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Heat Transfer Study of Convective-Radiative Fin under the influence of Magnetic Field using Legendre Wavelet Collocation Method

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ABSTRACT

The development and production of high performance equipment necessitate the use of passive cooling technology. In this paper, heat transfer study of convective-radiative straight fin with temperature-dependent thermal conductivity under the influence of magnetic field is carried out using Legendre wavelet collocation method. The numerical solution is used to investigate the effects of magnetic, convective and radiative parameters on the thermal performance of the fin. From the results, it is established that increase in magnetic, convective and radiative parameters increase the rate of heat transfer from the fin and consequently improve the thermal performance of the fin. The results obtained are compared with the results established results in literature and good agreements are found. The analysis can help in enhancing the understanding and analysis of the problem. Also, they can provide platform for improvement in the design of extended surfaces in heat transfer equipment under the influence of magnetic field.

1. Introduction

The ultimate goals of improving the design of various thermal systems such as heat exchangers, economizers, super heaters, conventional furnaces, gas turbines, etc. have been to achieve high thermal performance with reduced size and cost. These goals are often meant by the use of passive components such as fins and spines. The study of thermal behaviors, the effects of various parameters on the thermal performance and the nonlinearities in the developed thermal models of the passive devices have attracted a large number of research works. The past few decades have witnessed the development of different solution techniques for the nonlinear equations arising in the heat transfer analysis of the devices. Aziz and Enamul-Huq [1] applied regular perturbation expansion to study a pure convection fin with temperature dependent thermal conductivity. Aziz [2] extended the previous analysis to include a uniform internal heat generation in the fin. A few years later, Campo and Spaulding [3] applied method of successive approximation to predict the thermal behaviour of uniform circumferential fins. Chiu and Chen [4] and Arslanturk [5] adopted the Adomian decomposition Method (ADM) to obtain the temperature distribution in a pure convection fin with variable thermal conductivity. The same problem was also solved by Ganji [6] with the aid of the homotopy perturbation method originally proposed by He [7]. Chowdhury and Hashim [8] applied the Adomian decomposition method to evaluate the temperature distribution of straight rectangular fin with temperature dependent surface flux for all possible types of heat transfer. In the following year, Rajabi [9] employed homotopy perturbation method (HPM) to calculate the efficiency of straight fins with temperature-dependent thermal conductivity. A year later, Mustapha [10] adopted homotopy analysis method (HAM) to find the efficiency of straight fins with temperature-dependent thermal conductivity. Also, Coskun and Atay [11] utilized variational iteration method (VIM) for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity while Languri *et al.*[12] applied both variation iteration and homotopy perturbation methods for the evaluation of efficiency of straight fins with temperature-dependent thermal conductivity. Coskun and Atay [13] applied variational iteration method to analyze the efficiency of convective straight fins with temperature-dependent thermal conductivity. In the same year, Atay and Coskum [14] employed variation iteration and finite element methods to carry out comparative analysis of power-law-fin type problems. Domairry and Fazeli [15] used Homotopy analysis method to determine the efficiency of straight fins with temperature-dependent thermal conductivity. Hosseini *et al.* [16] applied homotopy analysis method to provide approximate but accurate solution of heat

