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Analysis of Flow of Nanofluid through a Porous Channel with Expanding or Contracting Walls using Chebychev Spectral Collocation Method.

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Abstract

In this work, we applied Chebychev spectral collocation method to analyze the unsteady two-dimensional flow of nanofluid in a porous channel through expanding or contracting walls with large injection or suction. The solutions are used to study the effects of various parameters on the flow of the nanofluid in the porous channel. From the analysis, It was established that increase in expansion ratio and Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases near the surface of the channel during contraction. Moreover, it was also established that an increase in injection rate leads to a higher axial velocity near the center and the lower axial velocity near the wall. On the verification of the results, it is shown that the results obtained from Chebychev spectral collocation method are in good agreement when compared to the results obtained using other numerical methods.

Keywords: Nanofluid, Porous Channel; Expanding or Contracting walls, Chebychev Spectral Collocation method.

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1. Introduction

The vast applications of fluid flow in a porous channel through expanding or contracting walls with large injection or suction as in synthetic respiratory systems, artificial circulatory systems and several industrial processes have attracted a lot of research interests in the past few years. The pioneer work on the flow phenomenon was carried out by Berman [1]. Some years later, Terrill [2, 3] improved the work of Berman on laminar flow in channels with porous walls. Providing solutions to the inherent nonlinear equations governing the flow process have led to the applications of different analytical, approximate analytical and numerical methods. Dauenhauer and Majdalani [4] developed exact self-similarity solution for the Navier-Stokes equations for a porous channel with orthogonally moving walls. Asymptotic formulations based on Wentzel-Krammers-Brillouin (WKB) and multiple-scale techniques was used by Majdalani [5] and Majdalani and Roh [6] to study the oscillatory channel flow with wall injection. Similar work using the multiple scales techniques was done by Jankowski and Majdalani [9] to analyze oscillatory channel flow with arbitrary suction. The same authors [10] used Liouville-Green transformation to develop an analytical solution for laminar flow in a porous channel with large wall suction and a weakly oscillatory pressure. Zhou and Majdalani [11] applied finite difference method and asymptotic technique (variation of parameters and small parameter perturbations) to investigate the mean flow for slab rocket motors with regressing walls. A similar analysis was done by Majdalani and Zhou [12] for moderate-to-large injection and suction driven channel flows with expanding or contracting walls. Multiple solutions associated with this problem have been reported by Robinson [13], Zarturska et al. [14] and Si et al. [15, 16]. Majdalani et al. [17] applied regular perturbation method to study two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. In a recent study, Dinarvand et al. [18] adopted homotopy analysis and homotopy perturbation methods to solve Berman's model of two-dimensional viscous flow in porous channels with wall suction or injection. Using the homotopy analysis method, Xu et al. [19] developed highly accurate series approximations for two-dimensional viscous flow between two moving porous walls and obtained multiple solutions associated with this problem. Also, the same method was adopted by Dinarvand and Rashidi [20] to analyse two dimensional viscous flow in a rectangular domain bounded by two moving porous walls.

The approximate analytical methods as applied by past researchers solve the differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. However, most of the approximate methods give accurate predictions only when the nonlinearities are weak or for small values of the fin thermo-geometric parameter, they fail to predict accurate solutions for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to analytic expression involving a large number terms and when they are routinely implemented, they can sometimes lead to erroneous results [21, 22]. Moreover, in practice, approximate analytical solutions with large number of terms are not easily convenient for use by engineers and designers. This however, necessitate the application of simple yet accurate expressions for the determine the flow process an essential requisite.

