



Analytical Solutions for Bending Analysis of Functionally Graded Porous Beam Subjected to Various Mechanical Loads

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Abstract

In this study, a new three-variable shear deformation theory is proposed to investigate the bending behaviour of porous functionally graded FGP beams subjected to different loading conditions and characterized by two distinct porosity distributions. Within the framework of the proposed formulation, the number of unknown variables is reduced to three through the introduction of an indefinite integral. The developed theory naturally satisfies the boundary conditions requiring zero transverse shear stress at the upper and lower surfaces of the beam, thereby eliminating the need for a shear correction factor. The governing equilibrium equations for simply supported PFG beams are formulated based on the principle of virtual work. Navier's method is subsequently employed to derive analytical solutions for the various porosity configurations and loading conditions considered. The accuracy and reliability of the obtained results are assessed through comparisons with available solutions reported in the literature. Finally, a parametric study is conducted to examine the effects of the power-law index, length-to-thickness ratio, porosity distribution, and loading type on the bending response of PFG beams.

Keywords: bending behaviour , FGP beams, Property Gradient Materials; Principle of virtual work; porosity distributions.

1. Introduction

Functionally graded materials (FGMs) have emerged as advanced engineering materials owing to their

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continuously varying material properties, which enable improved mechanical performance and a gradual transition between dissimilar constituents. By appropriately controlling the spatial distribution of ceramic and metallic phases, FGMs can provide enhanced stiffness-to-weight ratios, thermal resistance, and resistance to localized stress concentrations compared with conventional homogeneous and laminated composites. These characteristics have promoted their application in aerospace, automotive, civil, mechanical, and energy-related structures. Among the different structural configurations, functionally graded beams have received considerable attention because of their simple geometry, efficient load-carrying capability, and relevance to lightweight structural components. In addition to material gradation, manufacturing processes may inevitably generate microscopic voids and defects within functionally graded components. These internal voids, commonly referred to as porosity, modify the effective elastic and inertial properties of the material and may significantly influence the structural response. Consequently, functionally graded porous (FGP) beams have attracted increasing research interest because the simultaneous consideration of material gradation and porosity provides a more realistic representation of advanced lightweight structures. The mechanical response of such beams depends not only on the power-law index and geometric parameters but also on the magnitude and spatial distribution of porosity through the beam thickness. Recent investigations have consequently considered different porosity distributions and demonstrated their significant effects on the bending, buckling, vibration, and wave-propagation characteristics of FGP beams. A considerable number of analytical and numerical studies have been devoted to the mechanical behaviour of porous FG beams using different beam theories and solution techniques.

Nguyen et al.[1] developed a higher-order shear deformation theory for the buckling and free-vibration analysis of [2, 3]functionally graded porous beams, considering symmetric and asymmetric porosity configurations and avoiding the use of a shear correction factor. Abdelaziz Timesli, Foroutan et Dai [2, 3], provided a comprehensive analysis model for the buckling behaviour of porous FGM cylindrical shells embedded within an elastic foundation. Mota et al[4] examined the effect of porosity on the static response of functionally graded (FG) plates using first-order shear deformation theory (FSDT) combined with the finite element method.

Ait Atmane et al. [5] investigated the dynamic response of porous FG beams resting on variable elastic foundations using a three-variable HSDT, with particular emphasis on the effects of the gradient index, porosity coefficient, slenderness ratio, and foundation parameters. Dahmane et al. [6] examined wave propagation in bidirectionally graded porous FG cantilever beams using a higher-order shear deformation model, accounting for material gradation along both the thickness and width directions. Mesmoudi et al. [7]proposed a highly efficient mesh-free approach for the nonlinear bending analysis of porous FG beams and sandwich beams with FG face sheets, highlighting the influence of geometrical and material parameters on the nonlinear response. Ikinici et al. [8] investigated the free-vibration characteristics of functionally graded porous beams using a hyperbolic shear deformation theory and different porosity distributions. Himeur et al. [9] developed a simple integral quasi-3D theory for the buckling and static bending of porous FG beams, incorporating thickness stretching and satisfying the zero-transverse shear-stress conditions without a shear correction factor. Furthermore, recent finite-element developments have incorporated bidirectional material gradation and porosity within quasi-3D higher-order shear deformation formulations to simultaneously investigate flexural torsional, and vibrational responses Amine Z et al. [10]. Several recent studies have investigated the behaviour of functionally graded material (FGM) beams under various material configurations, loading conditions, and boundary constraints. [11-15].

