JCAMECH

Vol. 47, No. 2, December 2016, pp 209-218
DOI: 10.22059/jcamech.2017.222578.122

Unsteady Magneto Hydro Dynamic Flow of a Second Order Fluid over an Oscillating Sheet with a Second Order Slip Flow Model

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Received: 12 Sep. 2016 , Accepted: 20 Nov. 2016

Abstract

Unsteady slip-flow of second grade non-Newtonian electrically conducting fluid over an oscillating sheet has been considered and solved numerically. A second-order slip velocity model is used to predict the flow characteristic past the wall. With the assumption of infinite length in x-direction, velocity of the fluid can be assumed as a function of y and t, hence, with proper variable change partial governing equations are converted to ordinary differential equations, and resulting equations are solved numerically. Fourth-order finite difference scheme is used to solve the transformed governing equations. The effects of magnetic field applied on surface, slip flow parameters, frequency of oscillating, mass suction or injection and elastic second number on the velocity distribution are shown graphically and discussed. With increase of slip flow parameter, unlike that of other parameters, thickness of the fluid affected by motion of boundary will decrease. It is also realized that both injection and suction of mass on the sheet, will increase amplitude of velocity.

Keywords: *Unsteady Flow; MHD flow; Second order fluid; Slip parameters*

1. Introduction and literature survey

The flow on stretching surface has found many applications in a number of process technologies including polymer processing industry. The classical problem has especially been used in manufacturing process of artificial film, artificial fibers, polymer extrusion, drawing of plastic films and wires, glass fiber and paper production, manufacture of foods, crystal growing, liquid film in condensation process, etc. [1,2,3]. Due to its applications, stretching flow has gained considerable interest in literature. The flow in the boundary layer of an incompressible viscous fluid, on the stretching surface with constant speed, has been investigated by Sakiadis[4,5]. Extensions of this problem were reported by many investigators [6-13].

Magneto hydro dynamics (MHD) of an electrically conducting fluid is encountered in many problems such as in engineering and industrial applications. Since the flow can be regulated by external means through a magnetic field, an electrically conducting polymeric liquid can prove highly advantageous in such industrial applications as polymer technology and metallurgy.

Nomenclature

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Alphabetic symbols

A1	slip parameter of Wu's model
B	strength of constant magnetic field ($N/m.A$)
B1	slip parameter of Wu's model
G	dimensionless pressure ($\rho U_c^2/P_c$)
Ha	Hartman number ($\sigma_o B^2 L_c/\rho U_c$)
Kn	Knudsen number (λ/L)
ko	elastic second order number ($\alpha/\rho L_c^2$)
K1	dimensionless elastic second order number ($m_1/\rho L_c^2$)
K2	dimensionless elastic second order number ($m_2/\rho L_c^2$)
l	$\min(1/Kn, 1)$
Lc	characteristic length (m)
m1	non-Newtonian second order constant
m2	non-Newtonian second order constant
N	Number of nodes
P	pressure (N/m^2)
Pc	characteristic pressure (N/m^2)
Re	Reynolds number
t	Time (s)
t_{max}^*	Maximum time choose for numerical calculation (s)
Uc	characteristic velocity (m/s)
U∞	infinite velocity (m/s)
v	y-velocity(m/s)
vo	injection or suction velocity (m/s)
x	distance (m)
x*	dimensionless distance
y	distance (m)
y*	dimensionless distance

Greek Symbols

α	thermal diffusivity(m^2/s)
α_1	dimensionless first slip coefficient(B_1/L_c^2)
α_2	dimensionless second slip coefficient(B_2/L_c^2)
ζ	Womersly number($\omega L_c/U_c$)
λ	Molecular mean free path
μ	fluid viscosity(Pa/s)
ρ	fluid density(Kg/m^3)
σ_o	Electrical conductivity (A^2s^3/m^3kg)
ω	Angular frequency (rad/s)

Acronyms

MEMS	Micro-Electro-Mechanical Systems
MHD	Magneto hydro dynamics

Many metallurgical processes involve the cooling of continuous strips or filament by drawing, these strips is sometimes stretched. Another interesting application of hydro-magnetics to metallurgy lies in purification by the application of a magnetic field. So, the MHD flow over stretching sheet has been the focus of attention of many researchers [14-18].

