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On the onset of triple-diffusive convection in a layer of nanofluid

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Abstract

On the onset of triple-diffusive convection in a horizontal layer of nanofluid heated from below and salted from above and below is studied both analytically and numerically. The effects of thermophoresis and Brownian diffusion parameters are also introduced through Buongiorno model in the governing equations. By using linear stability analysis based on perturbation theory and applying normal modes analysis method, the dispersion relation accounting for the effect of various parameters is derived. The influences of solute-Rayleigh number, analogous solute-Rayleigh number, thermo-nanofluid Lewis number, modified diffusivity ratio and nanoparticle Rayleigh number on the stability of stationary convection are presented analytically and graphically. The solute Rayleigh number and analogous solute Rayleigh number have stabilizing effects on the onset of stationary convection for both top-heavy and bottom-heavy arrangements. The thermo-nanofluid Lewis number and diffusivity ratio have stabilizing effects on the onset of stationary convection while nanoparticle Rayleigh number has destabilizing effect on the onset of stationary convection. The necessary conditions for the existence of oscillatory modes are also obtained. A very good agreement is found between the results of present paper and earlier published results.

Keywords:

Convection, Triple-Diffusive, Nanofluid, Nanoparticles, Rayleigh

1. Introduction

Double-diffusive convection is a natural phenomenon that has various applications in different areas such as geophysics, soil sciences, food processing, oil reservoir modeling, oceanography, limnology and engineering, among others. Double-diffusive convection is a mixing process of two fluid components which diffuse at different rates. Brakke [1] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose. Double-diffusive convection problems related to different types of fluids and geometric

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configurations have been extensively studied [2-11].

All the above researchers have considered only two fluid component systems. However, there are many situations in which more than two fluid components involved. Examples of such multiple diffusive convection fluid systems include the solidification of molten alloys, geothermally heated lakes and sea water etc. Griffiths [12], Turner [13], Pearlstein *et al.* [14] and Lopez [15] studied the triply diffusive convection fluid (where the density depends on three independently diffusing agencies with different diffusivities). These researchers found that small concentrations of a third component with a smaller diffusivity can have a significant effect upon the nature of diffusive instabilities and 'oscillatory' and direct 'salt finger' modes are simultaneously unstable under a wide range of conditions, when the density gradients due to components with the greatest and smallest diffusivity are of the same signs. Some fundamental differences between the double and triple diffusive convection are noticed by these researchers. The presence of more than one chemical dissolved in fluid mixtures is very often requested for describing natural phenomena (contaminant transport, underground water flow, acid rain effects, worming of stratosphere) (see, Rionero [16]). Recently, Chand [17] studied Linear stability of triple-diffusive convection in micropolar ferromagnetic fluid saturating porous medium while triple-diffusive convection in Walters' (model B') fluid with varying gravity field saturating a porous medium studied by Kango et al. [18].

In recent years, much research has focused on the study of nanofluids with a view to applications in several industries such as the automotive, pharmaceutical and energy supply industries. A nanofluid is a colloidal suspension of nano sized particles. Common fluids such as water, ethanol or engine oils are typically used as base fluids in nanofluids. Among the variety of nanoparticles that have been used in nanofluids, it can be found oxide ceramics such as $Al_2\,O_3$ or CuO, nitride ceramics like AlN or SiN, and several metals such as Al or Cu. Choi [19] was first who coined the term nanofluid. Nanofluids are being looked upon as great coolants due to their enhanced thermal conductivities, and suspensions of nanoparticles are being developed medical applications including cancer therapy. Buongiorno [20] proposed that the absolute nanoparticle velocity can be viewed as the sum of the base fluid velocity and a relative slip velocity. Thus, convection of nanofluids based on Buongiorno's model has attracted great interest.

A considerable number of double-diffusive convection problems in a horizontal layer saturated by a nanofluid have also been numerically and analytically investigated [21-27]. In this paper, the study is extended to triple-diffusive convection in a layer of nanofluid heated from below and salted from above and below by salt S' and S'', respectively. To the best of researchers knowledge, this original study has not been published yet.

2. Mathematical Model and formulation

We consider an infinite horizontal layer of nanofluid of thickness d, bounded by the planes $z = 0$ and $z = d$ heated from below and salted from above and below by salt S' and S'' respectively as shown in Figure 1. Each boundary wall is assumed to be impermeable and perfectly thermal conducting. The layer is acted upon by a gravity force $\mathbf{g} = (0, 0, -g)$ aligned in the z direction. The temperature T, concentrations C', C'' and the volumetric fraction of nanoparticles φ at the lower (upper) boundary is assumed to take constant values T_0, C_0', C_0'' and φ_0 (T_1, C_1', C_1'' and φ_1), respectively.