Spectral methods have been widely applied in computational fluid dynamics, electrodynamics and magnetohydrodynamics [23-30]. Also, the collocation method is much more suitable for treating with non-linear problems. Recent numerical work concerned with the solution of non-linear differential equations has also provided more and more evidence of the applicability and accuracy of the Chebyshev collocation method [31-34]. The main advantage of spectral methods lies in their accuracy for a given number of unknowns. For smooth problems in simple geometries, they offer exponential rates of convergence/spectral accuracy [35-38]. Chebychev spectral collocation method is a numerical approach that solves nonlinear integral and differential equations without linearization, discretization, closure, restrictive assumptions, perturbation, approximations, round-off error and discretization which often results in massive numerical computations. Chebychev spectral collocation method reduces the complexity of expansion of derivatives and the computational difficulties of the other traditional approximation analytical or perturbation methods. The method provides excellent approximations to the solution of non-linear equation with high accuracy, minimal calculation, and avoidance of physically unrealistic assumptions. It is not affected by computation round off errors and one is not faced with necessity of large computer memory and time. Thus, when compared with other numerical methods, Chebychev spectral collocation method offers fast rate of convergence with a very large converging speed. The converging speed of the approximated numerical solution to the primitive problem is faster than one expressed by any power-index of *N-1*. Nevertheless, despite the high accuracy and efficiency of this method, it has not been significantly applied to nonlinear flow problems. Therefore, in this work, Chebychev spectral collocation method is applied to analyze the unsteady two-dimensional flow of nanofluid through a porous channel with expanding/contracting walls. Moreover, the developed solutions are used to study the effects of the flow parameters in the expanding or contracting porous channel. From the present analysis, the results obtained by the method for solving the problem under investigation are compared with the numerical solution for the non-linear case and very good agreements are established.

2. Problem Formulation

Fig. 1 below shows the schematic diagram of a fully developed unsteady, laminar, isothermal, and incompressible flow in a two-dimensional porous channel bounded by two permeable surfaces or walls that enable the nanofluid to enter or exit during successive expansions or contractions. One end of the channel is closed by a compliant solid membrane. Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate, $\dot{a}(t)$. Also, a coordinate system is chosen with the origin at the center of the channel as shown in the figure.

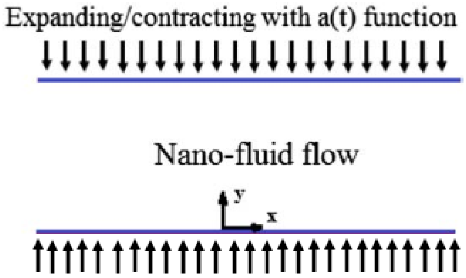


Fig. 1. The model of the porous channel with expanding or contracting walls.

Based on the assumptions, the equations for continuity and motion are

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)= -\frac{\partial \bar{p}}{\partial \bar{x}}+\mu_{nf}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+v\frac{\partial \bar{v}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{y}}+\mu_{nf}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{3}$$

where

$$\rho_{nf}=\rho_f\left(1-\phi\right)+\rho_s\phi \tag{4}$$

$$\mu_{nf}=\frac{\mu_f}{\left(1-\phi\right)^{2.5}} \tag{5}$$

Assuming no slip condition, the appropriate boundary conditions are given as

$$\bar{y}=a(t),\quad \bar{u}=0,\quad \bar{v}=-V_w=-\frac{a}{c}$$

$$\bar{y}=0,\quad \frac{\partial \bar{u}}{\partial \bar{y}}=0,\quad \bar{v}=0$$

$$\bar{x}=0,\quad \bar{u}=0 \tag{6}$$

where $c=\frac{\dot{a}}{V_w}$ is the wall presence or injection/suction coefficient, which is the measure of permeability

Introducing the following stream functions and the mean flow vorticity

$$\bar{u}=\frac{\partial\bar{\psi}}{\partial\bar{y}},\quad \bar{v}=-\frac{\partial\bar{\psi}}{\partial\bar{x}} \tag{7}$$

The pressure term in Eqs. (2) and (3) can be eliminated and the vorticity transport equation is obtained as

$$\rho_{nf}\left(\frac{\partial \bar{\xi}}{\partial t}+\bar{u}\frac{\partial \bar{\xi}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\xi}}{\partial \bar{y}}\right)=\mu_{nf}\left(\frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{\xi}}{\partial \bar{y}^2}\right)$$

(8)

where

$$\bar{\xi}=\frac{\partial \bar{v}}{\partial \bar{x}}-\frac{\partial \bar{u}}{\partial \bar{y}} \tag{9}$$

Also, the above partial differential equation can be converted to ordinary differential equation using the following similarity variables

$$\bar{u}=\frac{\mu_{nf}x}{\rho_{nf}H^2}\bar{f}'(\eta,t),\ \bar{v}=\frac{-\mu_{nf}}{\rho_{nf}H}\bar{f}\left(\eta,t\right) \tag{10}$$

where

$$\eta=\frac{\bar{y}}{H},\quad \bar{f}'(\eta,t)=\frac{\partial\bar{f}\left(\eta,t\right)}{\partial\eta}$$