Non-Newtonian fluids are widely used in industries, and many materials; such as drilling mud, polymer solutions, emulsions, grease and certain oil are classified as non-Newtonian fluids. The stretching sheet problem was also extended to non-Newtonian fluids [18-23]. Given the complexity of non-newtonan fluids, it is very difficult, if not impossible, to integrate all properties of such fluid into a single model. This has led most authors' attention toward proposing empirical and semi empirical models. non-newtoninan fluids are generally classified into two classes; namely the different-type fluids and rate-type fluid, among which, the differential type fluids have received more attention from researches in the past decades. Moreover, due to its relative mathematical simplicity, the second order slip flow has proven to be more acceptable than the third and fourth order fluids in this subclass of non-Newtonian fluids [24].

In the recent decades, a micro scale fluid flow in Micro-Electro-Mechanical Systems (MEMS) has become a popular research topic. Flow behavior in micro scale deviate from the traditional flows and belong to slip flow. In this case, the fluid's molecular structure becomes more significant and the continuum assumption is no longer valid. Fluid motion in slip flow regime, obeys Navier-Stokes equations with slip velocity and temperature jump at the fluid-solid interface [25]. However, the no-slip situation is insufficient in a number of cases where the fluid is particulate [3]. Emulsions, suspensions, foams and polymer solutions are among such fluids, which are frequently applied in technology such as in polishing of artificial heart valves and internal cavities [26]. In problem of stretching surface, the assumption of no-slip, in certain situations, is not applicable and slip boundary conditions should be used. Non-Newtonian fluids such as polymer melts exhibit wall slip condition [22, 27].

The main objective of the current paper is to investigate the influences of slip boundary condition and magnetic field on velocity distribution of non-Newtonian fluid over an oscillating sheet which is not investigated in the previous studies. Stokes second problem of oscillating surface [28] is a simplified of this paper.

2. Mathematical formulation

In this paper, two dimensional unsteady MHD slip flow of non-Newtonian fluid of second grade over a continuously oscillating surface in a quiescent fluid has been investigated. The surface (as shown in figure 1) at t=0 oscillates with the velocity Uw(t) along the x-axis. Since the boundary at y=0 is oscillating in time, it is predictable that fluid will also oscillate in the x direction in time. By comparison to Stokes second problem [28], the flow is governed by the following equations:

From the continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

It can be assumed that when the sheet start to oscillation at t>0, due to infinite length of the sheet, all fluid particles will oscillate in horizontal direction similarly. This assumption will lead to simplified continuity equation as follow:

$$\frac{\partial}{\partial x}=0\xrightarrow{(1)}\frac{\partial v}{\partial y}=0\rightarrow v=v(x,\mathrm{t}) \tag{1-a}$$

Therefore the velocity of fluid is only a function of time and horizontal distance (t and x). Applying constant injection or suction boundary condition on the surface, the fluid velocity in y direction will be as below equation:

$$@\ y=0\ \rightarrow v_{y=0}=v_0=cte \tag{1-b}$$

Where is suction or injection velocity at wall and assumed to be constant. Now, momentum equation in horizontal and vertical directions will be as following [29]:

X-momentum:

$$\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+ \frac{m_1}{\rho}\left(\frac{\partial^3 u}{\partial t\partial y^2}+v_0\frac{\partial^3 u}{\partial y^3}\right)-\frac{\sigma_o B^2}{\rho}u \tag{2}$$

Y-momentum:

$$-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{m_1}{\rho}\left(4\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)+ \frac{m_2}{\rho}\left(2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right)=0 \tag{3}$$