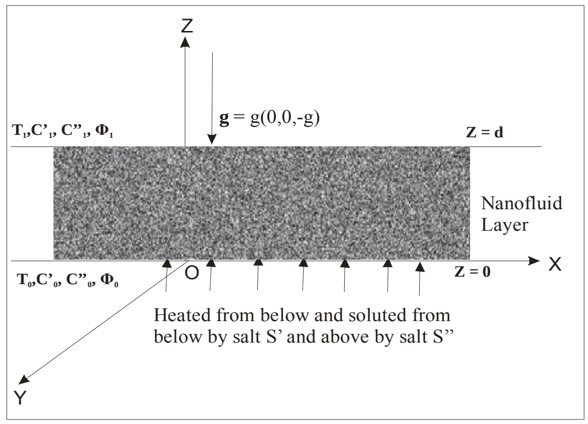

Figure 1. Physical configuration

2.1. *Governing Equations*

Let ρ, μ, p and $\mathbf{q}$ (u, v, w), denote respectively, the density, viscosity, pressure and Darcy velocity vector. Then, the governing equations of conservation of mass and momentum for nanofluid (Boungiorno [20], Nield and Kuznetsov [21], Chand [17], Kango et al. [18] and Rana and Chand [27]) in a triple-diffusive convection are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\rho_f \frac{d\mathbf{q}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{q} + \left(\phi\rho_p + (1-\phi) \left\{ \rho_f \begin{pmatrix} 1-\beta_T (T-T_0) \\ -\beta_{C'}\left(C'-C_0'\right) + \beta_{C''}\left(C''-C_0''\right) \end{pmatrix} \right\} \right) \mathbf{g}, \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q} \cdot \nabla)$ stands for convection derivative, φ is the volume fraction of nano particles, ρ_p is the density of nano particles and ρ_f is the density of base fluid, β_T is the uniform temperature gradient , $\beta_{C'}$ and $\beta_{C''}$ uniform solute gradients and we approximate the density of nanofluid by that of base fluid (i.e., $\rho = \rho_f$) (Boungiorno [20], Nield and Kuznetsov [21], Sheu [22], and Rana and Chand [27]).

The researchers approximate the density of the nanofluid by that of the base fluid that is to be considered $\rho = \rho_f$ (Boungiorno [20], Nield and Kuznetsov [21], Sheu [22], and Rana and Chand [27]). The continuity equation for the nanoparticles (Buongiorno [20]) is

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}.\nabla\varphi = D_B \nabla^2 \varphi + \frac{D_T}{T_1} \nabla^2 T. \tag{3}$$

The thermal energy equation for a nanofluid is

$$(\rho c)\left(\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T\right) = \kappa \nabla^2 T + (\rho c)_p \left(D_B \nabla\varphi.\nabla T + \frac{D_T}{T}\nabla T.\nabla T\right), \tag{4}$$

where (ρc) is heat capacity of fluid, $(\rho c)_p$ is heat capacity of nano particles.

The conservation equation for solute concentrations (Kuznetsov and Nield[21]) are

$$\frac{\partial C'}{\partial t} + \mathbf{q}.\nabla C' = D_{S'} \nabla^2 C'. \tag{5}$$

$$\frac{\partial C''}{\partial t} + \mathbf{q}.\nabla C'' = D_{S''} \nabla^2 C''. \tag{6}$$

where $D_{S'}$ and $D_{S''}$ are the solute diffusivities.

The boundary conditions

$$w = 0, \quad T = T_0, \quad \varphi = \varphi_0, \; C' = C'_0, \; C'' = C''_0, \frac{\partial w}{\partial z} + \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0 \text{ at } \quad z = 0, \tag{7}$$

$$w = 0, \quad T = T_1, \quad C' = C'_1, \; C'' = C''_1, \; \varphi = \varphi_1, \; \frac{\partial w}{\partial z} - \lambda_2 \frac{\partial^2 w}{\partial z^2} = 0 \text{ at } \quad z = 1. \tag{8}$$

We introduce non-dimensional variables as

$$(x^*, y^*, z^*,) = \left(\frac{x, y, z}{d}\right),$$

$$(u^*, v^*, w^*,) = \left(\frac{u, v, w}{\kappa_m}\right) d, \; t^* = \frac{t\alpha_f}{d^2},$$

$$p^* = \frac{pd^2}{\mu\alpha_f}, \; \varphi^* = \frac{(\varphi - \varphi_0)}{(\varphi_1 - \varphi_0)},$$

$$T^* = \frac{(T - T_0)}{(T_0 - T_1)}, \; C^* = \frac{(C' - C'_0)}{(C'_0 - C'_1)},$$

$$C^* = \frac{(C'' - C''_0)}{(C''_0 - C''_1)}, \quad \alpha_f = \frac{\kappa}{\rho c}$$

There after dropping the dashes (*) for convenience.

