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Analysis of Heat transfer in Porous Fin with Temperature-dependent Thermal Conductivity and Internal Heat Generation using Chebychev Spectral Collocation Method

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Abstract

In this work, analysis of heat transfer in porous fin with temperature-dependent thermal conductivity and internal heat generation is carried out using Chebychev spectral collocation method. The numerical solutions are used to investigate the influence of various parameters on the thermal performance of the porous fin. The results show that increase in convective parameter, porosity parameter, Nusselt, Darcy and Rayleigh numbers and thickness-length ratio of the fin, the rate of heat transfer from the base of the fin increases and consequently improve the efficiency of the fin. However, the rate of heat transfer from the base of the fin increases with decrease in thermal conductivity material. Also, from the parametric studies, an optimum value is reached beyond which further increase in porosity, Nusselt, Darcy and Rayleigh numbers, thermal conductivity ratio and thickness-length ratio has no significant influence on the rate of heat transfer. It is established that the temperature predictions in the fin using the Chebychev spectral collocation method are in excellent agreement with the results of homotopy perturbation method and that of numerical methods using Runge-Kutta coupled with shooting method.

Keywords: Porous Fin; Thermal performance**;** Temperature-Dependent Thermal Conductivity and Internal Heat Generation, Chebyshev spectral collocation method.

1. Introduction

The use of porous fin as a passive method for heat transfer enhancement in thermal equipment has attracted a lot of research interests following the pioneer work of Kiwan and Al-Nimr [1]. Also, the research interests have been greatly aroused following the further studies by Kiwan [2-4] on the thermal performance analysis of porous fin in natural convection environment. This is evident in the research works carried out many years later. Gorla and Bakier [5] studied the thermal analysis of natural convection and radiation in a rectangular porous fin while Kundu and Bhanja [6] presented analytical model for the analysis of performance and optimization of porous fins. In the following year, Kundu *et al.* [7] proposed a model for computing maximum heat transfer in porous fins. Meanwhile, Taklifi *et al.* [8] investigated the effects of magnetohydrodynamics (MHD) on the performance of a rectangular porous fin. In the work, it was stated that by imposing MHD in system except near the fin tip, heat transfer rate from the porous fin decreases. Bhanja and Kundu [9] analytically investigated thermal analysis of a constructal T-shape porous fin with radiation effects. An increase in heat transfer is found by choosing porous medium condition in the fin. Recently, Kundu *et al.* [10] applied Adomian decomposition method on the performance and optimum design analysis of porous fin of various profiles operating in convection environment transient heat transfer analysis of variable section pin fins. Saedodin and Sadeghi [11] analyzed the heat transfer in a cylindrical porous fin while Saedodin and Olank [12]. Darvishi *et al.* [13] studied the thermal performance of a porous radial fin with natural convection and radiative

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heat losses while Hatami and Ganji [14] investigated the thermal performance of circular convective-radiative porous fins with different section shapes and materials. Hatami *et al.* [15-18] presented various heat transfer studies in both dry and wet porous fins. All the studies on porous fin cited above are based on constant thermal conductivity. Such assumption might be correct because, for ordinary fins problem, the thermal conductivity of the fin might be taken to be constant. However, if large temperature difference exists within the fin, typically, between tip and the base of the fin, the thermal conductivity is not constant but temperature-dependent. Also, in their work, Gorla *et al.* [19] and Moradi *et al.* [20] established that for most materials, the effective thermal conductivity increases with temperature. Therefore, while analyzing the fin, the effects of temperature-dependent thermal conductivity must be taken into consideration. In carrying out such analysis, the thermal conductivity may be modeled for such and other many engineering applications by linear dependency on temperature. Such dependency of thermal conductivity on temperature renders the problem highly non-linear and difficult to solve exactly. It is also very realistic to consider the temperature-dependent internal heat generation in the fin (electric-current carrying conductor, nuclear rods or any other heat generating components of thermal systems). Most of the solutions for the analysis of heat transfer in porous fin are established using approximate analytical methods. Kundu [6-7, 10] applied Adomian decomposition method (ADM) on the performance and optimum design analysis of the fins while Saedodin and Sadeghi [11], Kiwan [1-5] applied Runge-Kutta for the thermal analysis in porous fin. Golar and Baker [5] and Gorla *et al*. [19] applied spectral collocation method (SCM) to study the effects of variable thermal conductivity on the natural convection and radiation in porous fin. Saedodin and Shahababaei [21] adopted homotopoy perturbation method (HPM) to analyse heat transfer in longitudinal porous fins while Darvishi *et al.* [13] and Moradi *et al.* [20] and Ha *et al.* [22] utilized homotopy analysis method (HAM) to provide solution to the natural convection and radiation in a porous and porous moving fins while Hoshyar *et al.* [23] used homotopy perturbation method and collocation method for Thermal performance analysis of porous fins with temperature-dependent heat generation. Hatami and Ganji [14] applied least square method (LSM) to study the thermal behaviour of convective-radiative in porous fin with different sections and ceramic materials. Also, Rostamiyaan *et al*. [24] applied variational iterative method (VIM) to provide analytical solution for heat transfer in porous fin. Ghasemi *et al.* [25] used differential transformation method (DTM) for heat transfer analysis in porous and solid fin while Ganji and Dogonchi [26] adopted DTM to analytically investigate convective heat transfer of a longitudinal fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. Also, Dogonchi and Ganji [27] applied DTM to carry out convection-radiation heat transfer study of moving fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. The approximate analytical methods as applied by past researchers solve the differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. However, the search for a particular value that will satisfy second the boundary condition or the determination of auxiliary parameters necessitated the use of software and such could result in additional computational cost in the generation of solution to the problem. Also, most of the approximate methods give accurate predictions only when the nonlinearities are weak or for small values of the fin thermo-geometric parameter, they fail to predict accurate solutions for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to analytic expression involving a large number terms and when they are routinely implemented, they can sometimes lead to erroneous results [28, 29]. Moreover, in practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers. Also, variational methods such as Ritz method and Rayleigh-Ritz method sometimes provide powerful results, such as upper and lower bounds on quantities of interest but require more mathematical manipulations than method of weighted residual and are not applicable to all problems, and thus suffer a lack of generality. Inevitably, simple yet accurate expressions are required to determine the fin temperature distribution, efficiency, effectiveness and the optimum parameter.

