h_b},\quad K=\frac{k}{k_a},\quad M^2=\frac{Ph_bL^2}{A_ck_a},$$
$$Q=\frac{q_aA_c}{h_bP(T_b-T_\infty)},\quad \gamma=\psi(T_b-T_\infty),\quad \beta=\lambda(T_b-T_\infty) \tag{7}$$

Therefore, the dimensionless form of the governing differential Eq. (6) and the boundary conditions can be expressed as:

$$\frac{d}{dX}\left[(1+\beta\theta)\frac{d\theta}{dX}\right]-M^2\theta^{n+1}+M^2Q(1+\gamma\theta)=0 \tag{8}$$

which can be rewritten as

$$\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-M^2\theta^{n+1}+M^2Q(1+\gamma\theta)=0 \tag{9}$$

The dimensionless boundary conditions are

$$X=0,\quad \theta=1$$
$$X=1,\quad \frac{d\theta}{dX}=0 \tag{10}$$

.

3. Haar Wavelet

Haar wavelet is a system of square wave where the first curve is marked up as $h_0(t)$ and the second curve is marked up as $h_1(t)$ as defined by:

$$h_0(x)=\begin{cases}1, & 0\le x<1\\ 0, & otherwise\end{cases} \tag{11a}$$

$$h_1(x)=\begin{cases}1, & 0\le x<1/2\\ -1, & 1/2\le x<1\\ 0, & otherwise\end{cases} \tag{11b}$$

where $h_0(t)$ and $h_1(t)$ are the scaling function and mother wavelet, respectively.

To carry out wavelet transform, Haar wavelet use dilations and translations of functions, such that for $x \in [0,1]$, the Haar wavelet function is defined as;

$$h_0(x) = \frac{1}{\sqrt{m}}$$

$$h_i(x) = \frac{1}{\sqrt{m}} \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le x < \frac{k-0.5}{2^j} \\ -2^{\frac{j}{2}}, & \frac{k-0.5}{2^j} \le x < \frac{k}{2^j} \\ 0, & otherwise \end{cases} \tag{12}$$

where $m = 2^j$ indicates the level of the wavelet.

Here, $i = 0,1,..., m-1$ and $k = 0,1,...,2^j - 1$ represents the translation parameters, whilst $j = 0,1,..., J$ is the dilatation parameter, and J is the maximum level of resolution. The index i, represents a number of wavelets is given by $i = m + k - 1 = 2^j + k - 1$ where the maximum value of i is given as $i = 2M = 2^{J+1}$. It is assumed that $i = 1$ corresponds to the scaling function for which $h_i \equiv 1$ in [0,1]. Thus, the collocation points are defined as:

$$x_j = \frac{j-0.5}{2M}, \qquad j = 1,2,\ldots,2M \tag{13}$$

and discretizing the Haar function $h_i(x)$, the Haar coefficient matrix $H(i, j) = h_i(x)$, which is of $2M\times2M$ dimension. In addition, the operational matrix of integration is obtained by integrating Eq. (12) as

$$p_{i,1}(x) = \int_0^x h_i(x')\,dx', \tag{14}$$

and

$$p_{i,v+1}(x) = \int_0^x p_{i,v}(x')\,dx', \qquad i = 2,3,\ldots \tag{15}$$

On evaluating these integrals by using Eq. (12), we have;

$$p_{i,1}(x) = \begin{cases} x - \frac{k}{2^j} & for\ x \in \left[\frac{k}{2^j}, \frac{k+0.5}{2^j}\right) \\ \frac{k+1}{2^j} - x & for\ x \in \left[\frac{k+0.5}{2^j}, \frac{k+1}{2^j}\right) \\ 0 & otherwise \end{cases} \tag{16}$$

$$p_{i,2}(x) = \begin{cases} \frac{1}{2}\left(x - \frac{k}{2^j}\right)^2 & for\ x \in \left[\frac{k}{2^j}, \frac{k+0.5}{2^j}\right) \\ \frac{1}{2^{2j+2}} - \frac{1}{2}\left(\frac{k+1}{2^j} - x\right)^2 & for\ x \in \left[\frac{k+0.5}{2^j}, \frac{k+1}{2^j}\right) \\ \frac{1}{2^{2j+2}} & for\ x \in \left[\frac{k+1}{2^j}, 1\right) \\ 0 & otherwise \end{cases} \tag{17}$$

Assume that

$$C_{i,1} = \int_0^1 p_{i,1}(x')\,dx'. \tag{18}$$

re-writing Eq. (9), we get

$$\theta''(X) = f(X, \theta, \theta'). \tag{19}$$

the Haar wavelet method for the nonlinear problem of Eq. (9) subject to the conditions is now discussed;

$$\theta(0) = \lambda_1, \qquad \theta'(1) = \lambda_2 \tag{20}$$

Assuming that;

$$\theta''(X) = \sum_{i=1}^{2M} a_i h_i(X) \quad , \tag{21}$$

where a_i is the Haar coefficient to be determined.