Substituting Eq. (9) and (10) into Eq. (8), we have a fourth order ordinary differential equation

$$\bar{f}_{\eta\eta\eta\eta}+\alpha(t)\left(\eta\bar{f}_{\eta\eta\eta}+3\bar{f}_{\eta\eta}\right)+\bar{f}\bar{f}_{\eta\eta\eta}-\bar{f}_{\eta}\bar{f}_{\eta\eta}-\frac{\rho_{nf}a^2}{\mu_{nf}}\bar{f}_{\eta\eta t}=0 \tag{11}$$

And the following boundary conditions becomes

$$\eta=1,\quad \bar{f}=Re,\quad \bar{f}_{\eta}=0$$
$$\eta=0,\quad \bar{f}=0,\quad \bar{f}_{\eta\eta}=0 \tag{12}$$

where $\alpha\left(t\right)=\frac{\rho_{nf}a\dot{a}(t)}{\mu_{nf}}$ is the non-dimensional wall dilation rate which is positive for expansion and negative for contraction. In addition, $Re=\frac{\rho_{nf}aV_w}{\mu_{nf}}$ is the permeation Reynolds number, which is positive for injection and negative for suction. Using the following variables,

$$\psi=\frac{\bar{\psi}}{a\dot{a}},\quad u=\frac{\bar{u}}{\dot{a}},\quad v=\frac{\bar{v}}{\dot{a}},\quad f=\frac{\bar{f}}{Re}\quad\Rightarrow\quad \psi=\frac{xf}{c},\quad u=\frac{xf'}{c},\quad v=\frac{-f}{c},\quad c=\frac{\alpha}{Re} \tag{13}$$

Eqs. (11) and (12) are normalized as

$$f_{\eta\eta\eta\eta}+\alpha(t)\left(\eta f_{\eta\eta\eta}+3f_{\eta\eta}\right)+Re\left(ff_{\eta\eta\eta}-f_{\eta}f_{\eta\eta}\right)-\frac{\rho_{nf}a^2}{\mu_{nf}}f_{\eta\eta t}=0 \tag{14}$$

With the boundary conditions

$$\eta=0,\quad f=0,\quad f_{\eta\eta}=0$$

$$\eta=1,\quad f=1,\quad f_{\eta}=0 \tag{15}$$

Assuming $\alpha(t)=\frac{\rho_{nf}a\dot{a}(t)}{\mu_{nf}}$ remains constant during the flow process, and $f=f(\eta)$, then $f_{\eta\eta t}=0$ and Eq. (14) reduces to

$$f_{\eta\eta\eta\eta}+\alpha\left(\eta f_{\eta\eta\eta}+3f_{\eta\eta}\right)+Re\left(ff_{\eta\eta\eta}-f_{\eta}f_{\eta\eta}\right)=0$$
(16)

The boundary conditions remain as Eq. (15).

If α=*0* in the above Eq. (16), the Berman's model [1] for channels with stationary walls is recovered.

3. Solution Procedure

The nonlinearity in governing equation Eq. (8) makes it very difficult to develop a closed-form solution to the non-linear equation. Therefore, in this work, a spectral collocation method of the Chebyshev type is employed to solve the heat transfer equation. The Chebyshev collocation spectral method is based on the expansion of Chebyshev polynomials. At first, it expands the variable at collocation points and seeks the variable derivatives at these points, then substitutes the expansions into the differential equations and finally seeks the approximated solution in physical space. This means that Chebyshev collocation spectral method is accomplished through, starting with Chebyshev approximation for the approximate solution and generating approximations for the higher-order derivatives through successive differentiation of the approximate solution. Looking for an approximate solution, which is a global Chebyshev polynomial of degree *N* defined on the interval [-1, 1], the interval is discretized by using collocation points to define the Chebyshev nodes in [-1, 1], namely

$$x_j=cos\left(\frac{j\pi}{N}\right),\quad j=0,1,2,...N \tag{17}$$

The derivatives of the functions at the collocation points are given by:

$$f^n\left(x_j\right)=\sum_{j=0}^{N}d_{kj}^n f\left(x_j\right),\quad n=1,2. \tag{18}$$