Eqs. (1)-(6) in non-dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \tag{9}$$

$$\frac{1}{P_r}\left(\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\nabla)\mathbf{q}\right) = -\nabla p + \nabla^2 \mathbf{q} - Rm\,\hat{e}_z + RaT\,\hat{e}_z - Rn\varphi\,\hat{e}_z - \frac{R_{S'}C'}{Le'}\hat{e}_z - \frac{R_{S''}C''}{Le''}\hat{e}_z, \tag{10}$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}.\nabla\varphi = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{11}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T = \nabla^2 T + \frac{N_B}{Ln}\nabla\varphi.\nabla T + \frac{N_A N_B}{Ln}\nabla T.\nabla T, \tag{12}$$

$$\frac{\partial C'}{\partial t} + \mathbf{q}.\nabla C' = \frac{1}{Le'}\nabla^2 C'. \tag{13}$$

$$\frac{\partial C''}{\partial t} + \mathbf{q}.\nabla C'' = \frac{1}{Le''}\nabla^2 C''. \tag{14}$$

where we have dimensionless parameters as:

Prandtl number $\Pr = \frac{\mu}{\rho\alpha_f}$; (15)

Thermosolutal Lewis number $Le' = \frac{\alpha_f}{D_{S'}}$; 16)

Analogous thermosolutal Lewis number

$$Le'' = \frac{\alpha_f}{D_{S''}}; \quad (17)$$

Thermo-nanofluid Lewis number $Ln = \frac{\alpha_f}{D_B}$; (18)

Thermal Rayleigh Number

$$Ra = \frac{\rho g \beta_T d^3 (T_0 - T_1)}{\mu \alpha_f}; \quad (19)$$

Solute Rayleigh Number $Rs' = \frac{\rho g \beta_{C'} (C_0' - C_1')}{\mu D_{S'}}$;

(20)

Analogous solute Rayleigh Number

$$Rs'' = \frac{\rho g \beta_{C''} (C_0'' - C_1'')}{\mu D_{S''}}; \quad (21)$$

Density Rayleigh number

$$R_m = \frac{(\rho_p \varphi_0 + \rho(1 - \varphi_0)) g d^3}{\mu \alpha_f}; \quad (22)$$

Nanoparticle Rayleigh number

$$Rn = \frac{(\rho_p - \rho)(\varphi_1 - \varphi_0) g d^3}{\mu \alpha_f}; \quad (23)$$

Modified diffusivity ratio $N_A = \frac{D_T (T_0 - T_1)}{D_B T_1 (\varphi_1 - \varphi_0)}$;

(24)

Modified particle- density ratio

$$N_B = \frac{(\rho c)_p (\varphi_1 - \varphi_0)}{(\rho c)_f}; \quad (25)$$

The dimensionless boundary conditions are

$$w = 0, \quad T = 1, C' = 1, C'' = 1, \quad \frac{\partial w}{\partial z} + \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0, \quad \varphi = 0 \quad \text{at} \quad z = 0 \quad (26)$$

$$w = 0, \quad T = 0, C' = 0, C'' = 0, \quad \frac{\partial w}{\partial z} - \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0, \varphi = 1 \quad \text{at} \quad z = 1. \quad (27)$$

2.2. *2.2. Basic Solutions*

Following Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

We assume a quiescent basic state that verifies

$$u = v = w = 0, p = p_b(z), C' = C_b'(z), C'' = C_b''(z), T = T_b(z), \varphi = \varphi_b(z). \quad (28)$$

Therefore, when the basic state defined in (28) is substituted into Eqs. (9) – (14), we get

$$0 = -\frac{dp_b(z)}{dz} - Rm + R_D T_b(z) - \frac{Rs'}{Le'} C_b'(z) - \frac{Rs''}{Le''} C_b''(z) - Rn\varphi_b(z), \quad (29)$$

$$\frac{d^2\varphi_b(z)}{dz^2} + N_A \frac{d^2 T_b(z)}{dz^2} = 0, \quad (30)$$

$$\frac{d^2 T_b(z)}{dz^2} + \frac{N_B}{Ln} \frac{d\varphi_b(z)}{dz} \frac{dT_b(z)}{dz} + \frac{N_A N_B}{Le} \left(\frac{dT_b(z)}{dz}\right)^2 = 0, \quad (31)$$

$$\frac{1}{Le'} \frac{d^2 C_b'(z)}{dz^2} = 0, \quad (32)$$

$$\frac{1}{Le''} \frac{d^2 C_b''(z)}{dz^2} = 0, \quad (33)$$

Using boundary conditions given in Eqs. (26), and (27) in the Eqs. (29) – (33), the solution is given by

$$\varphi_b(z) = z, T_b(z) = 1 - z, \quad C_b'(z) = 1 - z, C_b''(z) = 1 - z. \quad (34)$$