Integrating Eq. (21) from 0 to X, the solution of $\theta(X)$ is expressed in terms of the Haar functions and their integrals.

Therefore, integrating Eq. (21) and using the given boundary conditions, the derivative $\theta'(X)$ can be expressed as;

$$\theta'(X) = \theta'(0) + \sum_{i=1}^{2M} a_i p_{i,1}(X) \tag{22}$$

Putting $X = 1$ in Eq. (22) and using the second boundary condition, the value of

$$\theta'(0) = \lambda_2 - a_1 \tag{23}$$

Thus, Eq. (22) can be re-written as;

$$\theta'(X) = \lambda_2 - a_1 + \sum_{i=1}^{2M} a_i p_{i,1}(X) \tag{24}$$

Again, integrating Eq. (20) from 0 to X and using the first boundary condition; we arrived at

$$\theta(X) = \lambda_1 + (\lambda_2 - a_1)X + \sum_{i=1}^{2M} a_i p_{i,2}(X) \tag{25}$$

Substituting the values of $\theta(X)$, $\theta'(X)$ and $\theta''(X)$ in Eq. (16) and discretising using the collocation points given in Eq. (15), a nonlinear system of equation is arrived at as;

$$\sum_{i=1}^{2M} a_i h_i(X_j) = \left[f\left(X_j, \lambda_1\right) + (\lambda_2 - a_1)X_j + \sum_{i=1}^{2M} a_i p_{i,2}(X_j), (\lambda_2 - a_1) + \sum_{i=1}^{2M} a_i p_{i,1}(X_j) \right] \tag{26}$$

Solving the above $2M\times2M$ system using inexact Newton's method, we obtained the unknown Haar coefficient a_i's, $i = 1, 2,\ldots, 2M$ and then substituting these values in Eq. (25), we get the Haar wavelet collocation method which is used to find the approximate solution of the given nonlinear problem in Eq. (9).

$$\sum_{i=1}^{2M} a_i h_i(X_j) = \left[f\left(X_j, \lambda_1\right) + (\lambda_2 - a_1)X_j + \sum_{i=1}^{2M} a_i p_{i,2}(X_j), (\lambda_2 - a_1) \sum_{i=1}^{2M} a_i p_{i,1}(X_j) \right] \tag{27}$$

4. Fin efficiency and Optimisation

Here, we analyse fin efficiency as a performance indication parameter. Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature or is the amount of heat dissipated from the entire fin to the maximum heat dissipated obtained if the fin base temperature is kept constant throughout the fin and is given by:

$$\eta = \frac{\int_0^L Ph(T - T_\infty)^{n+1} dx}{Ph(T_b - T_\infty)} \tag{28}$$

After non-dimensionalising Eq. (28), we have the dimensionless form of the efficiency as:

$$\eta = \int_0^1 \theta^{n+1} dX \tag{29}$$

The optimization of the fin could therefore be achieved by either minimizing the volume (weight) for any required heat dissipation or maximizing the heat dissipation for any given fin volume. Thus, in this work, we adopt the latter by maximising the heat dissipated for any given fin volume. The constant fin volume is defined as $V=A_c b$.

The heat dissipation per unit volume can be written as:

$$\frac{q_f}{V} = \frac{\int_0^L Ph(T - T_\infty)^{n+1} dx}{A_c b} \tag{30}$$

The dimensionless form of Eq. (29) is given as

$$Q_f = \frac{q_f}{k_a (T_b - T_a)} \left(\frac{A_p}{V}\right) = \frac{PhA_p \int_0^1 \theta^{n+1} dX}{k_a A_c} \tag{31}$$

Eq. (31) could be written as;

$$Q_f = \xi M^{2/3} \int_0^1 \theta^{n+1} dX \tag{32}$$

where $A_p = \delta b$ $\quad \xi = \left(\frac{2h\sqrt{A_p}}{k_a}\right)^{2/3}$

The maximum heat dissipation value occurs at the condition when the optimum fin characteristics have been achieved. Therefore, the fin dimensions in this situation represent the optimum fin configuration per unit volume. With the volume constant, the optimization procedure is also carried out to fix the profile area A_p by first expressing Q_f/ξ as a function of the thermo-geometric parameter, M (or fin length b) and then searching for the optimum value of M or b.