Subject to the boundary conditions [29]:

$$u(0,t)=U_\infty\cos(\omega t)+ \frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha}-\frac{3}{2}\frac{l-l^2}{Kn}\right)\lambda\frac{\partial u(0,t)}{\partial y} -\frac{1}{4}\left(l^4+\frac{2}{Kn^2}(1-l^2)\right)\lambda^2\frac{\partial^2 u(0,t)}{\partial y^2}= U_\infty\cos(\omega t)+A_1\frac{\partial u(0,t)}{\partial y}+ B_1\frac{\partial^2 u(0,t)}{\partial y^2}\qquad t>0 \tag{4}$$

$$u(y,t)\rightarrow 0\ as\ y\rightarrow\infty\ and\ u(y,0)=U(y) \tag{5}$$

In the above equations, u is the velocity in x direction, v is velocity in y direction, and are non-Newtonian second grade fluid constant, B is strength of constant magnetic field applied perpendicular to the sheet and is electrical conductivity of the fluid. The Wu's slip velocity model is used in current paper (equation 4), which is valid for arbitrary Knudsen number (Kn) [30]. In Wu's slip model, $l=\min(\frac{1}{Kn},1)$, α is the momentum accommodation coefficient and λ is the molecular mean free path. According to definition of l, and , it is seen that and [31]. A1 and B1 are two slip parameters of Wu's model.

Since the boundary at y=0 is oscillating in time, it is predictable that fluid will also oscillate in the x direction in time. However, it is to be expected that the amplitude of motion and the phase shift relative to the motion of the boundary will depend upon on y. Thus, one can assume the velocity of fluid as below [28]:

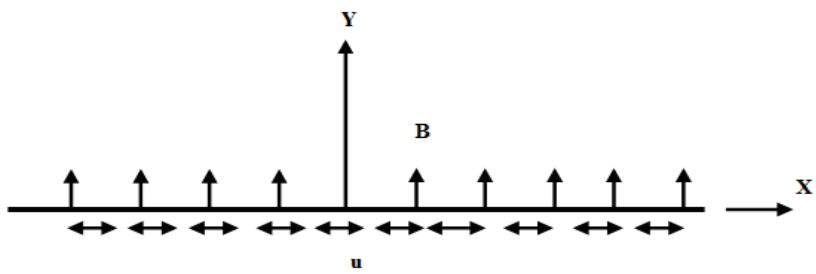

Figure 1 schematic of an oscillating sheet in magnetic field

$$u(y,t)=f(y)\cos(\omega t)+g(y)\sin(\omega t) \quad (6)$$

Partial differential equation x-momentum of (2) can be transformed to ordinary differential equation by substituting assuming velocity of (6) in (2), as follows:

X-momentum:

$$-\omega f+v_0\frac{dg}{dy}-\frac{\mu}{\rho}\frac{d^2g}{dy^2}-\frac{m_1}{\rho}(v_0\frac{d^3g}{dy^3}-\omega\frac{d^2f}{dy^2})+\frac{\sigma_o B^2}{\rho}g=0 \quad (7)$$

$$\omega g+v_0\frac{df}{dy}-\frac{\mu}{\rho}\frac{d^2f}{dy^2}-\frac{m_1}{\rho}(v_0\frac{d^3f}{dy^3}+\omega\frac{d^2g}{dy^2})+\frac{\sigma_o B^2}{\rho}f=0 \quad (8)$$

U_c, L_c are used as a characteristic velocity and length respectively, to rewrite equations in non-dimensional form as follows:

$$u^*(y^*,t^*)=f^*(y^*)\cos(\zeta \operatorname{Re} t^*)+g^*(y^*)\sin(\zeta \operatorname{Re} t^*) \quad (9)$$

$$-\zeta f^*+v_0^*\frac{dg^*}{dy^*}-\frac{1}{\operatorname{Re}}\frac{d^2g^*}{dy^{*2}}-k_0(v_0^*\frac{d^3g^*}{dy^{*3}}-\zeta\frac{d^2f^*}{dy^{*2}})+Ha\,g^*=0 \quad (10)$$

$$\zeta g^*+v_0^*\frac{df^*}{dy^*}-\frac{1}{\operatorname{Re}}\frac{d^2f^*}{dy^{*2}}-k_0(v_0^*\frac{d^3f^*}{dy^{*3}}-\zeta\frac{d^2g^*}{dy^{*2}})+Ha\,f^*=0 \quad (11)$$