According to Buongiorno (2006), for most nanofluid investigated so far $L_n/(\varphi - \varphi_0)$ is large, of order 10^5-10^6 and since the nanoparticle fraction decrement $(\varphi_1 - \varphi_0)$ in not smaller than 10^{-3} which means L_n is large. Typical value of N_A is not greater than about 10. Then, the exponents in equation (35) are small. Using boundary conditions given in Eqs. (26), and (27) in the Eqs. (29) – (33), by expanding and retaining up to the first order is negligible and so to a good approximation for the solution

$$\varphi_b(z) = z, T_b(z) = 1\text{-}z, \quad C'_b(z) = 1\text{-}z, C''_b(z) = 1\text{-}z. \tag{35}$$

These results are identical with the results obtained by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

2.3. *Perturbation Solutions*

To study the stability of the system, the researchers superimposed infinitesimal perturbations on the basic state, so that

$$\mathbf{q}(u,v,w) = 0 + q^{\bullet}(u,v,w), \quad T = (1-z) + T^{\bullet}, \mathrm{C}'_{\mathrm{b}} = (1-z) + C'^{\bullet}, \quad \mathrm{C}''_{\mathrm{b}} = (1-z) + C''^{\bullet}, \quad \varphi = z + \varphi^{\bullet}, p = p_b + p^{\bullet}. \tag{36}$$

Using Eq. (36) into Eqs. (9) – (14), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes ($\bullet$) for convenience, the following equations are obtained, as follows:

$$\nabla \cdot \mathbf{q} = 0, \tag{37}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial q}{\partial t} = -\nabla p + \nabla^2 \mathbf{q} + \mathrm{R_D T}\,\hat{e}_z + \frac{Rs}{Le} C\hat{e}_z - \mathrm{Rn}\varphi\,\hat{e}_z - \frac{R_{S'} C'}{Le'}\hat{e}_z - \frac{R_{S''} C''}{Le''}\hat{e}_z, \tag{38}$$

$$\frac{\partial \varphi}{\partial t} + w = \frac{1}{Ln}\nabla^2\varphi + \frac{N_A}{Ln}\nabla^2 T, \tag{39}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Ln}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2\mathrm{N}_A N_B}{Ln}\frac{\partial T}{\partial z}. \tag{40}$$

$$\frac{\partial C'}{\partial t} - w = \frac{1}{Le'}\nabla^2 C'. \tag{41}$$

$$\frac{\partial C''}{\partial t} - w = \frac{1}{Le''}\nabla^2 C''. \tag{42}$$

Boundary conditions for Eqs. (37) - (42) are

$$w = 0, \quad \mathrm{T} = 0, \mathrm{C}' = 0, \mathrm{C}'' = 0, \quad \frac{\partial \mathrm{w}}{\partial \mathrm{z}} + \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0, \quad \varphi = 0 \quad \text{at } z = 0, \tag{43}$$

$$w = 0, \ T = 0, \mathrm{C}' = 0, \mathrm{C}'' = 0, \quad \frac{\partial \mathrm{w}}{\partial \mathrm{z}} - \lambda_1 \frac{\partial^2 w}{\partial z^2} = 0, \ \varphi = 0 \ \text{at } z = 1. \tag{44}$$

The parameter *Rm* is not involved in Eqs. (37)-(42), it is just a measure of the basic static pressure gradient.

The eight unknown's *u, v, w, p, T, C', C''* and φ can be reduced to five by operating Eq. (38) with e_z.curl curl, which yields

$$\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\nabla^2 w = \nabla^4 w + R_D \nabla_H^2 T - Rn\nabla_H^2\varphi - \frac{Rs'}{Le'}\nabla_H^2 C' - \frac{Rs''}{Le''}\nabla_H^2 C'', \tag{45}$$

where $\nabla_H^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$. is the two-dimensional Laplace operator on the horizontal plane and $\zeta = \hat{e}_z.curl\ \mathbf{q}$ is the z-component of vorticity.