Despite significant advances, the static bending behavior of porous functionally graded (FG) beams remains insufficiently investigated, particularly regarding the coupled effects of material gradation, porosity distribution, slenderness ratio, and transverse loading. To address this gap, the present study develops an analytical formulation based on a higher-order shear deformation theory that accounts for transverse shear deformation. Two characteristic porosity distributions are considered, and the formulation is validated against available results from the literature. A comprehensive parametric study is then conducted to examine the effects of the power-law index, slenderness ratio, porosity coefficient, porosity distribution, and loading type on the deflection and stress responses. The proposed approach provides an accurate and computationally efficient framework for analyzing the static behavior of porous FG beams.

2. Functionally graded porous beam

Consider a functionally graded FGP beam of length (l) and rectangular cross-section characterized by a width (b) and thickness (h). The Cartesian coordinates x , y , and z are defined along the longitudinal, transverse, and thickness directions of the beam, respectively, as illustrated in Fig. 1.

The effective material properties of the FGM beam, including Young's modulus (E) and mass density (ρ), are assumed to vary continuously and smoothly along the thickness direction (z). These properties can be expressed as follows Benachi et al [16, 17].

$$P(z) = P_m V_m + P_c V_c \quad -\frac{h}{2} \leq z \leq \frac{h}{2} \quad (1)$$

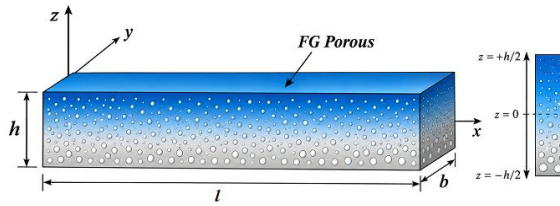


Fig. 1. Geometry of a functionally graded porous beam.

The volume fraction of the FGMs is supposed to obey a power-law function along the thickness direction

$$V(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k, \quad V_c + V_m = 1 \quad (2)$$

The distribution of the constituent phases through the beam thickness is characterized by the volume-fraction index k . In the present study, two representative porosity distribution models are considered for porous functionally graded material FG beams, as illustrated in Fig. 2. Accordingly, the effective material properties, namely the mass density $\rho(z)$ and Young's modulus $E(z)$, are assumed to vary continuously along the thickness coordinate z . These effective properties are defined by the following relations [18-21] :

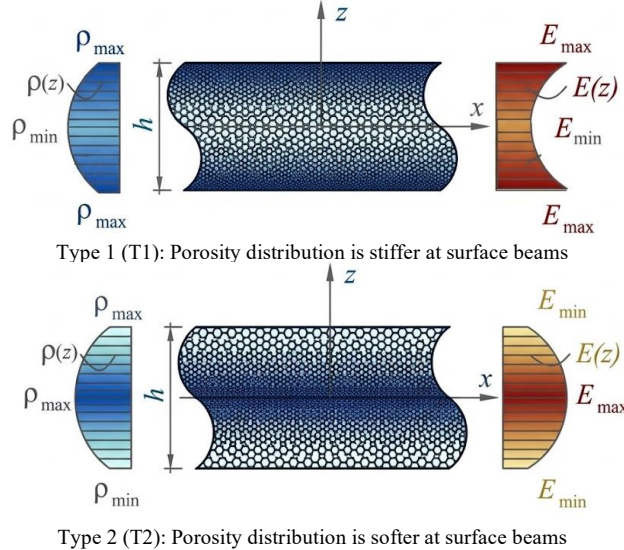


Fig 2: two typical porosity distribution profiles for functionally graded beam

Type 1 (P1): Non-Uniform Porosity distribution type (Stiffer Surfaces)

