Evaporation is one of the largest water losses from most of the dam lakes in Iran. Estimating the evaporation rate enables us to apply the proper evaporation mitigation technologies. In this study, the feasibility of different evaporation estimation methods was studied to find an optimum method with a fair tradeoff between cost and accuracy. The optimum method may vary depending on the climate. We found Penman, Montieth and Unsworth (PMU) method as the optimum estimation method applicable Karaj dam lake (located north west of Tehran, Iran). For validation, we used the filed measurements for 2005. The reason is that the PMU is highly sensitive to wind velocity and only for 2005 the meteorological data contained the wind velocity. For the sky clarity, we used the 22-year average sky clarity of Karaj dam lake in augusts (i.e. 80%). The PMU model is found to provide consistent results with filed measurements (less than 2% error). For example, from 2nd to 15th of August 2005, the PMU model predicts 7.98 ± 0.83 mm/day evaporation and field measurement for the same period was 8.13 ± 0.01 mm/day.

Keywords: Evaporation, Penman, Montieth and Unsworth method, Karaj dam lake, wind speed, radiation.

1. Introduction

Due to the extensive exposure of surface waters to sun radiation and wind, a huge volume of their water evaporates [1]. The first step to tackle this problem is estimating the evaporation rate. By knowing the order of magnitude for evaporation, the optimum evaporation mitigation technique can be suggested.

Most of the previous studies in Iran are focused on evaporation pan [2], GIS (SEBAL) [3] and water budget [4] methods. These studies measure and do not estimate the evaporation. In [2] different evaporation methods are
evaluated and the optimum estimation method is suggested for Doosti dam lake. In the same study, it was shown that for different climates, the optimum model may be different. In this paper, factors affecting the evaporation rate are discussed. Feasibility of the different evaporation estimation methods is evaluated. An optimum estimation method applicable to Karaj (Amirkabir) dam lake climate is recommended with a fair tradeoff between cost and accuracy.

Karaj (Amirkabir) dam is located at 48 km northwest of Tehran, with the altitude of 1297 m above the sea level. The dominant wind direction is from northwest to south and its daily average is 2.2 m/s. The optimum model is used to estimate the evaporation; then verified against the available filed results.

1. METHODS

Evaporation is governed by diffusion [3] (random walk) of the water molecules at any temperature above absolute zero [4], due to the excess of water molecules at the water surface. Factors affecting the evaporation rate can be categorized into: (i) environmental, and (ii) intrinsic factors [5]. Intrinsic factors are related to the water thermal properties and impurities. In [4,6] it was shown that impurities do not have much effect on the evaporation rate throughout the year. Environmental factors are wind, and solar radiation. Wind dries the air atop the water surface and increases the evaporation rate [6]. Solar radiation absorbed by the water surface is one of the most important factors affecting the evaporation rate [7]. The total (short-wave and long wave) radiation flux absorbed by water $R_n$ (W/m²) is found as:

$$ R_n = R_s - R_L \quad (1) $$

where $R_s$ (W/m²) is the net short-wave radiation energy flux absorbed by water from the sun (Eq. 2) and $R_L$ (W/m²) is the net long-wave radiation energy flux lost from the water (Eq. 4). Note that $R_s$ is not equal to the radiation flux emitted from the sun ($\bar{R}_i$) [8] due to reflection or absorption by clouds [7]:

$$ R_s = (a_s + b_s \left( \frac{T_{clear}}{T_{day}} \right)) \bar{R}_i (1 - \alpha) \quad (2) $$

where $a_s = 0.25$ and $b_s = 0.50$ are the empirical factors; $\frac{T_{clear}}{T_{day}}$ is a correction factor for cloudy sky [9] (where $T_{clear}$ is the actual duration of clear sky during the daylight (s), and $T_{day}$ (s) is the duration of daylight from the sunrise to the sunset [9]); $\bar{R}_i$ (W/m²) is the mean of radiation flux emitted from the sun in a day which depends on longitude ($\theta$) and day of the year (Fig. 1 shows the value of $\bar{R}_i$ throughout the year for the location of Karaj dam lake estimated using the relation in [9]); and $\alpha$ is the Albedo coefficient [8] which indicates the amount of radiation reflected by the water surface.