3. Normal Modes Analysis Method

The disturbances into normal modes of the form are expressed as

$$[w, T, \mathrm{C}', \mathrm{C}'', \varphi] = [W(z), \Theta(z), \Gamma(z), \Lambda(z), \Phi(z)]\exp(ilx + imy + \omega t), \tag{46}$$

where l, m are the wave numbers in the x and y direction, respectively, and ω is the growth rate of the disturbances.

Substituting Eq. (46) into Eqs. (45) and (39)-(42), we obtain the following eigen value problem

$$\frac{\omega}{\Pr}\left(D^2-a^2\right)W-\left(D^2-a^2\right)^2W+ a^2R_D\Theta-a^2Rn\Phi-\frac{Rs'}{Le'}a^2\Gamma- \frac{Rs'}{Le'}a^2\Lambda=0, \tag{47}$$

$$W+\left(\frac{1}{Le'}\left(D^2-a^2\right)-\omega\right)\Gamma=0, \tag{48}$$

$$W+\left(\frac{1}{Le''}\left(D^2-a^2\right)-\omega\right)\Lambda=0, \tag{49}$$

$$W+\begin{pmatrix} D^2+\frac{N_B}{Ln}D-\frac{2N_AN_B}{Ln}D \\ -a^2-\omega \end{pmatrix}\Theta- \frac{N_B}{Ln}D\Phi=0, \tag{50}$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{Ln}\left(D^2-a^2\right)\Theta -\left(\frac{1}{Ln}\left(D^2-a^2\right)-\omega\right)\Phi=0, \tag{51}$$

$$W=0,\quad D^2W=0,\Gamma=0,\Lambda=0,\ \Theta=0,\Phi=0 \text{ at } \quad z=0 \quad \text{and} \tag{52}$$

$$W=0,\ D^2W=0,\ \Gamma=0,\ \Lambda=0,\ \Theta=0,\ \Phi=0 \text{ at } z=1. \tag{53}$$

where $D=\frac{d}{dz}$ and $a^2=l^2+m^2$ is the dimensionless horizontal wave number.

4. Linear Stability Analysis and Dispersion Relation

Considering solutions W, Θ, Γ, Λ and Φ of the form

$$W=W_0\sin(\pi z),\ \Theta=\Theta_0\sin(\pi z),\ \Gamma=\Gamma_0\sin(\pi z),\ \Lambda=\Lambda_0\sin(\pi z),\Phi=\Phi_0\sin(\pi z)\ . \tag{54}$$

Substituting (54) into Eqs. (47) – (51) and integrating each equation from $z = 0$ to $z = 1$, the researchers obtain the dispersion relation

$$Ra=\frac{J^2}{a^2\Pr}\left(\mathrm{Pr}J^2+\omega\right)\left(\omega+J^2\right).+ \left(\omega+J^2\right)\left(\frac{Rs'}{\omega Le'+J^2}+\frac{Rs''}{\omega Le''+J^2}\right) -\frac{\left(\omega+J^2\right)Ln+NaJ^2}{\omega Ln+J^2}Rn \tag{55}$$

Eq. (56) is the required dispersion relation accounting for the effect of Prandtl number, thermo-solutal Lewis number, analogous thermo-solutal Lewis number, thermo-nanofluid Lewis number, solute Rayleigh Number, analogous solute Rayleigh Number, nanoparticle Rayleigh number, and modified diffusivity ratio on the onset of triple diffusive convection in a layer of nanofluid.

To examine the stability of the system, the real part of ω is set to zero and we take $\omega=i\omega_i$ in Eq. (56), then we obtain

$$Ra=\frac{J^2\left(J^4\Pr-\omega_i^2\right)}{a^2\Pr}+\frac{J^4+\omega_i^2Le'}{J^4+\omega_i^2Le'^2}Rs'+ \frac{J^4+\omega_i^2Le''}{J^4+\omega_i^2Le''^2}Rs''-\frac{J^4\left(Ln+N_A\right)+\omega_i^2Ln^2}{J^4+\omega_i^2Ln^2} +i\omega_i\Delta_1, \tag{56}$$

where

$$\Delta_1=\frac{J^4(\Pr+1)}{a^2\Pr}-\frac{J^2\left(Le'-1\right)}{J^4+\omega_i^2Le'^2}Rs' -\frac{J^2\left(Le''-1\right)}{J^4+\omega_i^2Le''^2}Rs''+\frac{J^2\left(1-Ln-N_A\right)Ln}{J^4+\omega_i^2Ln^2}Rn. \tag{57}$$

Ra must be real as it is a physical quantity. Thus, it follows from Eq. (57) that either $\omega_i=0$ (exchange of stabilities, steady state) or $\omega_i\neq 0$, overstability or oscillatory onset).