Numerical methods such as Euler and Runge–Kutta methods are limited to solving initial value problems. With the aid of shooting method, the methods could be carried out iteratively to solve boundary value problems. However, these numerical methods are only useful for solving ordinary differential equations. On the other hand, numerical methods such as finite difference method (FDM), finite element methods (FEM) and finite volume method (FVM) can be adopted to analyze heat transfer in fins with single and multiple independent variables as they have been used to solve different linear and non-linear differential equations in literatures. On the other hand, the fast rate of convergence and a very large converging speed of spectral methods over most of the commonly used numerical methods have been established in the field of numerical simulations. The converging speed of the approximated numerical solution to the primitive problem is faster than any one expressed by any power-index of $N-1$. Numerical methods such as finite element method (FEM) and the finite volume method (FVM) provide linear convergence, while, the spectral methods provide exponential convergence [30]. Spectral methods have been widely applied in computational fluid dynamics [31, 32], electrodynamics [33] and magneto hydro dynamics [34, 35]. From the view of approximation to the original equation, the spectral method can be classified as the collocation method which presents discretization in physical space, the Galerkin method which seeks solution in spectral space, and the pseudo-spectral method which provides discrete integration in physical space at first and then presents transformation into spectral space for seeking the solution. Among the three methods, the collocation method is much more suitable for treating with non-linear problems. Recent numerical work concerned with the solution of non-linear differential equations has also provided more and more evidence of the applicability and accuracy of the Chebyshev collocation method [36-41]. The main advantage of spectral methods lies in their accuracy for a given number of unknowns. For smooth problems in simple geometries, they offer exponential rates of convergence/spectral accuracy [42-45]. Despite the high accuracy and efficiency of the method, it has not been significantly applied to nonlinear heat transfer problems. Therefore, in this work, analysis of heat transfer in porous fin with temperature-dependent thermal conductivity and internal heat generation is carried out using Chebychev spectral collocation method. Effects of various parameters on the thermal performance of the porous fin are investigated. The results obtained by the method are compared with the previous studies and excellent agreements are established.

2 Problem Formulation

Consider a straight porous fin of length L and thickness t exposed on both faces to a convective environment at temperature T_∞ as shown in Fig.1.The dimension x pertains to the height coordinate which has its origin at the fin tip and has a positive orientation from fin tip to fin base. In order to analyze the problem, the following assumptions are made.

1. Porous medium is homogeneous, isotropic and saturated with a single phase fluid
2. Physical properties of solid as well as fluid are considered as constant except density variation of liquid, which may affect the buoyancy term where Boussinesq approximation is employed.
3. Fluid and porous mediums are locally thermodynamic equilibrium in the domain.
4. Surface radiative transfers and non-Darcian effects are negligible.
5. The temperature variation inside the fin is one-dimensional i.e. temperature varies along the length only and remain constant with time.
6. There is no thermal contact resistance at the fin base and the fin tip is adiabatic type.

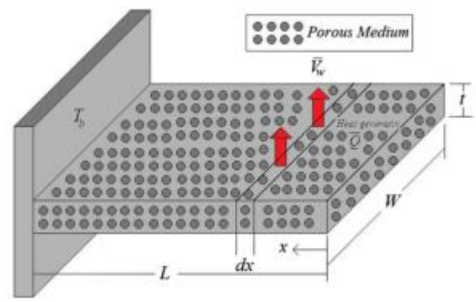


Fig. 1 Schematic of the longitudinal porous fin geometry with the internal heat generation

Based on Darcy's model and following the above assumptions, the governing equation for the heat transfer in the fin is given as

$$\frac{d}{dx}\left[k_{eff}(T)\frac{dT}{dx}\right]-\frac{hP(T-T_\infty)}{A}-\frac{\rho c_p g\beta' KP(T-T_\infty)^2}{A\nu_f}+q_a(T)=0 \tag{1}$$

The first, second, third and the fourth terms in the Eq. (1) are conductive, convective, porous and internal heat generation terms, respectively.

The boundary conditions are

$$x=L, \quad T=T_b$$
$$x=0, \quad \frac{dT}{dx}=0 \tag{2}$$

or many engineering applications, the thermal conductivity and the coefficient of heat transfer are temperature-dependent. Therefore, the temperature-dependent thermal properties and internal heat generation are given by

$$k_{eff}(T)=\phi k_f+(1-\phi)k_s=k_{eff,a}[1+\lambda(T-T_\infty)] \tag{3}$$

$$q_{\text{int}}(T)=q_a[1+\psi(T-T_\infty)] \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (1), we have

$$\frac{d}{dx}\left[[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right]-\frac{h(T-T_\infty)}{k_{eff,a}t}$$
$$-\frac{\rho c_p\, \mathrm{g} K\beta^{'}(T-T_\infty)^2}{k_{eff,a}t\nu_f}$$
$$+\frac{q_a}{k_{eff,a}}[1+\psi(T-T_\infty)]=0 \qquad (5)$$

On introducing the following dimensionless parameters in Eq. (6) into Eq. (5);

$$X=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\ Ra=Gr.\Pr=\left(\frac{\beta^{'} gT_b t^3}{\nu_f^2}\right)\left(\frac{\rho c_p \nu_f}{k_{eff,a}}\right),\quad Da=\frac{K}{t^2},\quad Q=\frac{q\nu_f t}{\rho\, \mathrm{c}_p\, \beta^{'}\, \mathrm{g} K(T_b-T_\infty)^2},\quad M^2=\frac{hL^2}{k_{eff,a}t}$$

$$S_h=\left(\frac{\beta^{'} g(T_b-T_\infty)t^3}{\nu_f^2}\right)\left(\frac{\rho c_p \nu_f K}{k_{eff,a}t^2}\right)\frac{(L/t)^2}{k_{eff,a}}=\frac{RaDa(L/t)^2}{k_{eff,a}},\quad \gamma=\psi(T_b-T_\infty),\quad \beta=\lambda(T_b-T_\infty) \qquad (6)$$

We arrived at the dimensionless governing differential Eq. (7) and the boundary conditions

$$\frac{d}{dX}\left[(1+\beta\theta]\frac{d\theta}{dX}\right]-M^2\theta-S_H\theta^2$$
$$+S_H Q\gamma\theta+S_H Q=0 \qquad (7)$$

If we expand Eq. (7), we have;

$$\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2-M^2\theta-S_H\theta^2+S_H Q\gamma\theta+S_H Q=0 \qquad (8)$$

The boundary conditions are

$$X=0,\quad \frac{d\theta}{dX}=0$$
$$X=1,\quad \theta=1 \qquad (9)$$

3 Solution Procedures

The nonlinearity in governing equation Eq. (8) makes it very difficult to develop a closed-form solution to the non-linear equation. Therefore, in this work, a spectral collocation method of the Chebyshev type is employed to solve the heat transfer equation. The Chebyshev collocation spectral method is based on the expansion by virtue of the Chebyshev polynomials. At first, it expands the variable at collocation points and seeks the variable derivatives at these points, then substitutes the expansions into the differential equations and finally seeks the approximated solution in physical space. This means that Chebyshev collocation spectral method is accomplished through, starting with Chebyshev approximation for the approximate solution and generating approximations for the higher-order derivatives through successive differentiation of the approximate solution.