$$\begin{aligned} E(z) &= E_{\max} \left(1 - \zeta \cos\left(\frac{\pi z}{h}\right) \right) \\ \rho(z) &= \rho_{\max} \left(1 - \zeta_m \cos\left(\frac{\pi z}{h}\right) \right) \end{aligned} \quad (3)$$

Type 2 (P2): Non-Uniform Porosity distribution type (Softer Surfaces)

$$\begin{aligned} E(z) &= E_{\max} \left(1 - \zeta \cos\left(\left|\frac{\pi z}{h}\right| - \frac{\pi}{2}\right) \right) \\ \rho(z) &= \rho_{\max} \left(1 - \zeta_m \cos\left(\left|\frac{\pi z}{h}\right| - \frac{\pi}{2}\right) \right) \end{aligned} \quad (4)$$

where $\rho_{\max}$ and $E_{\max}$ are the maximum values of mass density and Young's modulus, respectively; ζ and $\zeta_m = 1 - \sqrt{1 - \zeta}$ are the porosity parameters and mass density, respectively.

3. Kinematics and constitutive relations

In this study, the displacement field of the functionally graded FGP beam is formulated within the framework of a higher-order shear deformation theory. This kinematic formulation provides an accurate description of the beam behaviour while accounting for transverse shear deformation effects. The corresponding displacement field is expressed as follows:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0(x, y)}{\partial x} + K_1 A f(z) \frac{\partial \theta(x, y)}{\partial x} \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (5)$$

Where u_0 , w_0 , and θ denote the three displacement variables associated with the mid-plane of the FGP beam. the coefficient K_i depend on the geometry.

$$\text{with} \quad K_1 = \lambda^2, \quad A = -\frac{1}{\lambda^2}, \quad \lambda = \frac{m\pi}{a}$$

The specific form of the shape function $f(z)$ is given by Chitour et al [22]

$$f(z) = \left(\frac{3}{25}\right) \pi z \left(\pi - \sqrt[3]{0.135} \cosh\left(\frac{\pi z}{h}\right) \right), \quad (6)$$

The kinematic relations derived from the displacement field given in equations (5), based on the small-strain elasticity theory, are expressed as follows:

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u_0(x, y)}{\partial x} - z \frac{\partial^2 w_0(x, y)}{\partial x^2} + K_1 A f(z) \frac{\partial^2 \theta(x, y)}{\partial x^2} \\ \gamma_{xz} &= K_1 A g(z) \frac{\partial \theta(x, y)}{\partial x} \end{aligned} \quad (7)$$

$$g(z) = \left(\frac{\partial f(z)}{\partial z} \right) \quad (8)$$

where ε_{xx} and γ_{xz} represent the normal and transverse shear strains, respectively, which are related to the corresponding stresses σ_{xx} and τ_{xz} through the linear elastic constitutive law as follows:

Based on the two-dimensional displacement field given in equations (7), the linear constitutive relations for the functionally graded porous FGP beam are expressed as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} \quad (9)$$

Where:

$$\begin{aligned} Q_{11} &= E(z) \\ Q_{55} &= \frac{E(z)}{2(1+\nu)} \end{aligned} \quad (10)$$

4. Equilibrium equations

The governing equations of equilibrium are derived by applying the principle of virtual displacements, and are expressed as follows: Ali Rachedi et al. Thuc P et al [23, 24]

$$\int_V \left[\sigma_{xx} \delta \varepsilon_{xx} + \tau_{xz} \delta \gamma_{xz} \right] dV - \int_A q \delta w(x) dA = 0 \quad (11)$$