5. The Stationary Convection

For stationary convection, putting $\omega_i = 0$ in equation (56), we obtain

$$\left(Ra\right)_s=\frac{\left(\pi^2+a^2\right)^3}{a^2}+\mathrm{Rs}'+ \mathrm{Rs}''-\left(\mathrm{Ln}+\mathrm{N_A}\right)\mathrm{Rn}. \tag{58}$$

Eq. (58) expresses the thermal Rayleigh number as a function of the dimensionless resultant wave number *a* and the parameters Rs', Rs'', Ln, *Rn*, N_A. Eq. (58) is identical to that obtained by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27] . Also, in Eq. (58) the particle increment parameter N_B does not appear, and the diffusivity ratio parameter N_A appears only in association with the nanoparticle Rayleigh number *Rn*. This implies that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

In the absence of the solute gradient parameter *Rs''*, Eq. (58) reduces to

$$(Ra)_s = \frac{(\pi^2 + a^2)^3}{a^2} + Rs' - (Ln + N_A)Rn, \tag{59}$$

Equation (59) is the same as the results derived by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

The critical cell size at the onset of instability is obtained by minimizing Ra with respect to *a*. Thus, the critical cell size must satisfy

$$\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c} = 0,$$

Equation (58) which gives

$$a_c = \frac{\pi}{\sqrt{2}}. \tag{60}$$

And the corresponding critical thermal Rayleigh number $(Ra)_c$ on the onset of stationary convection is given by

$$(Ra)_c = \frac{27\pi^4}{4} + \mathrm{Rs}' + \mathrm{Rs}'' - (\mathrm{Ln} + \mathrm{N_A})\mathrm{Rn}. \tag{61}$$

It is noted that if *Rn* is positive then *Ra* is minimized by a stationary convection. The result given in equation (61) is a good agreement with the result derived by Sheu [22] and Rana and Chand [27].

In order to study the effect of solute Rayleigh number (Rs'), analogous solute Rayleigh number

(Rs''), thermo-nanofluid Lewis number (Ln), diffusivity ratio (N_A) and nanoparticle Rayleigh number (Rn) on the stationary convection, the behaviour of $\frac{\partial (Ra)_s}{\partial Rs'}$, $\frac{\partial (Ra)_s}{\partial Rs''}$, $\frac{\partial (Ra)_s}{\partial Ln}$, $\frac{\partial (Ra)_s}{\partial N_A}$ and $\frac{\partial (Ra)_s}{\partial Rn}$ analytically are examined by the researchers.

From Eq. (58), we obtain

$$\frac{\partial (Ra)_s}{\partial Rs'} = +1, \tag{62}$$

which is positive; therefore, solute Rayleigh number (Rs') inhibits the onset of triple-diffusive stationary convection implying thereby solute Rayleigh number (Rs') has stabilizing effect on the system which is an agreement with the results derived by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

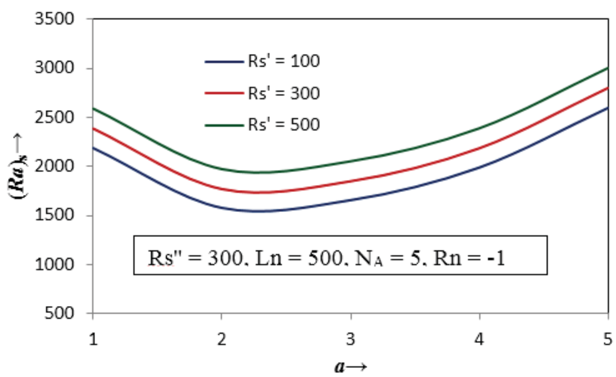

Figure 2. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* fordifferent values of solute Rayleigh number (Rs')

As shown in Figure 2, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of solute Rayleigh number (Rs'). This shows that as Rs' increases, the stationary thermal Rayleigh number $(Ra)_s$ also increases. Thus, solute Rayleigh number (Rs') has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (62).

It is evident from Eq. (58) that

$$\frac{\partial (Ra)_s}{\partial Rs''} = +1, \tag{63}$$

which is positive; therefore, analogous solute Rayleigh number (Rs'') inhibits the onset of triple-diffusive stationary convection implying thereby solute Rayleigh number (Rs'') has stabilizing effect on the system which is an agreement with the results derived by Chand [17] and Kango et al. [18].