where d_{kj}^n represents the differential matrix of order n and are given by

$$d_{kj}^1=\frac{4\gamma_j}{N}\sum_{n=0,l=0}^{N}\ \sum_{n+l=odd}^{n-1}\frac{n\gamma_n}{c_l}T_l^n\left(x_k\right)T_n\left(x_j\right),\quad k,j=0,1,...N, \tag{19a}$$

$$d_{kj}^2=\frac{2\gamma_j}{N}\sum_{n=0,l=0}^{N}\ \sum_{n+l=even}^{n-1}\frac{n\gamma_n\left(n^2-l^2\right)}{c_l}T_l^n\left(x_k\right)T_n\left(x_j\right),\quad k,j=0,1,...N, \tag{19b}$$

where $T_n\left(x_j\right)$ are the Chebyshev polynomial and coefficients γ_j and c_l are defined as:

$$\gamma_j=\begin{cases}\frac{1}{2} & j=0, or\ N\\ 1 & j=1,2,...\ N-1\end{cases} \tag{20a}$$

$$c_l=\begin{cases}2 & l=0, or\ N\\ 1 & l=1,2,...\ N-1\end{cases} \tag{20b}$$

As described above, the Chebyshev polynomials are defined on the finite interval [-1, 1]. Therefore, to apply Chebyshev spectral method to our equation (8), we make a suitable linear transformation and transform the physical domain [-1, 1] to Chebyshev computational domain [-1,1]. We sample the unknown function *w* at the Chebyshev points to obtain the data vector $w=\left[w\left(x_o\right),w\left(x_1\right),w\left(x_2\right),...w\left(x_N\right)\right]^T$.

The next step is to find a Chebyshev polynomial P of degree *N* that interpolates the data $\left(i.e.,\quad P\left(x_j\right)=w_j,\ j=0,1,...N\right)$ and obtains the spectral derivative vector w by differentiating P and evaluating at the grid points $\left(i.e.,\quad w_j^{'}=P'\left(x_j\right)=w_j,\ j=0,1,...N\right)$. This transforms the nonlinear differential equation into system nonlinear algebraic equations, which are solved by Newton's iterative method starting with an initial guess. Making a suitable transformation to map the physical domain [0, 1] to a computational domain [-1,1] to facilitate our computations.

$$f_{\eta\eta\eta\eta}+\alpha\left(\eta f_{\eta\eta\eta}+3f_{\eta\eta}\right)+Re\left(ff_{\eta\eta\eta}-f_{\eta}f_{\eta\eta}\right)=0 \tag{21}$$

With the boundary conditions

$$\tilde{f}(-1)=0,\quad \tilde{f}_{\eta\eta}(-1)=0$$

$$\tilde{f}(\eta)=1,\quad \tilde{f}_{\eta}(\eta)=0 \tag{22}$$

After applying CSCM, using Eq. (14), the governing equation and boundary conditions are transformed into a system of nonlinear algebraic equations:

$$\sum_{j=0}^{N} d_{kj}^{4}\tilde{f}(\eta_j)+\alpha\left\{\sum_{j=0}^{N}\eta d_{kj}^{3}\tilde{f}(\eta_j)+3\sum_{j=0}^{N}d_{kj}^{2}\tilde{f}(\eta_j)\right\}$$
$$+Re\left\{\sum_{j=0}^{N}\tilde{f}(\eta_j)d_{kj}^{3}\tilde{f}(\eta_j)-\sum_{j=0}^{N}d_{kj}^{1}\tilde{f}(\eta_j)\sum_{j=0}^{N}d_{kj}^{2}\tilde{f}(\eta_j)\right\}=0 \tag{23}$$

The boundary conditions are

$$\tilde{f}(x_0)=0,\quad \sum_{j=0}^{N}d_{0j}^{2}\tilde{f}(x_j)=0,$$
$$\tilde{f}(x_N)=1,\quad \sum_{j=0}^{N}d_{Nj}^{1}\tilde{\theta}(x_j)=0, \tag{24}$$