Substituting equations (7) and (9) into equation (11) and integrating through the thickness of the beams, equation (11) can be rewritten as:

$$\int \left[N_{xx} \frac{\partial u_0}{\partial x} - M_{xx} \frac{\partial^2 w_0}{\partial x^2} + K A S_{xx} \frac{\partial^2 \theta}{\partial x^2} + K A Q_{xz} \frac{\partial \theta}{\partial x} \right] dx - \int [q \delta w_0] dx = 0 \quad (12)$$

where q denotes the distributed transverse load. The stress resultants N_{xx} , M_{xx} , S_{xx} and Q_{xz} are defined as follows:

$$\begin{aligned} N_{xx} &= \int \sigma_{xx} dz \\ M_{xx} &= \int \sigma_{xx} z dz \\ S_{xx} &= \int \sigma_{xx} f(z) dz \\ Q_{xz} &= \int \tau_{xz} f'(z) dz \end{aligned} \quad (13)$$

By substituting equations (7) and (9) into equations (13), the stress and moment resultants N_{xx} , M_{xx} , S_{xx} and Q_{xz} are obtained as follows:

$$\begin{aligned}
N_{xx} &= A_{11} \frac{\partial u_0}{\partial x} - B_{11} \frac{\partial^2 w_0}{\partial x^2} + K A C_{11}^s \frac{\partial^2 \theta}{\partial x^2} \\
M_{xx} &= B_{11} \frac{\partial u_0}{\partial x} - D_{11} \frac{\partial^2 w_0}{\partial x^2} + K A F_{11} \frac{\partial^2 \theta}{\partial x^2} \\
S_{xx} &= C_{11}^s \frac{\partial u_0}{\partial x} - F_{11} \frac{\partial^2 w_0}{\partial x^2} + K A H_{11} \frac{\partial^2 \theta}{\partial x^2} \\
Q_{xz} &= K A G_{55} \frac{\partial \theta}{\partial x}
\end{aligned} \tag{14}$$

Where the stiffness components are given as follows:

$$\begin{aligned}
\{A_{11}, B_{11}, C_{11}^s, D_{11}, F_{11}, H_{11}\} &= \int C_{11} \{1, z, f(z), z^2, z f(z), f^2(z)\} dz, \\
G_{55} &= \int C_{55} g^2(z) dz
\end{aligned} \tag{15}$$

The principal differential equations of equilibrium (16) are obtained by integrating equation (12) by parts and setting $(\delta u_0, \delta w_0, \delta \theta)$ equal to zero:

$$\begin{aligned}
\delta u_0 : \quad \frac{\partial N_{xx}}{\partial x} &= 0 \\
\delta w_0 : \quad \frac{\partial^2 M_{xx}}{\partial x^2} + q &= 0 \\
\delta \theta : \quad -(K A) \frac{\partial^2 P_{xx}}{\partial x^2} + (K A) \frac{\partial Q_{xz}}{\partial x} &= 0
\end{aligned} \tag{16}$$

By substituting equations (14) and equations (15) into equations (16), the principal equations are stated as follow

$$\begin{aligned}
&A_{11} \frac{\partial^2 u_0}{\partial x^2} - B_{11} \frac{\partial^3 w_0}{\partial x^3} + K A C_{11}^s \frac{\partial^3 \theta}{\partial x^3} \\
&B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_0}{\partial x^4} + K A F_{11} \frac{\partial^4 \theta}{\partial x^4} + q = 0 \\
&-C_{11}^s \frac{\partial^3 u_0}{\partial x^3} + F_{11} \frac{\partial^4 w_0}{\partial x^4} - K A H_{11} \frac{\partial^4 \theta}{\partial x^4} + K A G_{55} \frac{\partial^2 \theta}{\partial x^2} = 0
\end{aligned} \tag{17}$$