In the above equations $y^*=y/L_c$, $t^*=t/(\rho L_c^2/\mu)$ is dimensionless form of y and t and u*, f* and g* are dimensionless form of u, f and g respectively. Also, $v_0^*=v_0/U_c$ is dimensionless suction or injection velocity, $\zeta=\omega L_c/U_c$ is the Womersly number for oscillating behavior, $\operatorname{Re}=\rho U_c L_c/\mu$ is the Reynolds number, $k_o=\alpha/\rho L_c{}^2$ is elastic second order number for elastic properties of fluid and $Ha=\sigma_o B^2 L_c/\rho U_c$ is Hartmann number for magnetic property of fluid.

Similar to the governing equations, the boundary conditions can be transformed as follows:

$$f^*(0)=U_\infty^*+\alpha_1\frac{df^*(0)}{dy^*}+\alpha_2\frac{d^2f^*(0)}{dy^{*2}} \quad (12)$$

$$g^*(0)=\alpha_1\frac{dg^*(0)}{dy^*}+\alpha_2\frac{d^2g^*(0)}{dy^{*2}} \quad (13)$$

$$g^*(\infty)=f^*(\infty)=0 \quad (14)$$

$\alpha_1=A_1/L_c$, $\alpha_2=B_1/L_c{}^2$ are dimensionless first and second order slip coefficients for fluid velocity in wall. In present work, $U_c=U_\infty$ is used for convenient.

After solving x-momentum equation, u is obtained. By substituting u in y-momentum equation, the distribution of pressure can be obtained. y-momentum equation in dimensionless form is as follow:

$$0=-\frac{\partial p^*}{\partial y^*}+4K_1G\frac{\partial u^*}{\partial y^*}\frac{\partial^2u^*}{\partial y^{*2}}+2K_2G\frac{\partial u^*}{\partial y^*}\frac{\partial^2u^*}{\partial y^{*2}} \quad (15)$$

Where, $G=\rho U_c^2/P_c$ is dimensionless pressure (P_c is characteristic pressure), $K_1=m_1/\rho L_c^2$ and $K_2=m_2/\rho L_c^2$ are dimensionless elastic second grade numbers in second order fluid model. Integrating of equation (15) respect to y yields to:

$$p^*=(2K_1+K_2)G\left(\frac{\partial u^*}{\partial y^*}\right)^2+h(t^*) \quad (16)$$

Finally, h(t*) may be evaluated by applying boundary condition in large enough distance(infinity), as follows:

$$at\ \ y\to\infty: p(y^*,t^*)=p_0^*\to p^*=(2K_1+K_2)G\left(\frac{\partial u^*}{\partial y^*}-\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*\to\infty}\right)^2+p_0^* \quad (17)$$

In equations (16) and (17) $p_0^* = 0$ (dimensionless reference pressure) and $G = 1$ are considered to be solved numerically.

3. Numerical procedure and validation

A finite difference fourth order scheme is used to cover the third order derivatives existing in the ODEs of equations 7 and 8. Also, due to the existence of third order derivatives, forward second order procedure has been used in wall, backward second order procedure has been used in places sufficiently out of the boundary layer (infinity), and a combination of central and backward fourth order procedure has been used in other places. This helps the stability of the solution, due to dissipative nature of fourth order scheme in interior domain and second order scheme in boundary. in other words, using second order scheme in boundary and fourth order scheme in computation domain are numerical tricks for damping unstable effect of third order derivative (which exists in governing equations) of solution fields. Furthermore, it is important to note that using a combination of central and backward finite difference procedures increases the effect of periodic boundary condition for wall in solution field.