Looking for an approximate solution, which is a global Chebyshev polynomial of degree N defined on the interval [-1, 1], the interval is dicretized by using collocation points to define the Chebyshev nodes in [-1, 1], namely

$$x_j=cos\left(\frac{j\pi}{N}\right),\quad j=0,1,2,...N \qquad (10)$$

The derivatives of the functions at the collocation points are given by:

$$f^n\left(x_j\right)=\sum_{j=0}^{N} d_{kj}^n f\left(x_j\right),\quad n=1,2. \qquad (11)$$

where d_{kj}^n represents the differential matrix of order n and are given by

$$d_{kj}^1=\frac{4\gamma_j}{N}\sum_{n=0,l=0}^{N}\sum_{n+l=odd}^{n-1}\frac{n\gamma_n}{c_l}T_l^n\left(x_k\right)T_n\left(x_j\right),$$
$$k,j=0,1,...N, \qquad (12a)$$

$$d_{kj}^2=\frac{2\gamma_j}{N}\sum_{n=0,l=0}^{N}\sum_{n+l=even}^{n-1}\frac{n\gamma_n\left(n^2-l^2\right)}{c_l}T_l^n\left(x_k\right)T_n\left(x_j\right),\quad k,j=0,1,...N, \qquad (12b)$$

where $T_n\left(x_j\right)$ are the Chebyshev polynomial and coefficients γ_j and c_l are defined as:

$$\gamma_j = \begin{cases} \frac{1}{2} & j = 0, or\ N \\ 1 & j = 1, 2, \ldots N-1 \end{cases} \tag{13a}$$

$$c_l = \begin{cases} 2 & l = 0, or\ N \\ 1 & l = 1, 2, \ldots N-1 \end{cases} \tag{13b}$$

As described above, the Chebyshev polynomials are defined on the finite interval [-1, 1]. Therefore, to apply Chebyshev spectral method to our equation (8), we make a suitable linear transformation and transform the physical domain [-1, 1] to Chebyshev computational domain
[-1,1]. We sample the unknown function w at the Chebyshev points to obtain the data vector $w = \left[w\left(x_o\right), w\left(x_1\right), w\left(x_2\right), \ldots w\left(x_N\right)\right]^T$. The next step is to find a Chebyshev polynomial P of degree N that interpolates the data $\left(i.e.,\quad P\left(x_j\right) = w_j,\ j = 0,1,\ldots N\right)$ and obtains the spectral derivative vector w by differentiating P and evaluating at the grid points $\left(i.e.,\quad w_j^{'} = P'\left(x_j\right) = w_j,\ j = 0,1,\ldots N\right)$. This transforms the nonlinear differential equation into system nonlinear algebraic equations, which are solved by Newton's iterative method starting with a initial guess.

Making a suitable transformation to map the physical domain [0, 1] to a computational domain
[-1,1] to facilitate our computations.

$$\begin{aligned} &\frac{d^2\tilde{\theta}}{dX^2} + \beta\tilde{\theta}\frac{d^2\tilde{\theta}}{dX^2} + \beta\left(\frac{d\tilde{\theta}}{dX}\right)^2 - M^2\tilde{\theta} - S_H\tilde{\theta}^2 \\ &+ S_H Q\gamma\tilde{\theta} + S_H Q = 0 \end{aligned} \tag{14}$$

The boundary conditions are

$$\tilde{\theta}'(-1) = 0, \quad \tilde{\theta}(1) = 1 \tag{15}$$

After applying CSCM, using Eq. (14), the governing equation and boundary conditions are transformed into a system of nonlinear algebraic equations:

$$\begin{aligned} &\sum_{j=0}^{N} d_{kj}^2\tilde{\theta}\left(X_j\right) + \beta\sum_{j=0}^{N}\tilde{\theta}\left(X_j\right)d_{kj}^2\tilde{\theta}\left(X_j\right) \\ &+\beta\sum_{j=0}^{N} d_{kj}^1\tilde{\theta}\left(x_j\right)\sum_{j=0}^{N} d_{kj}^1\tilde{\theta}\left(x_j\right) \\ &-M^2\tilde{\theta}\left(x_j\right) - S_H\left\{\tilde{\theta}\left(x_j\right)\right\}^2 \\ &+S_H Q\gamma\tilde{\theta}\left(x_j\right) + S_H Q = 0 \end{aligned} \tag{16}$$

The boundary conditions are

$$\sum_{j=0}^{N} d_{kj}^1\tilde{\theta}\left(x_j\right) = 0, \quad \tilde{\theta}\left(x_j\right) = 1 \tag{17}$$

The above system of nonlinear algebraic equation is solved using Newton's method to determine the temperature distribution in the fin

3.1 Heat flux of the Fin and rate of heat transfer per unit area from the porous fin

The fin base heat flux is given by Eq. (18)

$$q_b = A_c k(T)\frac{dT}{dx} \tag{18}$$

The dimensionless heat transfer rate at the base of the fin is given by

$$Q_b = \frac{qL}{k_a A_c(T_b - T_\infty)} = \left[(1+\beta\theta)\frac{d\theta}{dX}\right]_{X=1} \tag{19}$$

3.2 Analysis of Heat transfer augmented in porous fin

In order to make a comparison between the heat transfer from a porous fin with that from a solid fin, the ratio of heat transfer rate between the two fins are given by

$$\frac{q_b}{q_s} = \frac{k_{eff}(T)A_b\left(\frac{dT}{dx}\right)_{x=o}}{hA_s(T_b - T_\infty)} \tag{20}$$

where, the denominator represents the maximum possible heat transfer rate obtained using a solid fin. Writing the above equation in terms of the dimensionless temperature and axial distance, yields

$$\frac{q_b}{q_s} = \frac{A_r}{Nu}\left[(1+\beta\theta)\frac{d\theta}{dX}\right]_{X=1} \tag{21}$$

4 Results and Discussion

The solutions are reported in figures and the effects of various parameters such as convective parameter, porosity parameter, Nusselt, Darcy and Rayleigh numbers and thickness-length ratio on the temperature distributions, rate of heat transfer and by extension on the thermal performance of the porous fin are investigated. Figs. 2a-d show the effects of nonlinear thermal conductivity parameters on the dimensionless temperature distribution and by extension on the rate of heat transfer. It is shown that as the non-linear thermal conductivity parameter increases, the dimensionless temperature distribution in the fin decreases.