5. Analytical Solution

Beams are commonly classified according to their boundary conditions. For simply supported functionally graded porous (FGP) beams, the Navier method, based on double Fourier series expansions, is employed to derive an analytical solution to the governing partial differential equations Chikh, A [25]. Accordingly, the displacement variables u_0 , w_0 and θ are represented in terms of the following assumed trigonometric functions:

$$\begin{aligned}
u_0 &= \sum_{n=1}^{\infty} U_m \cos(\lambda x) \\
v_0 &= \sum_{n=1}^{\infty} W_m \sin(\lambda x) \\
\theta &= \sum_{n=1}^{\infty} X_m \sin(\lambda x)
\end{aligned} \tag{18}$$

where $\lambda = m\pi/a$ and U_m , W_m and X_m denote unknown coefficients to be determined. The transverse load (q) is similarly represented by a Fourier series expansion, given by:

$$q_m = \frac{2}{a} \int_0^a q(x) \sin(\lambda x) dx \tag{19}$$

The values of q_m using equations (19) are set as follows [22]:

- *Sinusoidally distributed load*

$$q_m = q_0 \quad \text{and} \quad m=1 \tag{20}$$

- *Uniform distributed load*

$$q_m = \frac{4q_0}{m\pi} \tag{21}$$

- *Central patch load*

$$q_m = \frac{4q_0}{m\pi} \sin\left(m\pi 0.02\left(\frac{a}{2}\right)\right) \tag{22}$$

- *Central concentrated load*

$$q_m = \frac{4Q_0}{a} \sin\left(\frac{m\pi}{2}\right); \quad Q_0 = \frac{q_0}{a} \tag{23}$$

By substituting equations (18) and (19) into equations (17), the resulting system of equations can be written as follows:

$$\begin{aligned}
[L] \{\Delta\} &= \{P\} \\
[L] &= \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \quad [P] = \begin{Bmatrix} 0 \\ q_m \\ 0 \end{Bmatrix} \quad [\Delta] = \{U_m, W_m, X_m\}
\end{aligned} \tag{24}$$

Where ($i, j=1, 2, 3$), with L_{ij} denoting the components of the coefficient matrix, are defined as follows:

$$\begin{aligned}
L_{11} &= -4A_{11}\lambda^2 \\
L_{12} &= -8B_{11}\lambda^3 \\
L_{13} &= -8K_1 A C_{11}^S \lambda^3 \\
L_{22} &= -8D_{11}\lambda^4 \\
L_{23} &= 8K_1 A F_{11} \lambda^4 \\
L_{33} &= -8H_{11} K_1 A \lambda^4 - 2G_{55} K_1 A \lambda^2
\end{aligned} \tag{25}$$

6. Numerical results and discussion

In this section, several numerical examples are presented to assess the accuracy and reliability of the proposed formulation and to investigate the static bending behavior of isotropic and functionally graded porous (FGP) beams. Two configurations of FGP beams, denoted as Types T1 and T2, are considered. These beams are composed of a

combination of isotropic ceramic Al_2O_3 and metallic Al materials. The material properties of Al_2O_3 are $E_c=380\text{GPa}$, $\nu_c=0.3$, and $\rho_c=3960\text{kg/m}^3$, whereas those of Al are $E_m=70\text{GPa}$, $\nu_m=0.3$, and $\rho_m=2702\text{kg/m}^3$.

The effects of the power-law index, span-to-thickness ratio, porosity coefficient, and different loading types on the bending behavior of isotropic and functionally graded porous beams are investigated in detail. Several loading configurations are considered, including sinusoidally distributed, uniformly distributed, centrally distributed, and concentrated loads.

For convenience, the following dimensionless quantities are introduced to express the maximum transverse displacement and stresses in the functionally graded porous (FGP) beams:

$$\bar{w} = \frac{100h^3 E_0}{q_0 a^4} w\left(\frac{a}{2}, \frac{b}{2}, 0\right), \quad \bar{\sigma}_x = \frac{h}{q_0 a} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right),$$

$$\bar{\tau}_{xz} = \frac{h}{q_0 a} \tau_{xz}\left(0, \frac{b}{2}, \frac{z}{h}\right), E_0 = 1 \quad (26)$$

The numerical results reported in Tables 2–7 provide a comprehensive validation of the proposed formulation for the static analysis of simply supported functionally graded porous beams FGP. The maximum non-dimensional central deflections, axial stresses, and transverse shear stresses obtained using the present model are compared with several results available in the literature for different values of the power-law index k and slenderness ratio l/h . Overall, the results demonstrate a very good agreement between the present predictions and the reference solutions, confirming the accuracy and reliability of the proposed beam formulation.