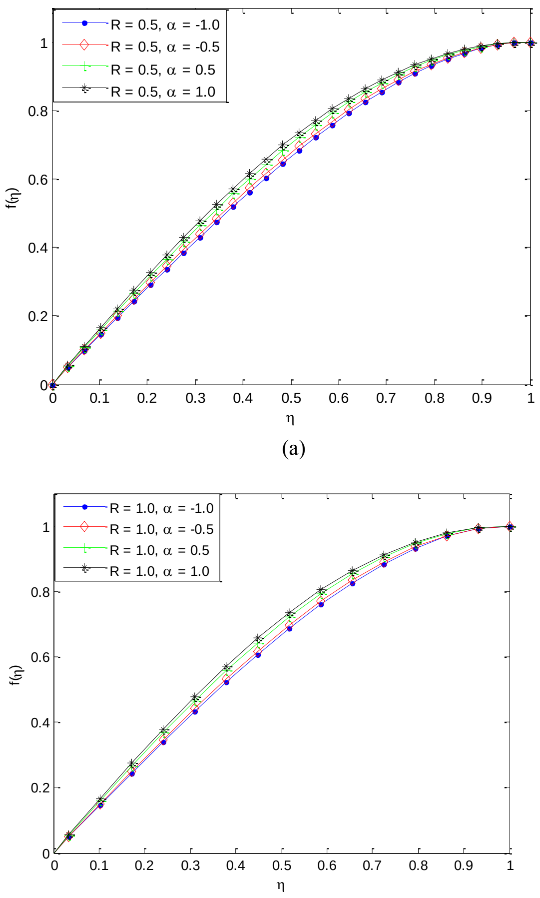
The above system of nonlinear algebraic equation is solved using Newton's method to determine the temperature distribution in the fin.

4. Results and Discussion

Table 1 shows the comparison between the results of CSCM and NM. The obtained results of velocity distributions using CSCM as compared with the numerical procedure using Runge-Kutta method coupled with shooting method are in good agreements. The high accuracy of CSCM gives high confidence about validity of the method in providing solutions to the problem.

Table 1: Comparison of results of flow for large Reynolds number and suction

$f(\eta)$	Re = 5, α=0.5		
η	NM	CSCM	Error
0.0	0.00000	0.00000	0.00000
0.1	0.15287	0.15288	0.00001
0.2	0.30155	0.30157	0.00002
0.3	0.44261	0.44262	0.00001
0.4	0.57320	0.57320	0.00000
0.5	0.69088	0.69087	0.00001
0.6	0.79337	0.79335	0.00002
0.7	0.87837	0.87836	0.00001
0.8	0.94330	0.94331	0.00001
0.9	0.98509	0.98508	0.00001
1.0	1.00000	1.000000	0.00000



(b)

Fig. 2 Variation of f (η) for different expansion and contraction ratio, α and different small values of Re

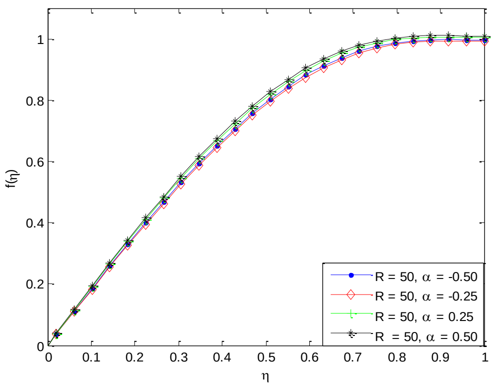
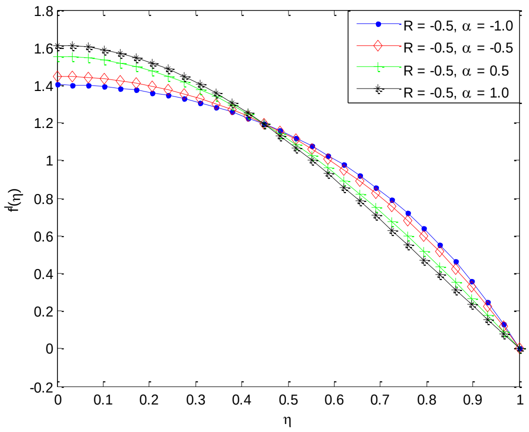
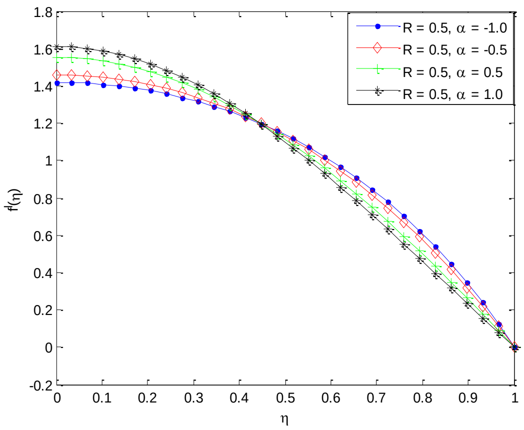