Verification has been done by extracting our numerical results of this scheme for computing unsteady velocity profile in Newtonian fluid obtained by setting zero non-Newtonian parameters (i.e. ko & Ha), no suction or injection and also no-slip boundary condition in numerical code. Exact agreement has been observed between this results and unsteady velocity profile for stokes second problem for Newtonian fluids. Numerical results showed that 250 equal distance nodes in the domain will satisfy the grid independency for all cases. The result of grid independency is shown in figure 2 for the horizontal velocity component at the half of the domain of numerical solution (ymax/2) and maximum time of calculation (tmax).

4. Results and discussions

By using a fourth order finite difference scheme, governing equations (10) and (11) are solved numerically subject to the boundary conditions (12)-(14). In present work we use $\zeta = 1, \mathrm{Re} = 10, k_o = 1, Ha = 1, v_0 = 5, \alpha_1 = 1, \alpha_2 = -1$ as local values for dimensionless parameters. Furthermore, $t^*_{\max}$ is used to represent the maximum time chosen for numerical calculation. Since the governing equations are first order in time, an initial condition is needed at (t=0), so choosing maximum time is optional and depends on dimensionless time given to the problem in order to study unsteady

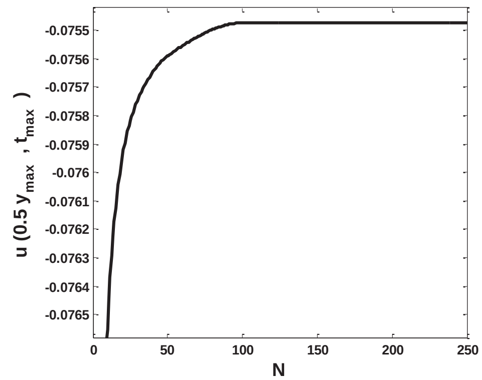

Figure 2 Axial velocity at specific spatial point (ymax/2) at maximum time (tmax) as a function of number of computational point.

behavior. In the present study, $t^*_{\max} = 6$ is used to in the numerical calculation. Figure 3 show the velocity of oscillating surface along the y-coordinate in time. The amplitude of oscillating fluid reaches its maximum value at y=0 and decrease quickly as y increase. In Figure 4, velocity at $t^* = t^*_{\max}/2$, f and g are plotted in different Hartmann number, while other parameters are fixed at constant values (shown in figure). As the value of Hartmann number increased in magnitude, the amplitude of oscillating fluid gradually increased and the fluid velocity decreased [18] and also fluid was distorted in y-direction, due to increase in the magnetic force in x-momentum equations. Fluid near the wall of sheet is majorly influenced by oscillating surface; therefore, the distance away from the oscillating sheet within which the fluid is influenced by the motion of the boundary is increased, thus fluid reached quiescent at higher distance from the surface.

Figure 5 shows variations of velocity at $t^* = t^*_{\max}/2$, f* and g* of fluid in a different Womersly number while the other parameters kept constant. It is seen from the figure that, by increasing Womersly number, the amplitude of oscillating fluid in y=0 is decreased, and so is the distance away from the moving boundary within which the fluid is influenced by the motion of the boundary. Note that, this behavior also depends on the other values which are used.

Figure 6 represents variations of velocity at $t^* = t^*_{\max}/2$, f* and g* in a different k0 while the other parameters kept constant. As the elastic second order number of fluid k0 increases, the amplitude of velocity and also the thickness of affected fluid by the motion of boundary are increased. Increasing the elastic second order number; which results in the increase of fluid elastic force due to normal stress, causes the further displacement of elements of the fluid in horizontal direction; i.e. increase in velocity.