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transfer in fin with temperature-dependent internal heat generation and thermal conductivity. Joneidi *et al.* [17], Moradi and Ahmadikia [18] Moradi [19], Mosayebidorcheh *et al.* [20], Ghasemi*et al.* [21], Sandri *et al.* [22], Ganji and Dogonchi [23] presented analytical solution for fin with temperature dependent thermal coefficient using differential transform method (DTM).
The above reviewed approximate analytical methods solve nonlinear differential equations without linearization, without restrictive assumptions or any perturbation, without discretization or approximation of the derivatives. However, most of the approximate methods give accurate predictions only when the nonlinearities are weak and they fail to predict accurate solutions for strong nonlinear models. Also, when they are routinely implemented, they can sometimes lead to erroneous results [24-26]. Additionally, some of them require more mathematical manipulations and are not applicable to all problems, and thus suffer a lack of generality. For example, DTM proved to be more effective than most of the other approximate analytical solutions as it does not require many computations as carried out in ADM, HAM, HPM, and VIM [27]. However, the transformation of the nonlinear equations and the development of equivalent recurrence equations for the nonlinear equations using DTM proved somehow difficult in some nonlinear system such as in rational Duffing oscillator, irrational nonlinear Duffing oscillator, finite extensibility nonlinear oscillator. Moreover, the determination of Adomian polynomials as carried out in ADM, the restrictions of HPM to weakly nonlinear problems as established in literatures, the lack of rigorous theories or proper guidance for choosing initial approximation, auxiliary linear operators, auxiliary functions, and auxiliary parameters in HAM and the search Langrange multiplier as carried in VIM, and the challenges associated with proper construction of the approximating functions for arbitrary domains or geometry of interest as in Galerkin weighted residual method (GWRM), least square method (LSM) and collocation method (CM) are some of the difficulties in applying these approximate analytical methods [27]. Therefore, the quest for comparatively simple, flexible, generic and highly accurate analytical solutions continues. In other to reduce the computation cost and time in the analysis of nonlinear problems, different wavelet collocation methods such as Legendre, Haar, Chebyshev, Leibnitz-Haar, cubic B-spline, sympletic, multi-sympletic, adaptive, multi-level, interpolating, rational, spectral, ultraspherical, first split-step, sine-cosine and semiorthogonal B-spline wavelet collocation methods have adopted to solve different nonlinear equations. The ease of use, simplicity and fast rate of convergence have in recent times made these methods gain popularity in nonlinear analysis of systems and they have been applied to nonlinear problems in heat transfer analysis of fins [28-32]. Also, the ability of these wavelet collocation methods to solve the nonlinear differential equations directly without simplification, linearization, perturbation, Taylor's series expansion, mesh independent study, determination of auxiliary parameters, functions, Lagrange multiplier, Adomian polynomials and recursive relations as carried out HAM, VIM, ADM, VIM, DTM etc. Therefore, in this paper, effects of magnet field on the thermal performance of convective-radiative straight fin with temperature-dependent thermal conductivity using wavelet collocation method. The wavelet collocation method is mathematically very simple, easy and fast. It is an efficient and powerful in solving wide class of linear and nonlinear differential equations. In recent times, the method has gained the popularity and reputation of being a very effective tool for many practical applications. From the computational simulation point of view, it has been established that the numerical approximate solution provided by the method is much closer to the exact solutions in many practical applications of the method. The results of obtained by LWCM are in excellent agreements with exact analytical solutions (for the linear model) and the direct numerical solutions (for the nonlinear model).

2. Problem formulation

Consider a convective-radiative straight fin of temperature-dependent thermal conductivity *k(T),* length *L* and thickness δ that is exposed on both faces to a convective environment at temperature T_a and with heat transfer co-efficient *h* and subjected to magnetic field as shown in Fig.1, assuming that the heat flow in the fin and its temperature remain constant with time, the temperature of the medium surrounding the fin is uniform, the fin base temperature is uniform, there is no contact resistance where the base of the fin joins the prime surface, also the fin thickness is small compared with its width and length, so that temperature gradients across the fin thickness and heat transfer from the edges of the fin may be neglected. The dimension *x* pertains to the length coordinate which has its origin at the tip of the fin and has a positive orientation from the fin tip to the fin base. Based on Darcy's model and following the above assumptions, the thermal energy balance could be expressed as stated in Eq. (1)

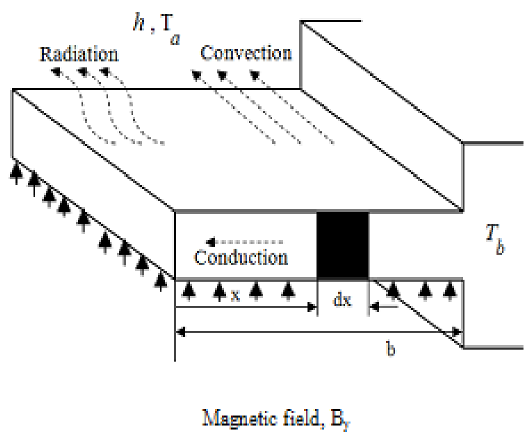

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Fig. 1 Schematic of the convective-radiative longitudinal porous fin with magnetic field