Fig. 3 Variation of f (η) for different expansion and contraction ratio, α and large value of Re

Effects of the permeation Reynolds number and non-dimensional wall dilation rate on the dimensionless flow velocities are shown in Fig. 2 and 3 while Figs. 4 and 5a and 5b show the effects of Reynolds number, Re, on the velocity at constant non-dimensional wall dilation rate on the dimensionless axial velocity. Increase in the Reynolds number decreases the axial velocity at the centre of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate.



(a)



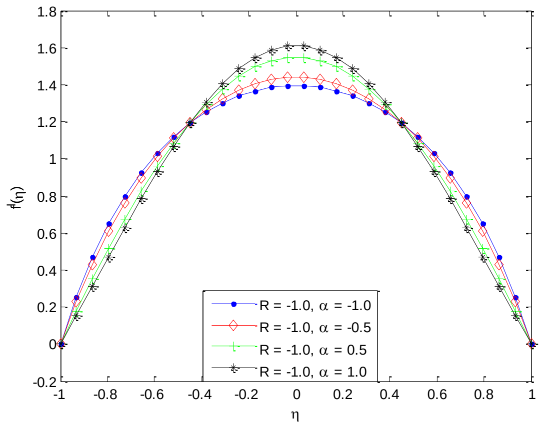
(b)

Fig. 4 Variation of f' (η) for different expansion and contraction ratio, α and different small values of Re

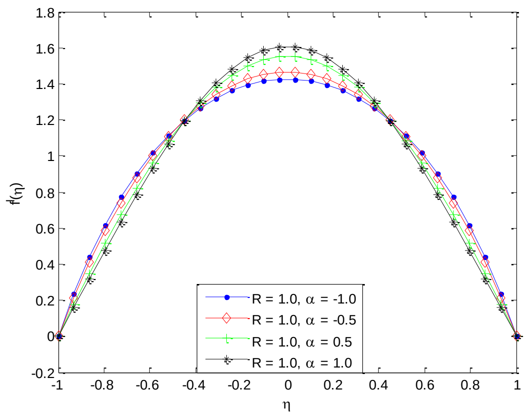
The behaviour of axial velocity for different permeation Reynolds number, over a range of non-dimensional wall dilation rate were plotted in Figs. 2-5. The figures depict that, for every level of injection or suction, the velocity is maximum at the centre of the channel and near the point, the velocity is increased when the channel is expanding and decrease when the channel contracts. As the wall expansion ratio increases, the velocity at the centre decreases and increases near the wall. Similarly, for the case of contracting wall as shown in Fig. 5a-b, increasing contraction ratio leads to lower axial velocity near the centre and the higher near the wall because the flow toward the wall becomes greater and as a result the axial velocity near the wall becomes greater. So, both the expansion and suction through the wall reinforce the flow through the channel and similarly does the wall contraction and injection through the surface. The results of the present study show that for every level of injection or suction, in the case of expanding wall, increasing *a(t)* leads to higher axial velocity near the centre and the lower axial velocity near the wall.

5. Conclusion

In this work, Chebychev spectral collocation method has been applied to analyze the unsteady two-dimensional flow of nanofluid in a porous channel through expanding or contracting walls with large injection or suction. The solutions are used to study the effects of model parameters on the flow of the nanofluid in the porous channel. The obtained results of velocity distributions using CSCM as compared with the numerical procedure using Runge-Kutta method coupled with shooting method are shown to be in good agreements. Therefore, the high accuracy of CSCM gives high confidence about validity of the method in providing solutions to the flow problems.



(a)



(b)

Fig. 5 Variation of f' (η) for different expansion and contraction ratio, α and different small values of Re

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