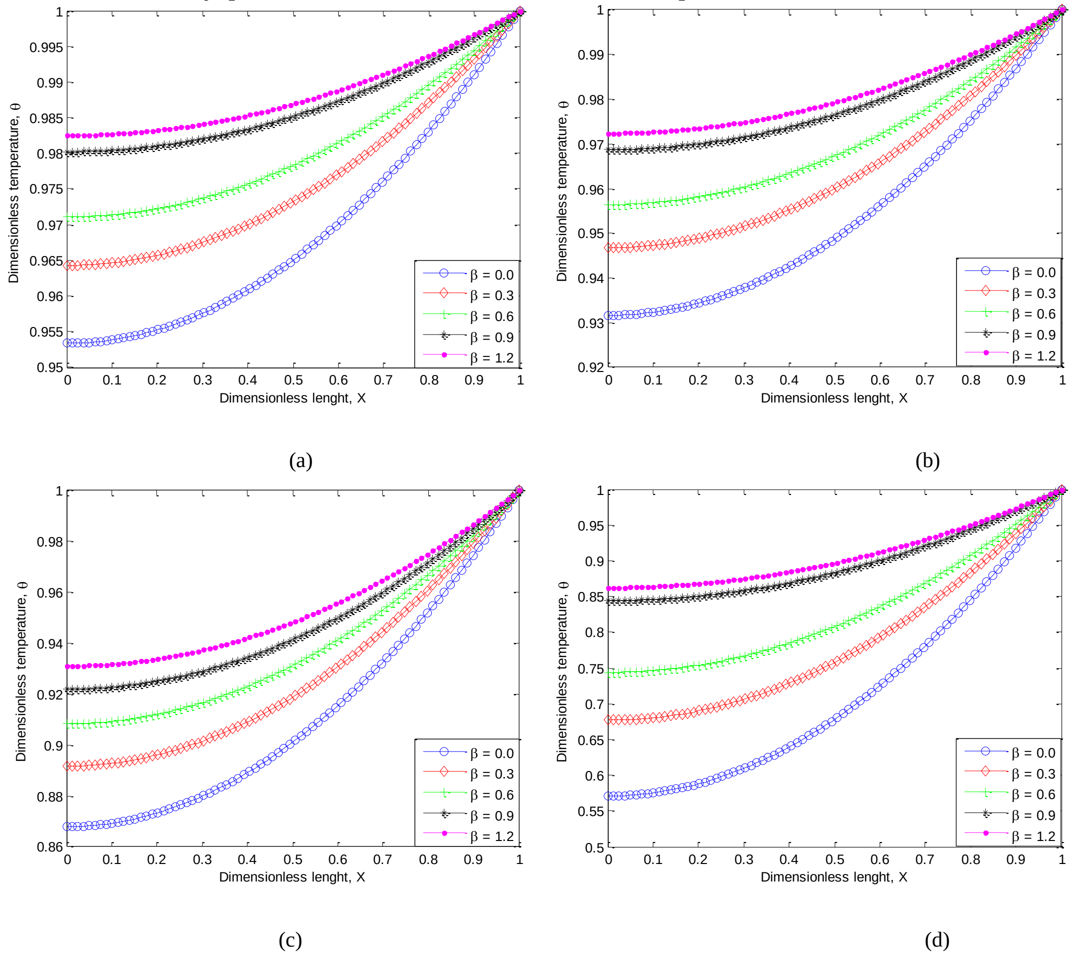


Fig. 2 Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) M=0.3, S_h,= 0, Q=0.4, γ=0.2 (b)) M=0.3, S_h,=0.1, Q=0.4, γ=0.2 (c) S_h,=0.5, M=0.3, Q=0.4, γ=0.2 (d)) M=0.8, S_h,=0.1, Q=0.4, γ=0.2

Figs. 3a-d show the effects of porous parameter or porosity on the temperature distribution in the porous fin are shown. As the porosity parameter increases, the temperature decreases rapidly and the rate of heat transfer (the convective heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. The rapid decrease in fin temperature due to increase in the porosity parameter is because as porosity parameter, S_h increases and in consequent, the Darcy and Raleigh number increase, the permeability of the porous fin increases and therefore the ability of the working fluid to penetrate through the fin pores increases, the effect of buoyancy force increases and thus the fin convects more heat, the rate of heat transfer from the fin is enhanced and the thermal performance of the fin is increased. Therefore, increase in the porosity of the fin improves fin efficiency due to increasing in convection heat transfer.

The effects of the internal heat generation on the thermal stability of the fin is shown in Fig. 4a-d and Fig. 5a-b. It is obvious that as porous parameter, S_h increases to a certain value, the dimensionless temperature distribution decreases. The effects of the internal heat generation on the thermal stability of the fin is shown in Fig. 4a-b, it is obvious that as porous parameter, S_h increases to a certain value, the dimensionless temperature distribution at the fin tip results in negative value (which shows thermal instability) at $x=0$, contradicting the assumption made in the analysis. However, value of porosity parameter for the thermal stability increases with increase in internal heat generation parameter, Q (Fig. 4c) and thermal conductivity parameters, β. This fact was not established in the Kiwan [3] numerical analysis of the same problem for the large values of S_h.