![Fig. 1. The value of $\bar{R}_i$ in Eq. 2 throughout the year for the location of the Karaj dam (i.e. $\theta = 36.0^\circ$) is estimated using the relation in [10].](image-url)
Assuming water is pure and its surface is planar and under direct radiation, \( \alpha \) can be approximated using Fresnel Albedo coefficient as [10]:

\[
\alpha \approx 0.5 \left[ \left( \frac{\cos(e + \sin^{-1}(\frac{\cos(e)\cos(\theta)}{n_w}))}{\cos(e - \sin^{-1}(\frac{\cos(e)\cos(\theta)}{n_w}))} \right)^2 + \left( \frac{\cot(e + \sin^{-1}(\frac{\cos(e)\cos(\theta)}{n_w}))}{\cot(e - \sin^{-1}(\frac{\cos(e)\cos(\theta)}{n_w}))} \right)^2 \right]
\]

where \( e \) is the solar elevation angle measured from horizon to the sun location and is a function of longitude, latitude, date and time of the year (the value of \( e \) is calculated for the location of Karaj dam lake and shown in Fig. 2); and \( n_w \) is the refractive index of water (=4/3) [11]. In [10], it is shown that Eq. 3 is not very accurate for \( e < 10^\circ \). As shown in Fig. 2, for a major portion of the daytime for the Karaj dam lake, Eq. 3 is valid, i.e. \( e > 10^\circ \). Using Eq. 3, the average of \( \alpha \) throughout the year and for the location of Karaj dam lake is approximately equal to 0.06 (FAO\(^2\) suggests 0.05 for this location [9]).

Regarding the value of \( R_L \) in Eq. 1, the following equation is suggested by FAO [9]:

\[
\sigma (W/m^2K^4) \text{ is Stefan-Boltzmann constant (} 5.670 \times 10^{-8} W/m^2 K^4 \text{), } T_{\text{max}} (K) \text{ and } T_{\text{min}} (K) \text{ are maximum and minimum temperatures in a day, respectively; and } P_v (Pa) \text{ is water vapor pressure in moist air which is calculated by multiplying the } RH \text{ (relative humidity) and } P_{v*} \text{ (saturated water vapor pressure at the temperature of water surface).}
\]

In the following, evaporation methods are discussed and their feasibilities are evaluated.

**Evaporation Pan Methods**

The evaporation from pans can be related to the real evaporation values through some coefficients. The coefficients depend on the environmental conditions and should be measured on site, i.e. may vary from 0.66 to 1.5 [12]. Famous pans are: Colorado Sunken Pan and Class A Pan of the U.S. Weather Bureau [12]. In general, the coefficients are not very accurate and readily available. Also, many factors may impact the accuracy of the measurements [4].

**Adaptive Network-Based Fuzzy Inference System (ANFIS)**

In this method, experimental data are used to train a fuzzy-neural system and the best prediction function is found, e.g. [13]. Typical parameters to train the system are solar radiation, temperature, and moisture [14].

**Bowen Ratio Energy Balance Method**

In this method the ratio of gradient of the temperature with respect to the height over the gradient of the pressure above the water surface with respect to the height (i.e. \( \frac{\partial T}{\partial z} / \frac{\partial P}{\partial z} \) should be measured to estimate the ratio of evaporation rate to the sensible heat flux lost from the water [4]. Although this method is very accurate, it requires special measuring equipment to accurately measure temperature and vapor pressure changes [4].

\(^2\) Food and Agriculture Organization of the United Nations
Fig. 2. Elevation angle (e) versus time for the location of Karaj dam is calculated at four different days evenly distributed throughout the year using the relation in [9]. Note that the summer time (daylight saving time) adjustment is neglected.

Eddy correlation method

In this method using fast and accurate sensors, parameters like air velocity, temperature and moisture should be measured accurately. This method is very accurate, but it is relatively expensive and hard to use [4].

Area Based Methods

Using the satellite images taken from surface vegetation and according to the mining algorithms, rate of evaporation and transpiration from vegetation is calculated [4], [15]. The Surface Energy Balance Algorithm for Land (SEBAL) method is one of these methods [16]. As these methods require satellite systems and precise measurements, they can be expensive [15].