5. Results and Discussion

As earlier pointed out, the exponent n represents the different transfer modes, such that when $n = -1/4$, it represents the laminar film boiling or condensation, it is laminar natural convection when $n = 1/4$, turbulent natural convection when $n = 1/3$, nucleate boiling when $n = 2$, and radiation when $n = 3$. However, when $n = 0$, it implies a constant heat transfer coefficient. Therefore, Fig. 2a and b shows the effects of boiling conditions on the dimensionless temperature of the fin. From Fig. 2a and b, it is shown that the dimensionless temperature distribution fall monotonically along the length of the fin at various boiling condition parameter, n. At lower boiling condition parameter, more heat convects from the fin through its length and therefore more energy is efficiently transferred through the length of the fin. It can therefore be implied that the performance of the fin is enhanced as the boiling condition parameter or the exponent decreases.

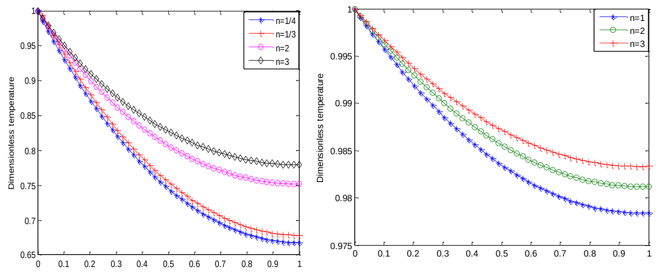
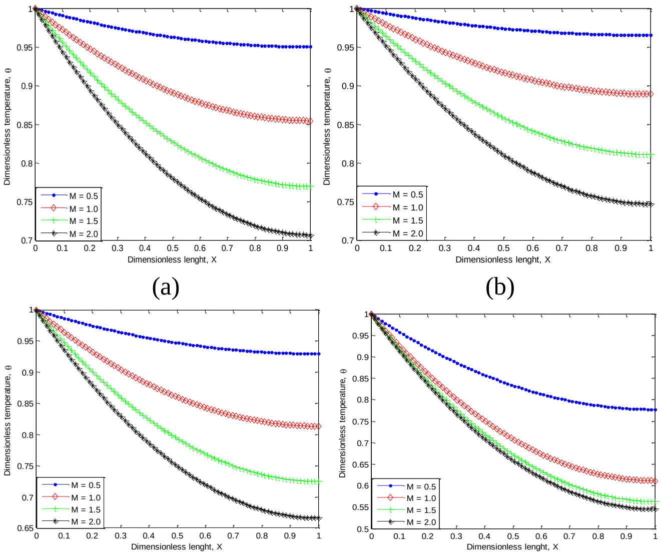


Fig. 2. Dimensionless temperature distribution in the fin at when: (a) M=1, β=0, Q=0, γ=0, (b) M=1, β=1, Q=0.8, γ=0.5



(a) (b)

(c) (d)

Fig.3 Dimensionless temperature distribution in the fin when (a) β=0.3, Q=0.2, γ=0.5 (b) β=0.9, Q=0.2, γ=0.5 (c) β=0.9, Q=0.2, γ=0.8 (d) β=0.3, Q=0.1, γ=0.2

Figs. 3(a) - (d) shows the effects of thermogeometric parameter, M (the ratio of convective heat transfer to conductive heat transfer at the base of the fin (h_b/k_b)) on the straight fin under different nonlinear and internal heat generation conditions. As the thermogeometric parameter increases, the rate of heat transfer (the convective heat transfer) through the fin increases as the temperature in the fin drops faster as depicted in the figures. This is because as convective heat transferred at the base increases (or the base thermal conductivity decreases), M increases and consequently the temperature along the fin, especially at the tip of the fin decreases. Thus, it can be implied that; as the fin convective heat transfer increases, more heat is transferred by conduction through the fin thereby increasing the temperature distribution in the fin and consequently the rate of heat transfer.