$$q_x - \left(q_x + \frac{\delta q}{\delta x} dx \right) + q(T) dx = hP(T - T_a) dx$$
$$+ \sigma \varepsilon P (T^4 - T_a^4) dx + \frac{J_c \times J_c}{\sigma} dx \tag{1}$$

where

$$J_c = \sigma (E + V \times B) \tag{2}$$

As $dx \rightarrow 0$, Eq. (1) reduces

$$-\frac{dq}{dx} = hP(T - T_a) + \sigma \varepsilon P (T^4 - T_a^4) dx + \frac{J_c \times J_c}{\sigma} \tag{3}$$

From Fourier's law of heat conduction, the rate of heat conduction in the fin is given by

$$q = -k(T) A_{cr} \frac{dT}{dx} \tag{4}$$

Where

$$k(T) = k_a \left[1 + \lambda (T - T_a) \right]$$

Following Rosseland diffusion approximation, the radiation heat transfer rate is

$$q = -\frac{4 \sigma A_{cr}}{3 \beta_R} \frac{dT^4}{dx} \tag{5}$$

Therefore, the total rate of heat transfer is given by

$$q = -k(T) A_{cr} \frac{dT}{dx} - \frac{4 \sigma A_{cr}}{3 \beta_R} \frac{dT^4}{dx} \tag{6}$$

Substituting Eq. (6) into Eq. (3), we have

$$\frac{d}{dx} \left(k_a \left[1 + \lambda (T - T_a) \right] A_{cr} \frac{dT}{dx} + \frac{4 \sigma A_{cr}}{3 \beta_R} \frac{dT^4}{dx} \right) =$$
$$hP(T - T_a) + \sigma \varepsilon P (T^4 - T_a^4) dx + \frac{J_c \times J_c}{\sigma} \tag{7}$$

Further simplification of Eq. (10) gives the governing differential equation for the fin as

$$\frac{d}{dx} \left[\left[1 + \lambda (T - T_a) \right] \frac{dT}{dx} \right] + \frac{4 \sigma}{3 \beta_R k_a} \frac{d}{dx} \left(\frac{dT^4}{dx} \right)$$
$$- \frac{h}{k_a t} (T - T_a) - \frac{\sigma \varepsilon}{k_a t} (T^4 - T_a^4) dx - \frac{J_c \times J_c}{\sigma k_a A_{cr}} = 0 \tag{8}$$

The boundary conditions are

$$x = 0, \quad \frac{dT}{dx} = 0$$
$$x = b, \quad T = T_b \tag{11}$$

But

$$\frac{J_c \times J_c}{\sigma} = \sigma B_o^2 u^2 \tag{12}$$

After substitution of Eq. (12) into Eq.(10), taking the magnetic field term as a linear function of temperature, we have

$$\frac{d}{dx} \left[\left[1 + \lambda (T - T_a) \right] \frac{dT}{dx} \right]$$
$$+ \frac{4 \sigma}{3 \beta_R k_a} \frac{d}{dx} \left(\frac{dT^4}{dx} \right) -$$
$$\frac{h}{k_a t} (T - T_a) - \frac{\sigma \varepsilon}{k_a t} (T^4 - T_a^4) dx \tag{13}$$
$$- \frac{\sigma B_o^2 u^2}{k_{eff} A_{cr}} (T - T_a) = 0$$

The case considered in this work is a situation where small temperature difference exists within the material during the heat flow. This actually necessitated the use of temperature-invariant physical and thermal properties of the fin. Also, it has been established that under such scenario, the term T^4can be expressed as a linear function of temperature. Therefore, we have

$$T^4 \cong 4 T_a^3 T - 3 T_a^4 \tag{14}$$

On substituting Eq. (14) into Eq. (13), we arrived at

$$\frac{d}{dx} \left[\left[1 + \lambda (T - T_a) \right] \frac{dT}{dx} \right]$$
$$+ \frac{16 \sigma}{3 \beta_R k_a} \frac{d^2 T}{dx^2} -$$
$$\frac{h}{k_a t} (T - T_a) - \frac{4 \sigma \varepsilon P T_a^3}{k_a t} (T - T_a) \tag{15}$$
$$- \frac{\sigma B_o^2 u^2}{k_a A_{cr}} (T - T_a) = 0$$