Water-Budget Method

This method is based on the conservation of mass [7]:

\[ Q_{in} + PR + \delta D = Q_{out} + S + E_{wb} \]  

(5)

where \( Q_{in} \) (mm/day) and \( Q_{out} \) (mm/day) are the inlet and outlet, \( PR \) (mm/day) is the precipitation rate, \( \delta D \) (mm/day) is the change in water surface, \( S \) (mm/day) is the rate of water seepage and \( E_{wb} \) (mm/day) is the corresponding rate of evaporation. As we cannot measure \( S \) easily, usually this relation is used to estimate the seepage, e. g. [17].

Energy-Balance Method

This method is based on the conservation of energy [7], [16], [18]–[20]:

\[ E = R_n - H - G + F_{in} - F_{out} + F_p \]

(6)

where \( E \) (W/m²) is the total evaporation rate, \( R_n \) is defined in Eq. 2, \( H \) (W/m²) is the sensible heat transfer which includes the effect of wind [21], \( G \) (W/m²) is the sum of fluxes absorbed by the ground (\( G_g \)) and stored in the water (\( G_s \)) and

\[ R_L = \sigma \left( \frac{T_{max}^4 + T_{min}^4}{2} \right) (0.34 - 0.14 \sqrt{P_v \times 10^{-3}})(1.35 \frac{a_s + b_s \left( T_{day/clear} \right)}{a_s + b_s} - 0.35) \]

(4)
\( F_{\text{in}}, F_{\text{out}} \) and \( F_p \) (W/m²) are the energy fluxes of inlet, outlet and precipitation, respectively (see Fig. 3). Assuming the balance between inlet and outlet flow rates, Eq. 6 simplifies to [7]:

\[
E = R_n - H - G
\]  

(7)

For shallow waters, the value of \( G \) can be estimated to be as a portion of \( R_n \): i.e. \( \sim 0.1 R_n \) for daytime and \( \sim 0.5 R_n \) for nighttime [9]. For deep waters, (i.e. deeper than 15 m), \( G \) can be neglected [22]. Regarding Eq. 7, it should be noted that some have also neglected the effect of wind (i.e. \( H \)) and estimated the evaporation rate with radiation, \( E \sim R_n \) [8]. Therefore, \( E \sim R_n \) is not recommended for windy situation.

\[\text{Fig. 3. Energy transfer in a water reservoir is shown.}\]

**Vapor Transmission Method**

This method neglects the radiation and only considers the effect of wind on evaporation rate. The wind speed can accelerate the evaporation as follows [23]:

\[
E_a = f(u) (1 - RH) P_v^* 
\]  

(8)

where \( E_a \) (W/m²) is the evaporation flux due to the wind and \( f(u) \) (W/Pa,m²) is the wind function that explains how wind affects the evaporation rate. Different theoretical and experimental values have been suggested for \( f(u) \), e.g [7], [18] and [24]. There is no uniformity in the literature for \( f(u) \). Also, radiation is neglected in this method. Therefore, this method is rarely used.

**Combinational Methods**

These methods combine the energy-balance (Eq. 6) and vapor transmission (Eq. 8) [20]. The result is typically a system of six equations with six unknowns and should be solved iteratively. The six equations are as follows:

\[
E = \left( \frac{\Delta}{\Delta + \gamma^*} \right) (R_n - G) + \left( \frac{\gamma^*}{\Delta + \gamma^*} \right) E_a
\]  

(9)

where \( \Delta \) is the slope of water vapor pressure against temperature (and is equal to 188.43 (Pa/K) at 25°C [9]) and \( \gamma^* \) is modified Psychrometric constant by Montieth [7], and is a function of air pressure (\( \gamma^* \equiv 55.68 \) (Pa/K) at 1 atm [9]). It should be noted that Eq. 9 was originally derived to estimate the evaporation resistance of a leaf [25]. It should be noted that for including the effect of any resisting layer on the water surface a correction factor should be used in Eq. 9 [25].