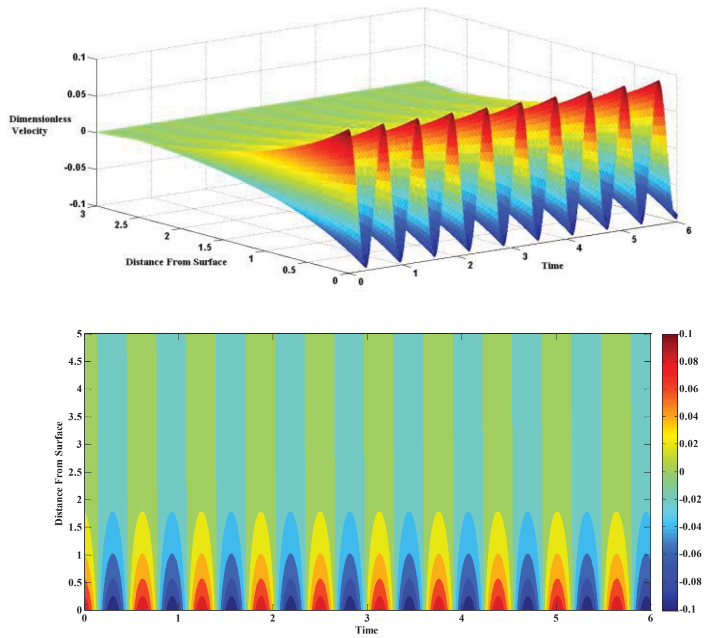

Figure 3 velocity along time and distance over the oscillating surface in the constant values of $\zeta = 1, \mathrm{Re} = 10, k_o = 1, Ha = 1, v_0 = 5, \alpha_1 = 1, \alpha_2 = -1$

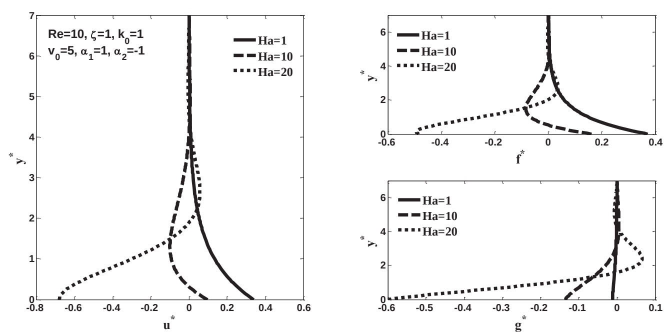

Figure 4 variations of velocity at $t^* = t^*_{max} / 2$, f^* and g^* for different Hartmann number

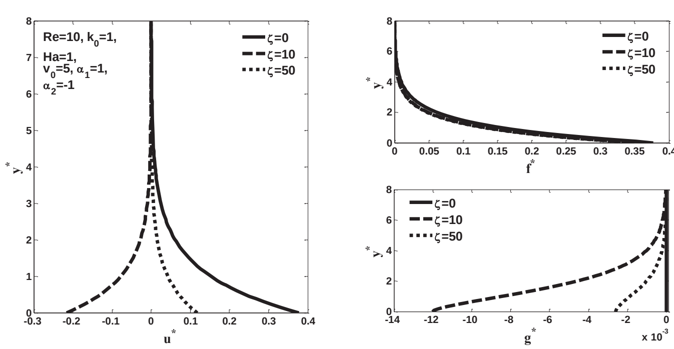

Figure 5 variations of veocity at $t^* = t^*_{max} / 2$, f^* and g^* for different Womersly number

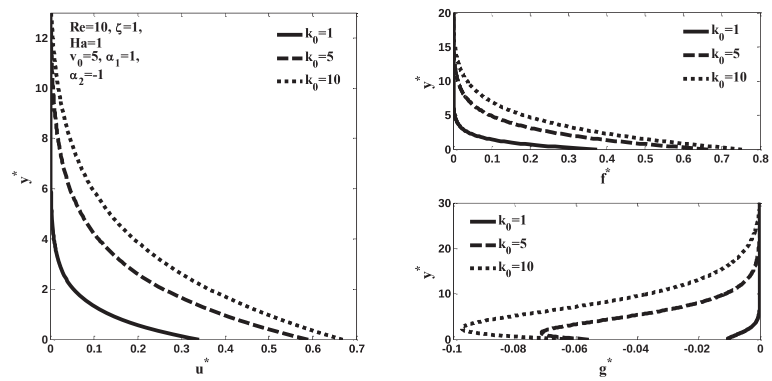

Figure 6 variations of velocity at $t^* = t^*_{\max} / 2$, f* and g* with y in different k_0