On introducing the following dimensionless parameters in Eq. (16) into Eq. (16),

$$\bar{x} = \frac{x}{b} \qquad \theta = \frac{T - T_a}{T_b - T_a}$$
$$\beta = \lambda (T_b - T_\infty) \quad M^2 = \frac{pbh}{A_b k_a}$$
$$Rd = \frac{4 \sigma_{st} T_\infty^3}{3 \beta_R k_a} \qquad N = \frac{4 \sigma_{st} b T_\infty^3}{k_a} \tag{16}$$
$$Ha = \frac{\sigma B_0^2 u^2}{k_a A_b}$$

we arrived at the dimensionless form of the governing Eq. (13) as

$$(1 + 4Rd) \frac{d^2 \theta}{d \bar{x}^2} + \beta \theta \frac{d^2 \theta}{d \bar{x}^2} + \beta \left(\frac{d \theta}{d \bar{x}} \right)^2 - M^2 \theta$$
$$- N \theta - Ha \theta = 0 \tag{17}$$

Then

$$\frac{d^2\theta}{d\bar{x}^2}+\frac{\beta}{(1+4Rd)}\theta\frac{d^2\theta}{d\bar{x}^2}+\frac{\beta}{(1+4Rd)}\left(\frac{d\theta}{d\bar{x}}\right)^2 -\frac{M^2}{(1+4Rd)}\theta-\frac{N}{(1+4Rd)}\theta-\frac{Ha}{(1+4Rd)}\theta=0 \quad (18)$$

Which is the same as

$$\frac{d^2\theta}{d\bar{x}^2}+\beta^*\theta\frac{d^2\theta}{d\bar{x}^2}+\beta^*\left(\frac{d\theta}{d\bar{x}}\right)^2-\left(M^*\right)^2\theta -N^*\theta-Ha^*\theta=0 \quad (19)$$

Where

$$\beta^*=\frac{\beta}{(1+4Rd)},\quad \left(M^*\right)^2=\frac{M^2}{(1+4Rd)} \quad N^*=\frac{N}{(1+4Rd)}\quad Ha^*=\frac{Ha}{(1+4Rd)}\theta=0 \quad (20)$$

and the dimensionless boundary conditions

$$\bar{x}=0,\quad \frac{d\theta}{d\bar{x}}=0 \qquad \bar{x}=1,\quad \theta=1 \quad (21)$$

For conveniences, the asterisk will be removed in the subsequent analysis

3. Method of Solution: Legendre Wavelet Collocation Method

There is a difficulty in developing an explicit exact analytical/closed-form solution for the above non-linear Eq. (19). Therefore, in this work, we apply Legendre wavelet collocation method. The wavelet algorithm is based on collocation method and the procedures for applications are described as follows.

Wavelets: Continuous wavelet are defined by the following formula

$$\psi_{a,b}(X)=|a|^{\frac{-1}{2}}\psi\left(\frac{X-b}{a}\right),a,b\in R,a\neq 0 \quad (22)$$

Where a and b are dilation and translation parameters, respectively.

The Legendre wavelets defined on the interval [0, 1] is given by

$$\psi_{n,m}(X)=\begin{cases}\sqrt{\left(m+\tfrac{1}{2}\right)}2^{k/2}\times \\ p_m\left(2^kX-\hat{n}\right) \quad ,\frac{\hat{n}-1}{2^k}\leq\frac{\hat{n}-1}{2^k} \\ 0 \qquad\qquad otherwise\end{cases} \quad (23)$$

Where m=0,1,…,M-1 and n=1,2,...2^{k-1}. $P_m(x)$ is the Legendre polynomial of order m

$$P_0(X)=1,P_1(X)=X, \quad P_{m+1}(X)=\frac{2m+1}{m+1}XP_m(X)-\frac{m}{m+1}P_{m-1}(X) \quad m=1,2,3......,M-1. \quad (24)$$