To solve Eq. 9, the value of \( E_a \) is needed. The value of \( E_a \) depends on the wind velocity (see Eq. 8). The following equation is an estimation to \( E_a \) [12]:

\[
E_a = K u_* \rho_a (q_{a^*} - q_a) \left[ \ln \left( \frac{z - d_{0v}}{z_{0v}} \right) \right.
\]

\[
- \left. \frac{1}{\psi_v} \left( \frac{z - d_{0v}}{L} \right) \right]^{-1}
\]  

(10)

where \( K = 0.41 \) is the von Kármán’s constant [26], \( u_* \) (m/s) is the friction velocity, \( \rho_a \) (kg/m³) is the air density, \( q_{a^*} \) and \( q_a \) are the saturated specific humidity³ and the specific humidity [12], \( z \) (m) is height at which wind velocity is measured, \( d_{0v} \) (m) is the displacement height for water vapor (height at which the mean velocity is zero [27]), \( z_{0v} \) (m) is the vapor density. But relative humidity is the ratio of water vapor pressure to saturated water vapor pressure.

³ Note that the specific humidity is different from relative humidity, as it is defined as the ratio of water vapor density to the mixed air
roughness height ~0.02 cm for water surface [9]. $\Psi_e$ is the similarity stability correction function of the vapor transmission [28], and $L$ is the Obukhov stability length described as [12]:

$$L = \frac{-u_*^3}{K \cdot g}\left[\frac{H}{\rho_a c_a T_a} + \frac{0.61 \mathcal{E}}{\rho_a}\right]$$

(11)

where $g$ (m/s$^2$) is the gravitational acceleration, $c_a$ (J/kgK) is the specific heat of air at constant pressure, and $T_a$ is the air temperature in Kelvin. Equation 12 relates the average horizontal wind speed $\bar{U}_{wind,x}$ (m/s) to $u_*$ as [28]:

$$\bar{U}_{wind,x} = \frac{u_*}{K} \left[\ln\left(\frac{Z - d_{0m}}{z_0}\right) - \Psi_m\left(\frac{Z - d_{0m}}{L}\right)\right]$$

(12)

where $\bar{U}_{wind,x}$ (m/s) is the mean wind speed at the measurement height $Z$ (m), $d_{0m}$ (m) is the momentum displacement length, $z_0$ (m) is the momentum roughness height (~0.002 cm for water surface [29]) and $\Psi_m$ is the similarity stability correction function of the momentum transmission [28]. Further investigation shows that the value of $E$ is not sensitive to $d_{0m}$ and $d_{0v}$ [28]. As such, Katul and Parlange [28] assumed zero for $d_{0m}$ and $d_{0v}$. $\Psi_e$ and $\Psi_m$ are functions of $\frac{Z}{z_0}$ [28]. $z_0$ and $z_{0v}$ can be estimated by Monin-Obukhov similarity [30][12]:

$$z_0 = \frac{u_*^2}{794.6}$$

(13)

$$z_{0v} = 7.4 \cdot z_0 e^{(-2.25\frac{z_0 u_*}{\nu})^1}$$

(14)

where $\nu$ (m$^2$/s) is the kinematic viscosity of water. The above combination method is known as Penman-Brutsaert (PB) [28]. In PB method, by simplifying the above six equations (Eqs. 9 to 14), a system of five equation with five unknowns (i.e. $E$, $H$, $u_*$, $L$ and $E_a$) will be derived. The PB method inputs are: $\bar{U}_{wind,x}$, $R_n$, and $G$. The five equations should be solved iteratively and usually within five to six iterations a solution with an accuracy of 0.1 W/m$^2$ is derived [28].

Another way of doing the combinational methods is done by Penman-Monteith-Unsworth (PMU). The PMU method neglects the $G$ in Eq. 9 and uses Eq. 15 instead of Eq. 10 [18]:

$$E_a = \frac{\rho_a c_a K^2 \bar{U}_{wind,x}}{\gamma \ln\left(\frac{Z - d_0}{z_0}\right) \ln\left(\frac{Z - d_{0v}}{z_{0v}}\right)} \left(1 - RH\right) P_v^*$$

(15)

As can be seen above, the PMU and PB methods are on the same basis. In [18], it is shown that the PB and PMU provide very close results. The main difference between PMU and PB is that PMU neglects the $G$. Therefore, as discussed in [9] [18] [31] [32] PMU is more accurate for long periods (e.g. weekly tests or longer). For shorter periods (e.g. hourly or daily tests), the PB model is recommended [26][29].