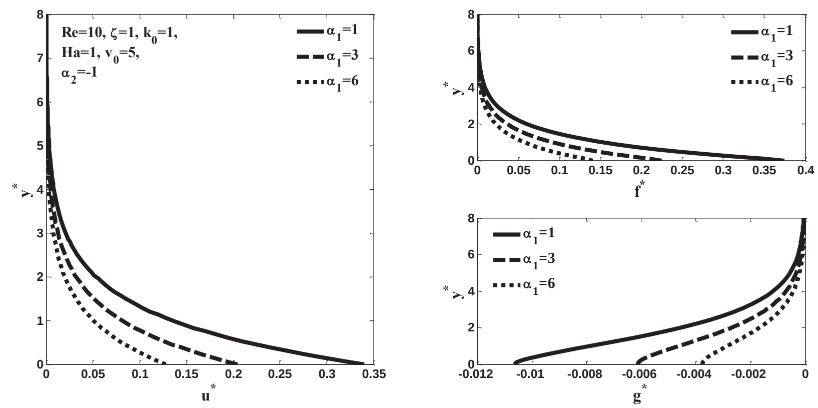

Figure 7 variations of velocity $t^* = t^*_{\max} / 2$, f* and g* with y in different α_1

In Figure 7 the values of velocity at $t^* = t^*_{\max} / 2$, f* and g* are plotted for various values of slip parameter α1 while other parameters are fixed at constant values (shown in figure). It is observed that as slip parameters increase in magnitude, more fluid is permitted to slip past the surface and also the thickness of the fluid affected by motion of boundary decreases. It is also seen that with the increase in slip parameter in this figure, velocity magnitude in the boundary layer decrease which in in agree with the results of Fang and Lee [13]. It can be observed that maximum shear stress has occurred in the surface of the wall [13, 27] .

Figure 8 shows variations of velocity at $t^* = t^*_{\max} / 2$, f* and g* with y in different slip parameters α2 with the other parameter kept constant. From this figure, fluid is permitted to slip more past the sheet and fluid velocity decreased as α2 is increased [14, 31]; also thickness of the fluid which is affected by motion of the boundary is decreased similar to that of figure 7.

Variation of velocity at $t^* = t^*_{\max} / 2$ and also f* and g* functions with different v0 velocities (suction and injection on the surface) are plotted in figure 9. It is seen from this figure that, with both injection and suction on surface, amplitude of velocity increases relative to that of no suction and injection applied on sheet [27]. The influence of Injection of mass in amplitude of velocity is greater than the suction of mass on oscillating surface. Note that, horizontal component of velocity will increase by increasing the suction or injection velocity values, due to a rise in horizontal inertia force which is implied in fluid's elements [31]. This force can have an effect similar to shear stress effect in fluid control volume. So by increasing the suction or injection values, fluid elements can be more tensile and have better displacements. This effect is meant to increase in values of horizontal velocity component.

Figure 10 shows the pressure over on oscillating surface along the y-coordinate in time. The amplitude of oscillating pressure over the surface has its maximum value at y=0 and decreases rapidly as y increases.

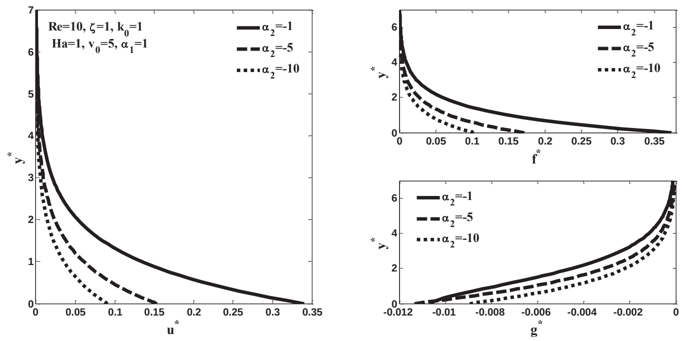

Figure 8 variations of velocity at $t^* = t^*_{max}/2$, f^* and g^* with y in different slip parameter α_2