Table 2 – The maximum non-dimensional central deflection of FG SS beams subjected to sinusoidal load

l/h	Method	k				
		0	1	2	5	10
5	Lazreg Hadji[26]	2.5019	4.9458	/	7.7715	/
	Sayyad et al[27]	2.5004	4.9432	6.3745	7.7830	8.6547
	present	2.5021	4.9454	6.3762	7.7752	8.6542
20	Lazreg Hadji[26]	2.2839	4.5774	/	6.9539	7.6421
	Present	2.2839	4.5772	5.8685	6.9545	7.6423

Table 3 – The maximum non-dimensional central deflection of FG SS beams subjected to Uniform load

l/h	Method	k				
		0	1	2	5	10
5	Li et al[28]	3.1657	6.2559	8.0602	9.7802	10.8979
	G. Chandra	3.1658	6.2603	8.0609	9.7806	10.8984
	Sayyad et al[27]	3.1635	6.2563	8.0666	9.8414	10.9400
	P. Bridjesh	3.1648	6.2506	8.0535	9.7759	10.8254
	Present	3.1655	6.2589	8.0682	9.8315	10.9400
20	Li et al[28]	2.8962	5.8049	7.4415	8.8151	9.6879
	G. Chandra	2.8964	5.8052	7.4419	8.8158	9.6884
	P. Bridjesh	2.8845	5.7914	7.4321	8.7958	9.5874
	Present	2.8963	5.8048	7.4424	8.8186	9.6908

Table 4- The maximum nondimensional axial stresses of FG SS beams subjected to sinusoidal load

l/h	Method	k				
		0	1	2	5	10
5	Lazreg Hadji[26]	3.0913	4.7851		6.6047	7.9069
	Sayyad et al[27]	3.0979	4.7964	5.6149	6.6281	7.9300
	Present	3.0929	4.7870	5.6032	6.6100	7.9119
20	Lazreg Hadji[26]	12.171	18.814		25.794	30.923
	Present	12.173	18.812	21.972	25.798	30.924

Tables 2 and 3 report the maximum dimensionless central deflections of simply supported FGP beams subjected to sinusoidal and uniformly distributed loadings, respectively. For an aspect ratio of $l/h=5$ and $l/h=20$, the results obtained using the present formulation are in very close agreement with the corresponding results available in the

literature. For the sinusoidal loading case, the results are compared with those reported by Lazreg Hadji and Sayyad et al [26, 27]. For $k=0$, the present formulation gives a dimensionless central deflection of 2.5021, whereas the values reported by Lazreg Hadji and Sayyad et al. are 2.5019 and 2.5004, respectively. Similarly, for $k=1$, the present result is 4.9454, compared with 4.9458 and 4.9432 reported by Lazreg Hadji and Sayyad et al [26, 27]., respectively. The same level of agreement is maintained for the other values of the volume fraction index k . For the uniformly distributed loading case, the results in Table 3 are compared with the solutions reported by Reedy, G. et al, Reddy, G.C.M. and N.V. Kuma, Sayyad et al., and Li et al [29, 30]. The differences between the present results and the corresponding reference solutions remain small for all investigated values of k .