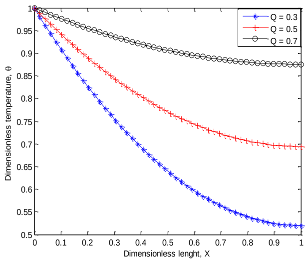


Fig. 4. Dimensionless temperature distribution in the fin parameter when β=0.2, M=2, γ=0.2

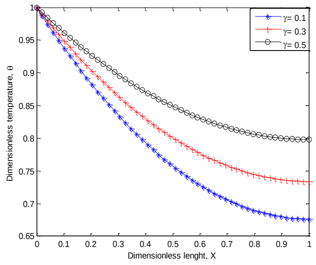


Fig. 5. Dimensionless temperature distribution in the fin parameters when M=2, β=0.3, Q=0.3

The effects of internal heat generation parameter on the temperature distribution are depicted in Figs. 4 and 5 while Fig. 6 shows the effects of internal heat generation on the fin thermal performance at different thermogeometric parameters. From these figures, internal heat generation parameter increases, as the temperature distribution of the fins decreases. Also, from Fig. 7, the effect of internal heat generation on the thermal efficiency of the fin is shown.

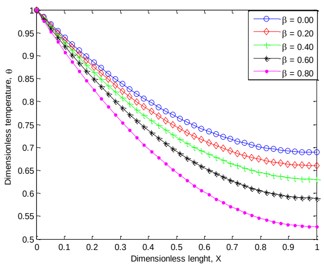


Fig. 6. Dimensionless temperature distribution in the fin parameters

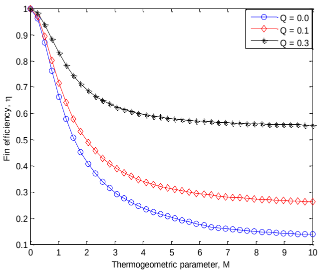


Fig. 7. Effects of internal heat generation on fin efficiency when β=0.1, γ=0.8

From Fig. 7, it can be seen that as *M* increases to a certain value, the dimensionless temperature distribution result in a negative value (i.e. thermal instability) at *x=0*, which contradicts an earlier assumption [19]. Maintaining the value of nonlinear thermal conductivity parameter, β while varying the value of the thermo-geometric parameter, M produces inaccurate characteristic for larger values of the thermo-geometric variable. As earlier established in the literature [19, 33], the increasing values of the thermo-geometric parameter tends towards negative values at the tip of the fin. The numerical solutions for the heat transfer in the fin, based on the assumption of the one-dimensional steady-state, heat conduction equation are not only inaccurate but are also thermally unstable as the fin thermo-geometric parameter, *M* exceeded a specific value [33]. Moreover, the thermal stability in the solution depicts that the adiabatic condition at the fin tip is not always maintained depending on the chosen parameter values as an investigation of their stability holds huge significance. Moreover, the decreasing value of *M* for thermal instability of the fin with constant thermal properties without internal heat generation is found to be around √2 [19]. Furthermore, the thermal instability value for any particular value of the thermogeometric parameter decreases with increasing values of the thermal conductivity and internal heat generation parameter. Fig. 8 shows the limiting/critical values of the thermo-geometric parameter, *M* for each value of *n* under different multi-boiling heat transfer models. From Fig. 8, it can be seen that the critical values of *M* and *n* are also the values for which the no-flux flow boundary condition is satisfied at the tip of the fin, and for which accurate solutions may be found. These thermal stability criteria as pointed out in this study are very important and of practical importance in the design of fins.

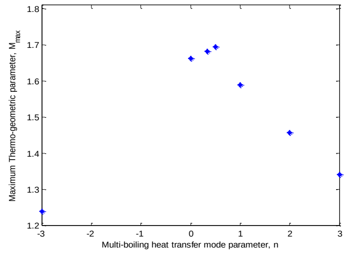


Fig. 8. Variation of thermogeometric parameter with multi-boiling heat transfer mode parameter

Table 1: Limiting values of thermo-geometric parameter, M for thermal stability of different multi-boiling heat transfer modes

n	**Numerical method (NM)**	**Present study (HWCM)**
-4	1.2540	1.2549
-3	1.2398	1.2398
0	1.6634	1.6634
1/3	1.6883	1.6832
½	1.6950	1.6951
1	1.589	1.5892
2	1.458	1.4583
3	1.341	1.3412