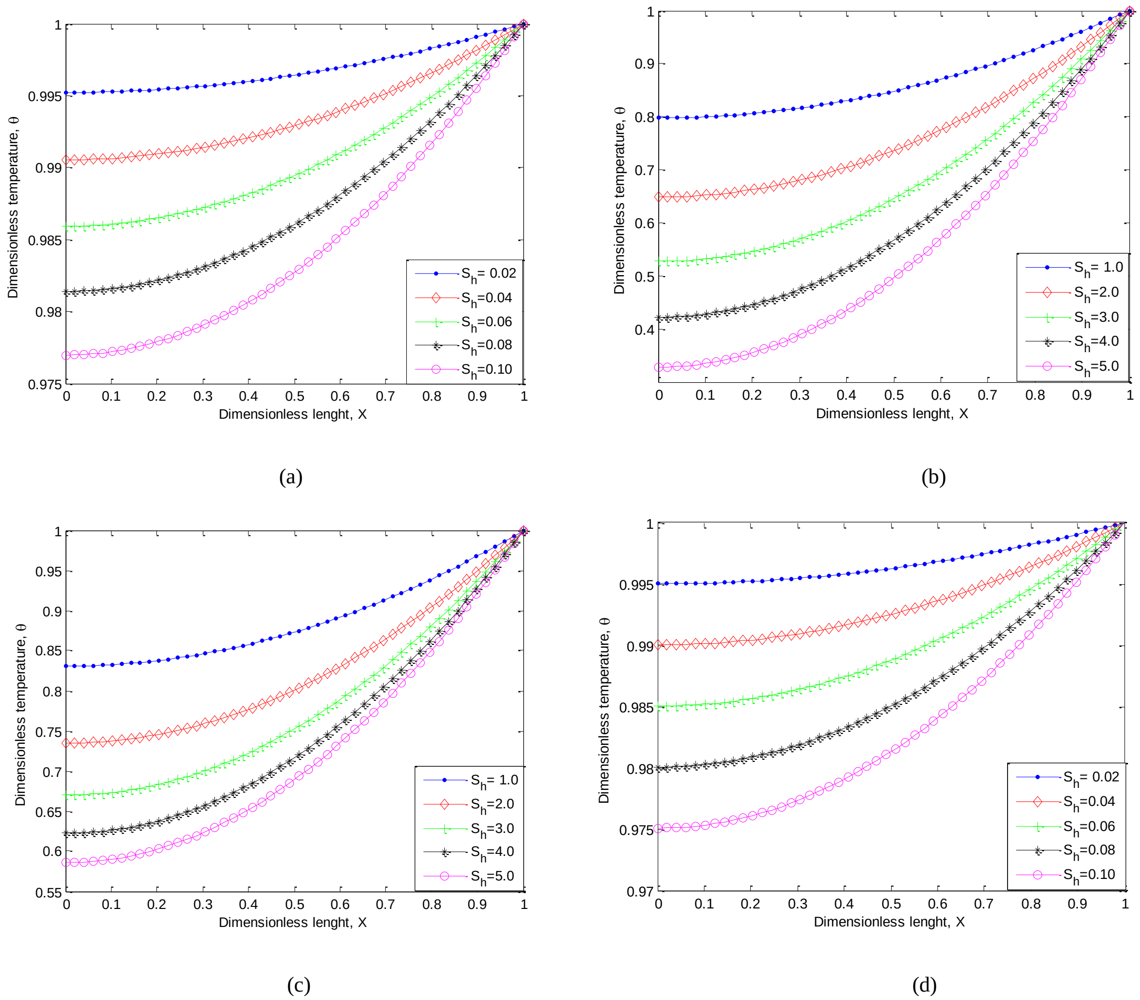


Fig. 3 Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) β=0.5, M=0.5, Q=0.2, γ=0.4 (b) β,=0.5, M=1.0, Q=0.2, γ=0.4 (c) β=0.5, M=2.0; Q=0.2, γ=0.4 (d)) β=0.5, M=10, Q=0.2, γ=0.4

Figs. 6a-b show the effects of temperature-dependent internal heat generation on the rate of heat transfer i.e. fin thermal performance at different porous parameters. From the figures, as the temperature-dependent internal heat generation parameter increases, the temperature gradient and consequently, the rate of heat transfer in the fin decreases. Also, the figures show that the rate of heat transfer at the base of the fin increases as the porous parameter or porosity increases.