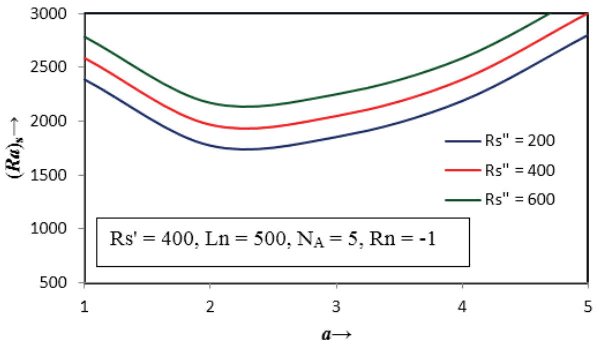

Figure 3. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* fordifferent values of analogous solute Rayleigh number (Rs'')

Figure 3 depicts that the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of solute Rayleigh number (Rs''). This shows that as Rs' increases, the thermal Rayleigh number $(Ra)_s$ also increases. Thus, solute Rayleigh number (Rs'') has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (63).

From Eq. (58), we obtain

$$\frac{\partial (Ra)_s}{\partial Ln} = -Rn, \tag{64}$$

implying thereby thermo-nanofluid Lewis number (Ln) inhibits the onset of triple-diffusive stationary convection. Thus, thermo-nanofluid Lewis number (Ln) has stabilizing effect on the system if *Rn < 0* (i.e., bottom heavy arrangement) which is a good agreement with the results derived by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

As shown in Figure 4, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of thermo-nanofluid Lewis number (Ln). This shows that as *Ln* increases, the thermal Rayleigh number $(Ra)_s$ also increases for bottom-heavy arrangements. Thus, of thermo-nanofluid Lewis number (Ln) has stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (64).

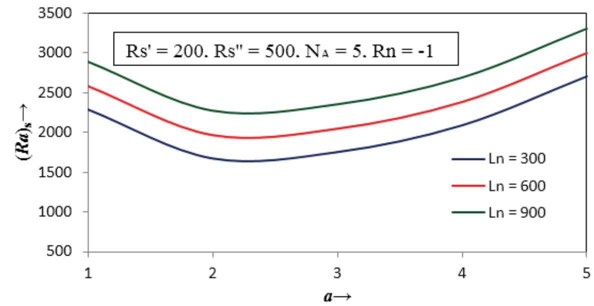

Figure 4. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* fordifferent values of thermo-nanofluid Lewis number (Ln)

From Eq. (58), we obtain

$$\frac{\partial (Ra)_s}{\partial N_A} = -Rn, \tag{65}$$

implying thereby diffusivity ratio (N_A) inhibits the onset of triple-diffusive stationary convection. Thus, diffusivity ratio (N_A) has stabilizing effect on the system if *Rn < 0* (i.e., bottom heavy arrangement) which is a good agreement with the results derived by Nield and Kuznetsov [21] , Sheu [22], Rana et al. [25,26] and Rana and Chand[27]

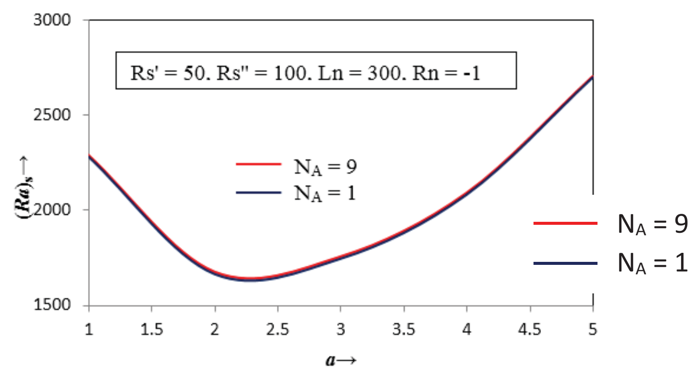

Figure 5. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* fordifferent values of diffusivity ratio (N_A)

In Figure 5, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* different values of diffusivity ratio (N_A) as shown. This shows that as *Ln* increases slightly, the thermal Rayleigh number $(Ra)_s$ also increases for bottom-heavy arrangements. Thus, diffusivity ratio (N_A) has low stabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (65).

It is evident from Eq. (58) that

$$\frac{\partial (Ra)_s}{\partial Rn} = -Ln - N_A, \tag{66}$$

which is negative implying thereby nanoparticle Rayleigh number (Rn) hastens the triple-diffusive convection implying thereby nanoparticle Rayleigh number (Rn) has destabilizing effect on the system which is a good agreement with the results derived by Nield and Kuznetsov [21], Sheu [22], Rana et al. [25, 26] and Rana and Chand [27].