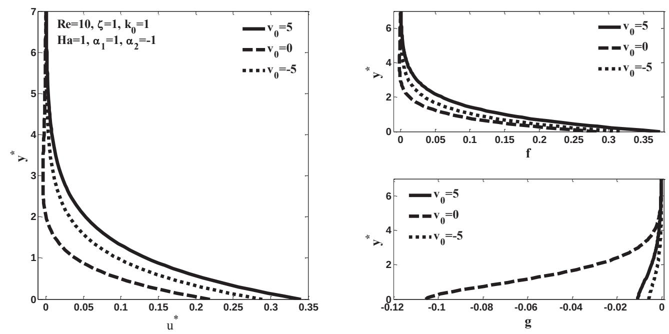

Figure 9 variations of velocity at $t^* = t^*_{max}/2$, f^* and g^* with suction or injection velocity on the surface

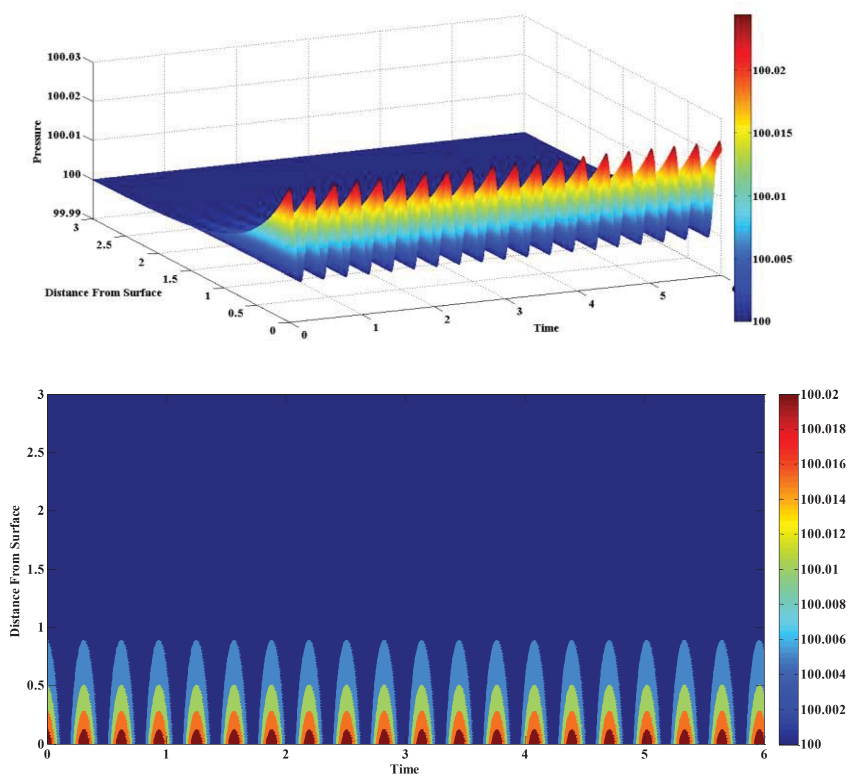

Figure 10 pressure along time and distance over the oscillating surface in the constant values of
$\zeta = 1, \mathrm{Re} = 10, k_o = 1, Ha = 1, v_0 = 5, \alpha_1 = 1, \alpha_2 = -1$

5. Conclusion

Unsteady slip flow of second grade non-Newtonian electrically conducting fluid of second order over an oscillating sheet has been considered. The results show that change of magnetic field applied on the sheet, slip flow parameters, frequency of oscillating, mass suction or injection and elastic second number will cause a change in amplitude and thickness of the fluid affected by motion of boundary. With increase of slip flow parameter, unlike that of other parameters, thickness of affected fluid by motion of boundary will decrease. It is also realized that both injection and suction of mass on the sheet, will increase the amplitude of velocity.

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