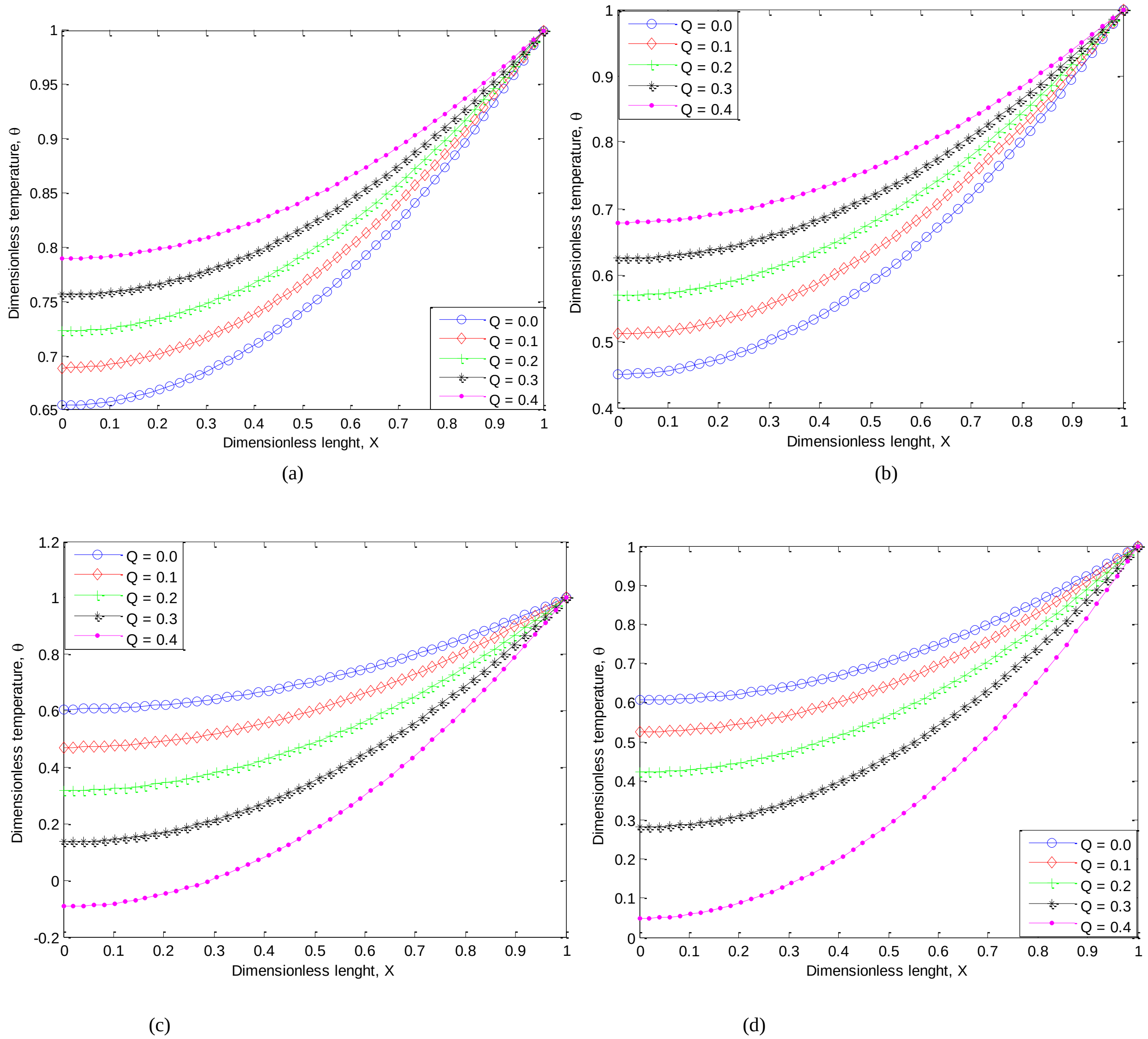
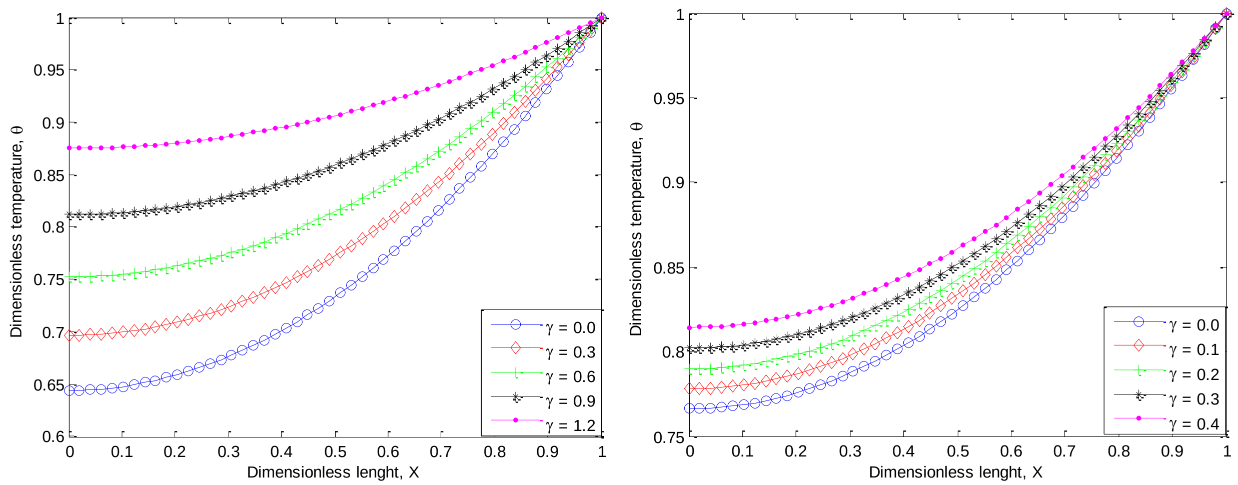
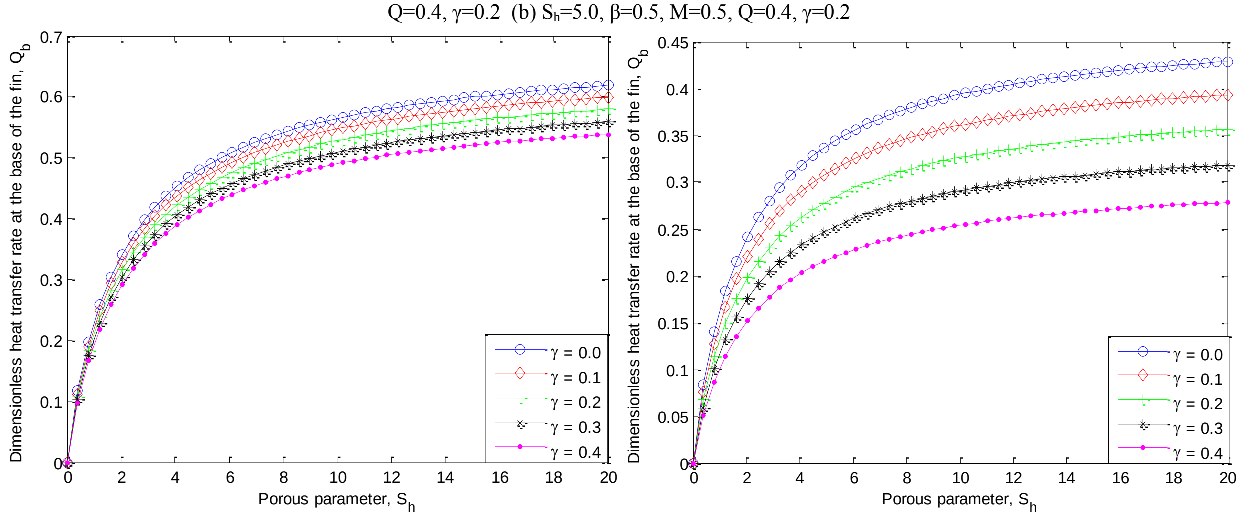


Fig. 4 Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) M=0.5, S_h=2.0, β=0.5, γ=0.2 (b) M=0.5, S_h=5.0, β,=0.5, γ=0.2, (c) M=2.0, S_h=5.0, β=0.5, γ=0.2 (d) M=2.0, S_h=5.0, β=0.5, γ=2.0



(a) (b)

Fig. 5 Dimensionless temperature distribution in the fin parameters for varying thermo-geometric parameter when (a) S_h=5.0,M=0.5, β= 0.5, Q=0.4, γ=0.2 (b) S_h=5.0, β=0.5, M=0.5, Q=0.4, γ=0.2



(a) (b)

Fig. 6 Effects of temperature-dependent internal heat generation parameter on the dimensionless heat transfer rate in the fin when (c) M=0,β= 0.4, Q=0.3 (d) M=0, β=0.4, Q=0.3