To use the combinational methods, only a few simple meteorological data are needed; i.e. relative humidity, air pressure, average wind speed, air temperature, and energy fluxes. The energy flux is approximately equal to the radiation. For clear sky, radiation can be calculated as a function of the location, date and time. Calculating the sky clarity for the desired location on earth requires accessing satellite images or using the NASA Surface meteorology and Solar Energy website which is an open access website [34].

### 2. Results and Discussion

As discussed above the combinational methods satisfy both the feasibility and accuracy conditions. Therefore, the combinational methods are suggested for calculating the evaporation rate from dam lakes or any locations where meteorological data are available and without having to setup

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Note that roughness in Monin-Obukhov similarity method is a distance used to extrapolate some property with log-profile relationship with its value at surface [35].
new instruments. We applied the combinational methods (PB and PMU) to calculate the evaporation rate from Karaj dam lake. The only year that wind velocity was recorded for the location was 2005. This should not be any concern as our goal is only to validate the applicability of the suggested models.

To perform an error analysis, we considered the following errors for relevant parameters: $\delta T_a = 2^\circ\text{C}, \delta RH = 5\%$, $\delta U_{\text{wind}} = 0.3\text{ m/s}$, $\delta\text{ clarity} = 10\%$, $\delta z = 1.0\text{ mm}$, $\delta \theta = 0.1^\circ$, $\delta J = 1\text{ day}$, $\delta z_0 = 0.002\text{ mm}$, $\delta z_{0v} = 0.0002\text{ mm}$, and $\delta \alpha_F = 0.01$, respectively; where $\delta$ indicates the associated error of each parameter, and $J$ is the day of the year. The error analysis shows 0.83 (mm/day) uncertainty in estimation of evaporation rate using PMU method.

Table 1 shows the 14-day and 21-day evaporation rates from Karaj dam lake in August 2005. Evaporations are estimated using PMU. The wind velocity, air temperature, and relative humidity are taken from the local weather station. For the value of sky clarity, 80% was used which is the 22-year average value for the location of Karaj dam lake and for the month of August [34].

<table>
<thead>
<tr>
<th>Period</th>
<th>Experiment</th>
<th>PMU Method</th>
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<tbody>
<tr>
<td></td>
<td>Evaporation (mm/day)</td>
<td>Evaporation (mm/day)</td>
</tr>
<tr>
<td>2nd to 15th of Aug 2005</td>
<td>8.13 ± 0.01</td>
<td>7.98 ± 0.83</td>
</tr>
<tr>
<td>2nd to 22nd of Aug 2005</td>
<td>8.07 ± 0.01</td>
<td>7.92 ± 0.83</td>
</tr>
</tbody>
</table>

According to Table 1, the dominant factor on the evaporation rate is radiation but as will be shown, evaporation variation during the day is usually due to the fluctuation of wind speed. In Fig. 6, the daily evaporation rate calculated using the PMU method is compared with the evaporation rate measured using evaporation pan method (experimental results are provided by Tehran Regional Water Authority).

![Fig. 4. Experimental (solid lines) and PMU (dashed lines) evaporation rates are shown for the timeframe of June 21st to September 21st 2005. Experimental results are provided by Tehran Regional Water Authority. Regarding the PMU, sky clarity, Fresnel Albedo, relative humidity, wind speed, radiation intensity, location, date and time of the year, and temperatures (maximum and minimum) in a day are the inputs.](image-url)
As shown in Fig. 4, the PMU is not very accurate for calculating the daily evaporation rates. To estimate the daily evaporations, PB is the better option. However, it requires more data points during the day [28]. The spikes in the PMU results in Fig. 4 are due to error in recording the wind speed. As shown in Fig. A1 evaporation rate is sensitive to the wind speed, temperature and sky clarity. So, any sharp change and error in any of these parameters may lead to a huge deviation from the exact rate. As shown in Fig. 5, the spikes are mainly due to error in reading the wind speed ($\overline{U_{\text{wind}}}$). In other words, when wind speed changes are sharp, we cannot use PMU method for calculating the daily evaporation [18].