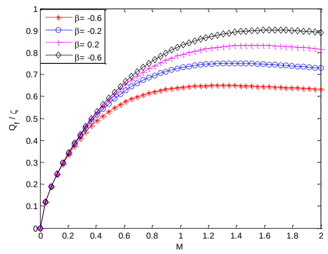


Fig. 9 Effects of non-linear thermal conductivity and thermo-geometric parameters on the dimensionless heat transfer, Q_f/ζ

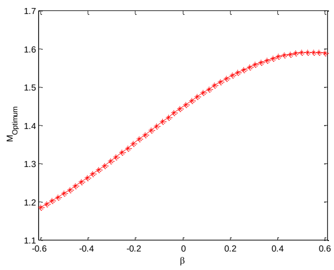


Fig. 10. Effects of non-linear thermal conductivity parameter on the optimum thermo-geometric parameter

Finally, to achieve optimum design parameter and criteria for the fin, Fig. 9 shows the dimensionless heat transfer *Q/ζ*, for a unit fin volume, with varying *M* from 1 and 2 for specified values of non-linear thermal conductivity terms. *β* for the case of *n=0*, under a given profile area, A_p, the heat transfer first rises and then falls as the fin length increases. From the figure, it can be seen that the optimum fin length (i.e. when *Q/ζ* reaches a maximum value) increases as the non-linear thermal conductivity term, *β* increases. This also shows that the optimum value of *M* can be obtained based on the value of the non-linear term. Therefore, from the analysis, the optimum dimensions of the convection fin with variable thermal conductivity are established and the relative values of optimum *M* and *β* are shown in Fig. 10.

Table 2: Comparison of results of fin-tip temperature

β	*M*	NM	HWCM
0.00	0.00	1.000000	1.000000
0.50	0.25	0.921090	0.921111
0.60	2.00	0.566280	0.566280
0.00	2.00	0.459098	0.459098

Table 3: Comparison of results of temperature gradient at the fin base

β	*M*	NM	HWCM
0.00	0.00	0.000000	0.000000
0.50	0.25	-0.158000	-0.157900
0.60	2.00	-0.886849	-0.886832
0.00	2.00	-1.256367	-1.256366

Tables 2 and 3 show comparison of results using a numerical method (NM) and Haar wavelet collocation method (HWCM) for the fin-tip temperature and temperature gradient at the fin base, respectively when $n=1$ and $Q=0$. From the computational results, as shown in the tables, it is established that there is good agreement between HWCM and the standard NM.

6. Conclusion

In this work, Haar wavelet collocation method is used to investigate the thermal performance, thermal stability and optimum design analyses of a longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation under multi-boiling heat transfer. The effects of the key controlling parameters on the thermal performance is investigated, established and discussed. The study establishes that the dimensionless temperature distribution falls monotonically along fin length for various boiling condition parameter, *n*. The study also establishes that as *M* increases to a certain value $(x=0)$, the dimensionless temperature distribution results in a negative value indicating thermal instability. In addition, we establish that as the convective heat transfer at the fin base increases, the thermogeometric parameter, *M* increases, which subsequently decreases the temperature along the fin, especially at the tip of the fin. Furthermore, the computational results obtained in this study are found to be in good agreement with the standard numerical solutions. The results of this work will enhance the optimum design of the fin for thermal performance.

Nomenclature

a_r aspect ratio
A cross-sectional area of the fins, m^2
Bi Biot number
h heat transfer coefficient
h_b heat transfer coefficient at the base of the fin
H dimensionless heat transfer coefficient at the base of the fin
j geometric parameter
k thermal conductivity of the fin material
k_b thermal conductivity of the fin material at the base of the fin
K dimensionless thermal conductivity of the fin material
L Length of the fin,
M dimensionless thermo-geometric fin parameter
M^2 thermo-geometric fin parameter
n convective heat transfer power
P perimeter of the fin,
T Temperature
T_∞ ambient temperature
T_b Temperature at the base of the fin
x fin axial distance
X dimensionless length of the fin
Q dimensionless heat transfer
q_i the uniform internal heat generation

Greek Symbols

β thermal conductivity parameter or non-linear parameter
δ thickness of the fin, m
δ_b fin thickness at its base.
γ dimensionless internal heat generation parameter
θ dimensionless temperature
θ_b dimensionless temperature at the base of the fin
η efficiency of the fin
ε effectiveness of the fin

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