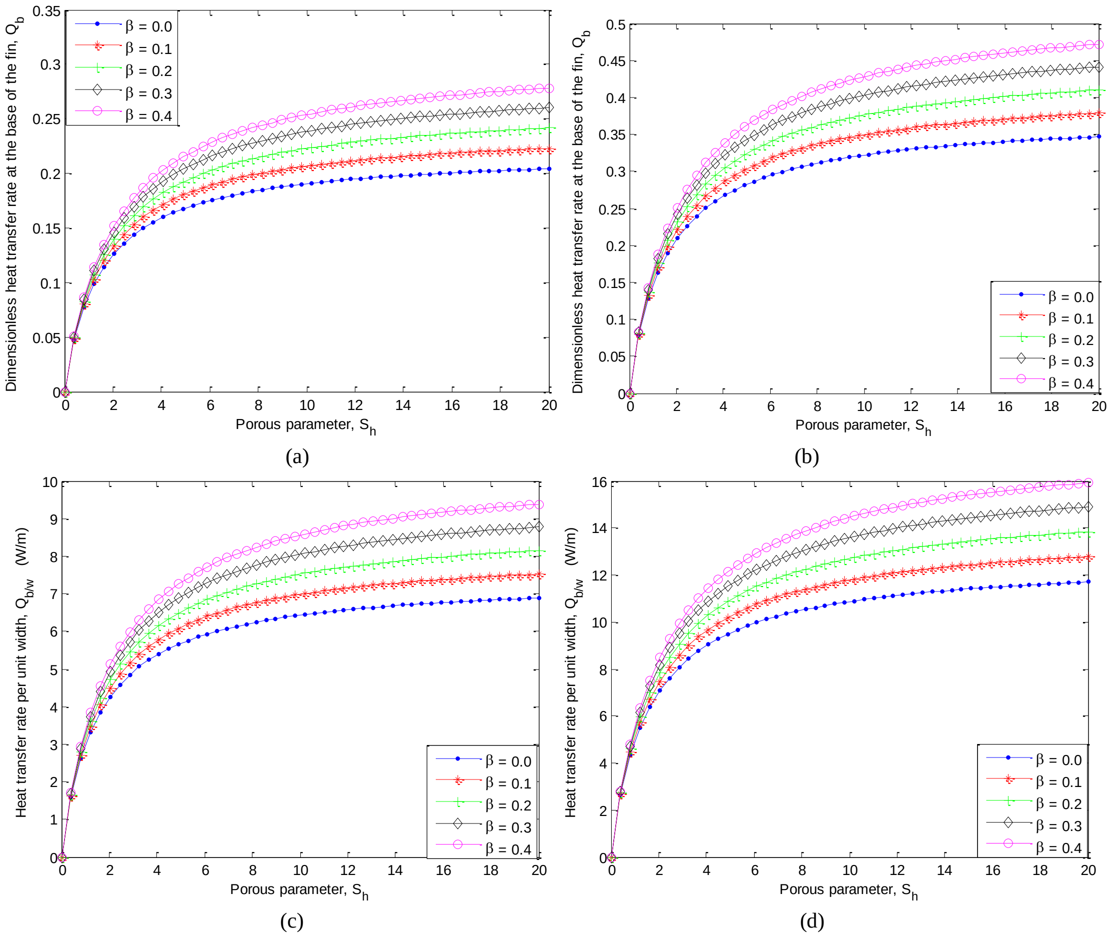


Fig. 7 Effects of temperature-dependent thermal conductivity parameter and fin thickness-lenght ratio on the dimensionless heat transfer rate at the base of the fin when (a) γ=-0.4, Q=0.5, M=0 (b) γ=0.7, Q=0.3 (c) t/L=1/1000 , k = 45 W/mK; Tb=373 K; Ta=298 K; γ=0.4, Q=0.5, M=0 (d) t/L=1/1000, , k=45 W/mK; Tb=373 K; Ta=298 K; γ=-0.4, Q=0.5 γ=0.7, Q=0.3, M=0

Actually, a major important analysis in the fin problem is the determination of the rate of heat transfer at the base of the fin. Figs. 7a-d effects of temperature-dependent thermal conductivity parameter and fin thickness-length ratio, *t/L*, on the dimensionless heat transfer rate at the base of the fin while Figs. 8a-d effects of temperature-dependent internal heat generation parameter and fin thickness-length ratio on the dimensionless heat transfer rate at the base of the fin at different porous parameters. From the figures, it could be deducted that the temperature-dependent thermal conductivity parameter, porosity and fin thickness ratio have direct and significant effects on the rate of heat transfer at the base of the fin. Increase in the dimensionless thickness parameter (fin thickness-length ratio) results in higher rate of heat transfer at the base of the fin.

Fig. 8 shows the effects of Darcy number on the dimensionless rate of heat transfer. Increasing Darcy number, Da causes an increase in the heat transfer rate from the fin. This is because when the Darcy number and consequently permeability reduces, collision among the fluid flow and the pores of the porous is increased. Thus the passing fluids gave more space to contact with the porous media which has internal heat generation. Consequently, the value of the fin temperature is increased by decreasing the Da number. Effects of Nusselt number on the rate of heat transfer at the base of the fin is depicted in Fig. 9.

It shows that as Nu increases more heat are drawn from the fin base. However, at high values of the porosity parameter S_h, increasing Nu has no significant influence on the heat transfer from the base of the fin. This is because as the porosity parameter S_h increases the temperature at the fin tip reaches the ambient temperature of the surrounding fluid and thus the driving force for heat transfer from the fin tip reduces. This leads to a significant reduction from the use of high values of Nu at the tip [3]. Increasing *t/L* or decreasing thermal conductivity parameter, K_r increases S_h and thus increasing the rate of heat transfer at the base of the fin. Moreover, increasing L or decreasing K_r tends to reduce the heat transfer rate from the fin. From the result, for the different values

of fin thicknesses, the respective optimum values (values beyond which a further increase on S_h or L has no significant change on the heat transfer rate) for S_h and L can be established as shown in Fig. 10.
Increase in fin thickness-length ratio, *t/L*, increases the rate of heat transfer from the base of the fin. However, as fin thickness-length ratio increases up to some certain values for the different fin thickness-length *ratio* considered, optimum points were reached where further increase in *t/L* has no significant influence on the heat transfer rate from the base of the fin. As the fin length increases, the temperature of the part far from the fin base approaches the working fluid temperature. This implies that the driving force for natural convection decreases and leads, in porous fins, to less fluid infiltrated through the pores of the porous domain. This implies that no significant improvements will attain if the fin length is further increased. This scenario is not only peculiar to porous fin, it also occurs in solid fin
In order to make a comparison between the heat transfer rates from a porous fin with that from a solid fin, the ratio of heat transfer rate between the porous fin and solid are established as given by Eq. (34). Fig. 11 shows the effects of porosity number on the ratio of heat transfer rate between the porous fin and solid. The increasing in porosity number, S_h, implies increase in Darcy and Rayleigh numbers. While the increasing Darcy number, Da increases the permeability of the fin, increase in the Ra number leads to more effects of buoyancy force and consequently heat transfer rate due to convection mechanism. Therefore, high values of S_h or Da and Ra lead high value of the ratio of heat transfer rate between the porous fin and solid and enhanced heat transfer between the fin and the air flow.