As shown in Figs. 4 and 5 (and further explained in the Appendix), the PMU model is highly sensitive to the inputs. Therefore, the PMU easily reflects the noise in the inputs. The sharp spikes and noises in the evaporation rates even out and PMU yields accurate results when the studied period becomes longer. Comparing the daily (Fig. 4), bi-weekly (Fig. 6a) and tri-weekly (Fig. 6b) periods, the PMU is more accurate for estimating the evaporation rate of tri-weekly periods. Regarding Fig. 6 it should be noted that the PMU results and experimental measurements are compared for the timeframe of June 21st to September 12th 2005.

As shown in Fig. 6, the average daily evaporation from Karaj dam lake and during summers is approximately 8.0 mm/day. Considering the area of the dam (764 km$^2$), daily evaporation during summers is more than 6 million cubic meters. As discussed in Table 1 more than 70% of evaporation is due to radiation. Therefore, the optimum evaporation suppression technique should minimize the radiation effects.

3. CONCLUSION

Different methods of estimating the evaporation from water reservoirs are explained. Some methods (e.g. Eddy correlation and Bowen’s ratio) are potentially accurate. However, they require expensive equipment and installations. The combinational methods (PB and PMU) are relatively accurate and require only relative humidity, wind speed, air temperature, and energy fluxes. The first three values can be obtained from a local weather station. Energy flux can be approximated by the radiation. Radiation can be easily calculated as a function of the lake location, time and day of the year, and sky clarity. Sky clarity can be found using satellite images. The PMU which is more accurate for periods of several days is used to calculate the evaporation rate from the Karaj dam lake. It was found that the daily average evaporation from the Karaj dam (located at northwest of Tehran) during the summer is approximately 8.0 mm/day.
This evaporation rate is equivalent to more than 6 million cubic meters of water loss per day. It was also found that more than 70% of this evaporation is due to radiation. As such the optimum evaporation mitigation technique should control the radiation.

4. ACKNOWLEDGMENT

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5. REFERENCES


APPENDIX

As shown in Eq. 6, evaporation rate is a function of radiation \( R_n \). Radiation depends on the followings: sky clarity, location, and date and time of the year. As shown in Fig. A1, evaporation rate is also sensitive to air temperature \( T_a \), relative humidity \( R_H \), and wind velocity \( \bar{U}_{\text{wind/\%}} \).

Regarding Fig. A1c, it should be noted that the PMU method is not recommended for high relative humidity values.

A similar sensitivity analysis is performed for \( z \) and it was found that the evaporation rate is not sensitive to that (see Fig. A2a). The sensitivity of evaporation rate to \( z_0 \) (and \( z_{0V} \) as they are related to each other \[9\]) values between 0.001 and 0.003 (which is the typical range \[29\]) and for high wind velocities evaporation rate is slightly sensitive to \( z_0 \).

Fig. A1. Evaporation rate (mm/day) for the location of Karaj dam lake and in August 1st is shown for different (a) sky clarity \( \frac{T_{\text{day/clere}}}{T_{\text{day}}} \) where \( R_H = 20\% \), and \( T_a = 25^\circ \text{C} \); (b) air temperature where sky clarity = 100% and \( R_H = 20\% \); (c) \( R_H \) where sky clarity = 100% and \( T_a = 25^\circ \text{C} \); (d) wind velocity where sky clarity = 100% and \( T_a = 25^\circ \text{C} \).
$25 \, ^\circ C$; and (d) wind speed where clarity = 100%, $T_a = 25 \, ^\circ C$, and $RH = 20\%$. For all of the cases $z = 10 \, m$, $z_0 = 0.002 \, cm$, $z_{0v} = 0.0002 \, cm$.

Fig. A2. Evaporation rate (mm/day) for the location of Karaj dam lake and in August 1st is shown at different (a) measurement height where $z_0 = 0.002 \, cm$, $z_{0v} = 0.0002 \, cm$ and (b) momentum roughness height where $z = 10 \, m$. For both cases $RH = 20\%$, sky clarity = 100\%, and $T_a = 25 \, ^\circ C$. 

150