A function f(x) defined in domain [0, 1] can be expressed as

$$f(X)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}c_{n,m}\psi_{n,m}(X) \quad (25)$$

where $c_{n,m}=<f(X),\psi_{n,m}(X)>$ in which <…> denotes the inner product

Taking some terms in infinite series, we can write Eq. (25) as

$$f(X)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n,m}(X)=C^T\psi(X) \quad (26)$$

Where C and $\psi(X)$ are M x1 matrices given by

$$C=\begin{bmatrix}c_1,0,c_1,1,.....,c_1,M-1,c_2,1,......,c_2\\ ,M-1,c_{2^{k-1}},1,...,c_{2^{k-1}},M-1\end{bmatrix}^T$$

$$\psi(X)=\begin{bmatrix}\psi_{1,0}(X),\psi_{1,1}(X),...,\\ \psi_{1,M-1}(X),\psi_{2,0}(X),......,\\ \psi_{2,M-1}(X),........,\\ \psi_{2^{k-1},0}(X)\psi_{2^{k-1},1}(X)\\ ,....,\psi_{2^{k-1},M-1}(X)\end{bmatrix}^T \quad (27)$$

(i) Property of the product of two Legendre wavelets

If E is a given wavelets vector, then we have the property

$$E^T\psi\psi^T=\psi^T\hat{E} \quad (28)$$

(i) Operational matrix of integration: The integration of wavelets $\psi(X)$ which is defined in Eq. (23) can be obtained as

$$\int_0^X\psi(s)ds=P\psi(X),X\in[0,1]$$

Where P is $2^{k-1}M$ x *$2^{k-1}M$,* the operational matrix of integration is given by

$$P=\frac{1}{2}\begin{pmatrix} 1 & \frac{1}{3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & & & \vdots \\ \vdots & \vdots & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \ddots & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \frac{-\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \end{pmatrix} \tag{29}$$

3.1 Legendre Wavelet Collation Method

$$Let \qquad \theta''(X)=C^T\psi(X) \tag{30}$$

Integrating Eq. (15) with respect to x from 0 to x, we have

$$\begin{aligned} &\theta'(X)=\theta'(0)+C^T P\psi(x) \\ &\Rightarrow \theta'(X)=C^T P\psi(x) \quad since \quad \theta'(0)=0 \end{aligned} \tag{31}$$

If we integrate Eq. (31) and use the boundary conditions, we arrived at

$$\theta(X)=\theta(0)+C^T P^2\psi(X) \tag{32}$$

Put $X=1$ in (31), we have

$\theta(0)=1-C^T P^2\psi(1)$, since $\theta(1)=1$, we obtain (33)

On substituting Eq. (33) into E.(32)

$$\theta'(X)=1-C^T P^2\psi(1)+C^T P^2\psi(X) \tag{34}$$

Again, integrating above equation, with respect to X from 0 to X, we obtain

$$\theta(X)=1-C^T P^2\psi(1)+C^T P^2\psi(X) \tag{35}$$

Substituting, $\theta''(X)$, $\theta''(X)$ and $\theta''(X)$ in Eq. (31), we arrived at

$$\begin{aligned} &\left(1+\beta^*\theta\right)C^T\psi(X)+\beta^*\left(C^T P\psi(X)\right)^2- \\ &\left(\left(M^*\right)^2+Ha^*+N^*\right)\times \\ &\left\{1-C^T P\psi(1)+C^T P\psi(X)\right\} \\ &=R\left(X,c_1,c_2,.....,c_n\right) \end{aligned} \tag{36}$$

On expanding Eq. (36), we have

$$\begin{aligned} &\left[1+\beta^*\left\{1-C^T P^2\psi(1)+C^T P^2\psi(X)\right\}\right] \\ &C^T\psi(X)+\beta^*\left\{C^T P\psi(X)\right\}^2 \\ &-\left(\left(M^*\right)^2+Ha^*+N^*\right) \\ &\left\{1-C^T P^2\psi(1)+C^T P^2\psi(X)\right\} \\ &=R\left(X,c_1,c_2,.....,c_n\right) \end{aligned} \tag{37}$$

Choosing n collocation points i.e. x_i, $i=1,2,3,...,n$ in the interval (0,1), at which residual $R(x, c_i)$ equal to zero. The number of such points gives the number of coefficient c_i, $i=1,2,3,..,n$.