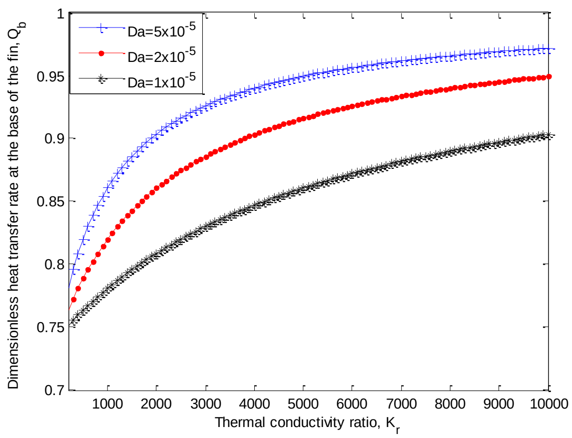


Fig. 8 Effects of Darcy number on dimensionless heat transfer rate

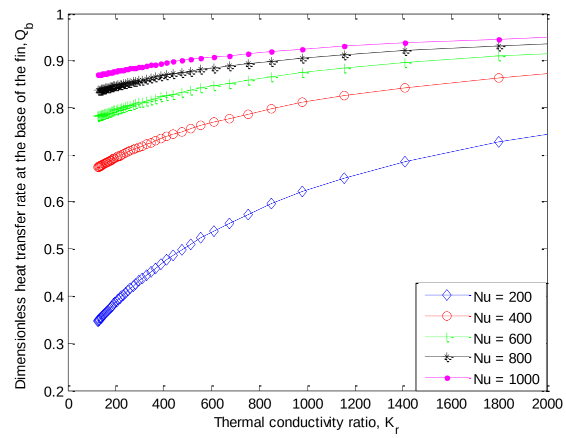


Fig. 9 Effects of Nusselt number on dimensionless heat transfer rate

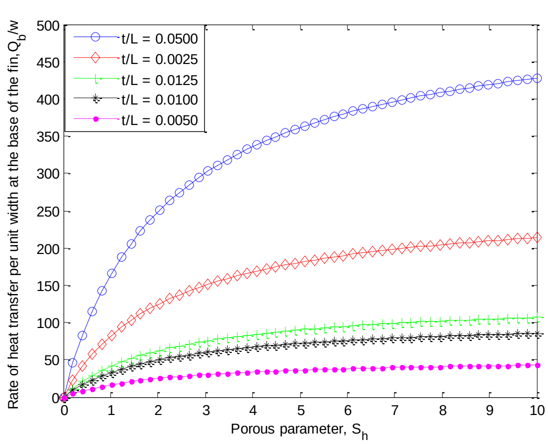


Fig. 10 Effects of length-thickness ratio on dimensionless heat transfer rate

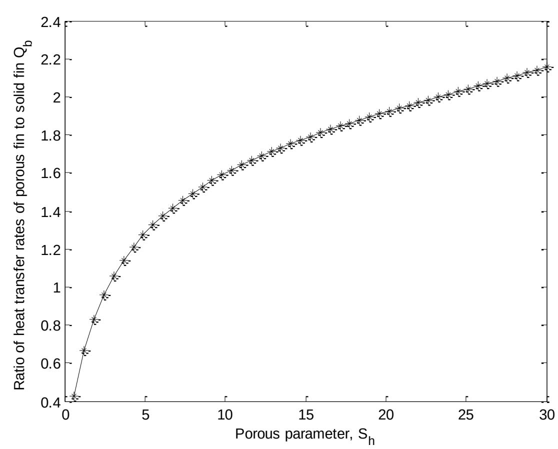


Fig. 11 Ratio of the porous fin to solid fin heat transfer rate with S_h

The numerical solution using CSCM for the non-linear thermal model is verified by the fourth-Order Runge Kutta coupled with shooting algorithm as presented by Kiwan [3]. The comparisons of the results are shown in Figs. 12. It is depicted that the CSCM is highly accurate and shows excellent agreement with the results of the other NM and the HPM. It was established that when S_h> 1, the HPM solutions for $\beta = \pm 0.4$ are very weak and provide unreasonable results because HPM is not applicable to these cases of high or strong nonlinearity. HPM solution fails when porosity parameter increases to a large number. This shortcoming in the solution method is not only peculiar to HPM, it is also experienced when using ADM [33] coupled with the additional task tasks of finding Adomian polynomials. The results show that the CSCM is very effective and it is a convenient tool to solve the nonlinear fin problems under different conditions.

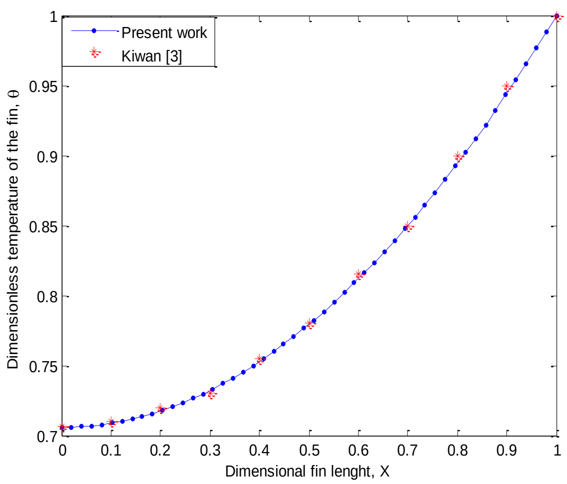


Fig. 12 Comparison of results for S_h=1

Table 1 shows comparison of results (*when M=0*) in the method used in this study. It could be inferred from the table that the CSCM is highly accurate and agrees very well with the results of homotopy perturbation method and that of numerical methods using Runge-Kutta coupled with shooting method.

Table 1: Comparison of results

X	NM	HPM (Petroudi *et al*, 2012)	CSC (The Present study)
0.0	0.9581	0.9581	0.9581
0.1	0.9585	0.9585	0.9585
0.2	0.9597	0.9597	0.9597
0.3	0.9618	0.9618	0.9618
0.4	0.9647	0.9647	0.9647
0.5	0.9685	0.9685	0.9685
0.6	0.9730	0.9730	0.9730
0.7	0.9785	0.9785	0.9785
0.8	0.9846	0.9846	0.9848
0.9	0.9919	0.9919	0.9919
1.0	1.0000	1.0000	1.0000

5. Conclusion

In this work, thermal performance analysis in a porous fin temperature-dependent thermal properties and internal heat generation has been analyzed using Chebychev spectral collocation method. The numerical solutions were used to investigate the effects of various parameters on the thermal performance of the porous fin. Increasing the porosity, Nusselt, Darcy and Rayleigh numbers and thickness-length ratio of the fin increase the rate of heat transfer from the base of the fin and consequently improve the efficiency of the fin. Also, decreasing thermal conductivity parameter, K_r results in increase in the rate of heat transfer from the base of the fin. However, an optimum value is reached beyond which further increase in porosity, Nusselt, Darcy and Rayleigh numbers, thermal conductivity ratio and thickness-length ratio has no significant influence on the rate of heat transfer. The CSCM used in the work was validated with the numerical method using Runge-Kutta method. The CSCM results used are in excellent agreement with results with the results of homotopy perturbation method and that of numerical methods using Runge-Kutta coupled with shooting method.

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