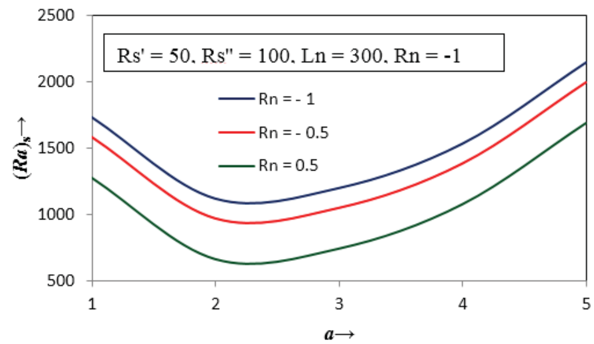

Figure 6. Variation of stationary thermal Rayleigh number $(Ra)_s$ with the wave number *a* for different values of nanoparticle Rayleigh number (Rn)

Figure 6 shows, the stationary thermal Rayleigh number $(Ra)_s$ is plotted against dimensionless wave number *a* for different values of nanoparticle Rayleigh number (Rn). This shows that as *Ln* increases, the thermal Rayleigh number $(Ra)_s$ decreases for bottom-heavy arrangements. Thus, nanoparticle Rayleigh number (Rn) has destabilizing effect on stationary convection which is in good agreement with the result obtained analytically from Eq. (66).

6. Oscillatory Convection

The oscillatory Rayleigh number is given by

$$(R_D)_{osc} = \frac{J^2\left(J^4\Pr - \omega^2\right)}{a^2\Pr} + \frac{J^4+\omega^2 Le'}{J^4+\omega^2 Le'^2}Rs' + \frac{J^4+\omega^2 Le''}{J^4+\omega^2 Le''^2}Rs'' - \frac{J^4\left(Ln+N_A\right)+\omega^2 Ln^2}{J^4+\omega^2 Ln^2}. \tag{67}$$

$$a_3\left(\omega_i^2\right)^3 + a_2\left(\omega_i^2\right)^2 a_1\left(\omega_i^2\right) + a_0 = 0, \tag{68}$$

where

$$a_0 = \left(\pi^2+a^2\right)^4\left(\left(\pi^2+a^2\right)^3(\Pr+1) - \left(Le'-1\right)a^2\Pr Rs' - \left(Le''-1\right)a^2\Pr Rs'' - \left(1-N_A-Ln\right)a^2 Ln\Pr Rn\right),$$

$$a_1 = \left(\pi^2+a^2\right)^2\begin{pmatrix}\left(\pi^2+a^2\right)^3(\Pr+1)\left(Le'^2+Le''^2+Ln^2\right) - \left(Le'-1\right)\left(Le''^2+Ln^2\right)a^2\Pr Rs' - \\ \left(Le''-1\right)\left(Le'^2+Ln^2\right)a^2\Pr Rs'' - \left(1-N_A-Ln\right)\left(Le'^2+Le''^2\right)a^2 Ln\Pr Rn\end{pmatrix},$$

$$a_2 = \left(\pi^2+a^2\right)^3\left(\left(\Pr+1\right)\left(Le'^2 Le''^2+Ln^2\right)+\left(\Pr+1\right)Ln^2\left(Le'^2+Le''^2\right)'\right) - \left(Le'-1\right)a^2\Pr Le''^2 Rs'$$
$$-\left(Le''-1\right)a^2\Pr Le'^2 Rs'' - \left(1-N_A-Ln\right)\left(Le'^2+Le''^2\right)a^2 Ln\Pr Le'^2 Le''^2 Rn,$$

$$a_3 = \left(\pi^2+a^2\right)Le'^2 Le''^2 Ln^2.$$

Since Ln is of order $10^2 - 10^3$, $1 < N_A < 10$ and so $\left(1 - N_A - Ln\right) < 0$. Thus, Eq. (68) does not admit positive value of ω_i^2 if $Le', Le'' > 1$. Hence, the necessary conditions for the occurrence of oscillatory convection are $Le', Le'' > 1$.

7. Conclusions

Triple-diffusive convection in a layer of Nanofluid heated from below, and soluted from below and above is investigated by using a linear stability analysis method. The main conclusions are as follows:

- The solute Rayleigh number (Rs') and analogous solute Rayleigh number (Rs'') have stabilizing effects on the onset of stationary convection for both top-heavy and bottom-heavy arrangements as shown in figures 2 and 3, respectively.

- The thermo-nanofluid Lewis number (Lr) and diffusivity ratio (N_A) have stabilizing effects on the onset of stationary convection for bottom-heavy arrangements as shown in figures 4 and 5, respectively.
- Nanoparticle Rayleigh number (Rn) has destabilizing effect on the onset of stationary convection as shown in figure 6.
- Necessary conditions for the occurrence of oscillatory convection are obtained and are given by $Le', Le'' > 1$.

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