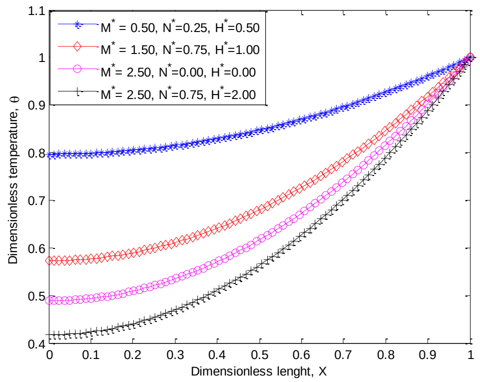
$$C=\begin{bmatrix} c_1,0,c_1,1,.....,c_1,M-1,c_2,1,......,c_2 \\ ,M-1,c_{2^{k-1}},1,...,c_{2^{k-1}},M-1 \end{bmatrix}^T$$

Thus, we get $R(X, c_1,c_2,c_3,...,c_n)=0$, $i=1,2,3,...,n$.

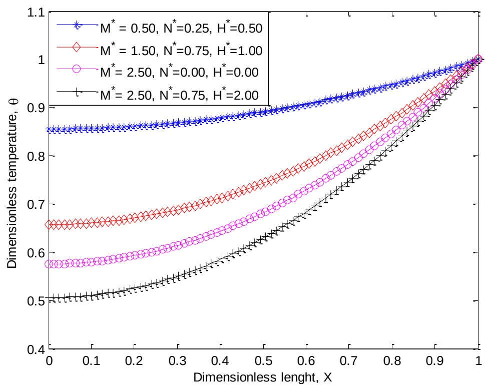
The above Eq. (37) gives system of nonlinear equations which are solved simultaneously using Newton-Raphson method and the values of C are obtained. Substituting the values of C in Eq. (20), the approximate solution of $\theta(X)$ is found.

4. Results and Discussion

Effects of thermo-geometric term, radiation number and magnetic number (Hartmann number) on the dimensionless temperature distribution and by extension on the thermal performance of the fin are shown in Figs 2 and 3. From the figures, as the magnetic parameter increases, the temperature decreases rapidly and the rate of heat transfer (the convective-radiative heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. The rapid decrease in fin temperature due to increase in the magnetic parameter is because as Hartmann number increases, the magnetic force increases and this in turn increases the magnetic field strength which consequently decreases the temperature of the fin as shown in the figures. Therefore, the thermal performance of the fin is increased by the presence of the magnetic field. This is in agreement with the deduction from works of Hoshyar et al. [33]. Also, the figures depict the effects of nonlinear parameter or temperature-dependent thermal conductivity term on the thermal performance of the fin. From the figures, it could be deducted that the nonlinear thermal conductivity parameter has direct and significant effects on the rate of heat transfer of the fin.

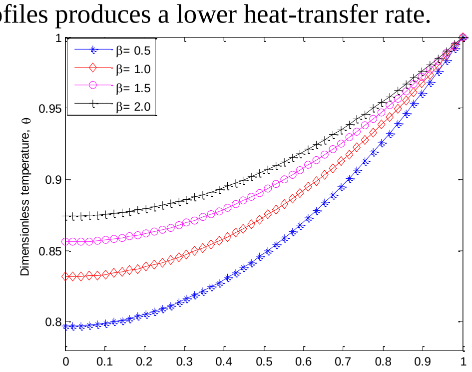


Figs. 2 Effects of radiation and Hartmann numbers on the temperature distribution in the fin when β=0.5

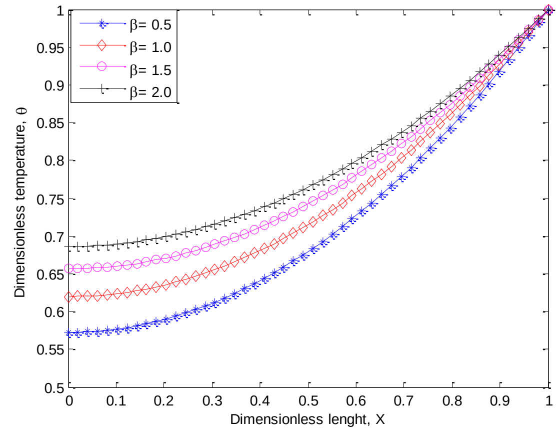


Figs. 3 Effects of radiation and Hartmann numbers on the temperature distribution in the fin when β=1.5

Figs. 4 and 5 show the effects of non-linear thermal conductivity on the fin thermal performance under different thermo-geometric parameter, radiation number and Hartmann number. The results show that performance of the fin temperature decreases with increase in the nonlinear term or the thermal conductivity number. Also, the figures depict that as the thermo-geometric parameter increases, the rate of heat transfer through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates). The profile has steepest temperature gradient at lower value of the thermo-geometric parameter, but its much higher value gotten from the lower value of thermal conductivity than the other values of *M* in the profiles produces a lower heat-transfer rate.



Figs. 4 Effects of non-linear parameter on the temperature distribution in the fin when M=0.50, N=0.25, H=0.50



Figs. 5 Effects of non-linear parameter on the temperature distribution in the fin when M=1.50, N=0.75, H=1.00

Table 1: Comparison of results

X	LWCM	NUM
0.00	0.863499240	0.863499231
0.05	0.863828561	0.863828568
0.10	0.864817079	0.864817090
0.15	0.866465751	0.866465743
0.20	0.868776255	0.868776261
0.25	0.871751107	0.871751104
0.30	0.875393406	0.875393404
0.35	0.879707016	0.879707010
0.40	0.884696498	0.884696500
0.45	0.890367175	0.890367181
0.50	0.896725092	0.896725096
0.55	0.903777057	0.903777060
0.60	0.911530656	0.911530658
0.65	0.919994252	0.919994259
0.70	0.929177055	0.929177056
0.75	0.939089074	0.939089079
0.80	0.949741206	0.949741203
0.85	0.961145186	0.961145189
0.90	0.973313762	0.973313764
0.95	0.986260543	0.986260549
1.00	1.000000000	1.000000000

5. Conclusion

In this work, thermal analysis of convective-radiative fin with thermal conductivity under the influence of magnetic field has been carried out using Legendre wavelet collocation method. The numerical solution is used to investigate the effects of magnetic, convective and radiative parameters on the thermal performance of the fin. From the results, it is established that increase in magnetic, convective and radiative parameters increase the rate of heat transfer from the fin and consequently improve the thermal performance of the fin. The results obtained are compared with the results established results in literature and good agreements are found. The analysis can serve as basis for comparison of any other method of analysis of the problem and they also provide platform for improvement in the design of fin in heat transfer equipment where the surrounding fluid is influenced by a magnetic field.

Nomenclature

A_{cr} cross sectional area of the fins
A_p profile area of the fins
b Length of the fin
B magnetic induction
B_o magnetic field intensity
E electric field
h heat transfer coefficient
H Hartmann number
J total current intensity
J_c conduction current intensity
k thermal conductivity of the fin material
k_a thermal conductivity of the fin material at ambient temperature
M dimensionless thermo-geometric fin parameter
m^2 thermo-geometric fin parameter
N dimensionless radiation number
q total rate of heat transfer
q_c rate of heat conduction transfer
q_r rate of heat radiation transfer

Ra Modified Rayleigh number
Rd radiation-conduction number
P perimeter of the fin
t thickness of the fin
T Temperature
T_a ambient temperature
T_b Temperature at the base of the fin
V Voltage
x fin axial distance, m
X dimensionless length of the fin

Greek Symbols

λ non-linear thermal conductivity parameter
β dimensionless non-linear thermal conductivity parameter
β_R Rosseland extinction coefficient
θ dimensionless temperature
ε emmisivity of the fin
σ Electric conductivity
σ_{st} Stefan-Boltmann constant

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