The effect of the gradation index k on the bending response is clearly demonstrated in Tables 2 and 3. As k increases, the dimensionless central deflection generally increases, indicating an increase in the beam deformability. In contrast, increasing the aspect ratio l/h results in a decrease in the dimensionless central deflection. When l/h increases from 5 to 20, a noticeable reduction in the deflection is observed. Under uniformly distributed loading, for example, and for $k=0$, the value predicted by the present formulation decreases from approximately 3.166 for $l/h=5$ to 2.896 for $l/h=20$. These results clearly demonstrate the opposite effects of the two parameters on the bending response of FG beams.

Table 5- The maximum nondimensional axial stresses of FG SS beams subjected to Uniform load

l/h	Method	k				
		0	1	2	5	10
5	Li et al [28]	3.8020	5.8837	6.8812	8.1030	9.7063
	G. Chandra[30]	3.8028	5.8843	6.8817	8.1035	9.7069
	Sayyad et al[27]	3.8084	5.8957	6.8971	8.1331	9.7345
	P. Bridjesh[29]	3.7958	5.8778	6.8719	8.1092	9.6973
	Present	3.8030	5.8889	6.8870	8.1144	9.7161
20	Li et al	15.0130	23.2054	27.0989	31.8112	38.1372
	G. Chandra[30]	15.0223	23.2059	27.0994	31.8117	38.1378
	P. Bridjesh[29]	15.0132	23.1832	27.1023	31.7812	38.1251
	Present	15.0090	23.2040	27.1200	31.8143	38.1380

Table 6- The maximum non-dimensional transverse shear stresses of FG SS beams subjected to sinusoidal load

l/h	Method	k				
		0	1	2	5	10
5	Lazreg Hadji[26]	0.4755	0.4755		0.3840	0.4208
	Sayyad et al[27]	0.5072	0.5430	0.5553	0.5024	0.4560
	present	0.4836	0.4835	0.4436	0.3928	0.4295
20	Lazreg Hadji[26]	0.4760	0.4760		0.3847	0.4215
	Present	0.4841	0.4841	0.4443	0.3935	0.4301

Table 7- The maximum nondimensional tangential stresses of FG SS beams subjected to Uniform load

l/h	Method	k				
		0	1	2	5	10
5	Li et al [28]	0.7500	0.7500	0.6787	0.5790	0.6436
	G. Chandra[30]	0.7339	0.7339	0.6698	0.5895	0.6379
	Sayyad et al[27]	0.7764	0.8288	0.8485	0.7654	0.6947
	P. Bridjesh[29]	0.7241	0.7311	0.6854	0.5799	0.6281
	Present	0.7436	0.7409	0.6811	0.5991	0.6534
20	Li et al [28]	0.7500	0.7500	0.6787	0.5790	0.6436
	G. Chandra[30]	0.7483	0.7483	0.6781	0.6052	0.6698
	P. Bridjesh[29]	0.7581	0.7442	0.6811	0.6102	0.6587
	Present	0.7530	0.7595	0.6893	0.6116	0.6705

Tables 4 and 5 further validate the proposed model through comparisons of the maximum nondimensional axial stresses. Under both sinusoidal and uniformly distributed loadings, the present results show very good agreement with the available reference solutions for $l/h=5$ and $l/h=20$. The small discrepancies observed with the results of Lazreg Hadji, Sayyad et al., Li et al., G. Chandra, and P. Bridjesh confirm the accuracy and reliability of the proposed formulation. The Tables 6 and 7 present the comparison of the maximum dimensionless shear stresses for two different transverse loading conditions. Table 6 corresponds to a sinusoidal transverse load, whereas Table 7

considers a uniformly distributed transverse load. For both loading cases, the results obtained using the present formulation show very good agreement with the analytical solutions reported in the literature for different values of the slenderness ratio l/h and the material gradation index k .

Overall, the comparisons presented in Tables 2–7 show excellent agreement between the results obtained using the proposed formulation and the analytical solutions available in the literature. This consistency is maintained for different values of the slenderness ratio l/h , material gradation index k , stress components, and transverse loading conditions, including sinusoidal and uniformly distributed loads. These results confirm the ability of the proposed formulation to accurately reproduce the static response of simply supported FGP porous beams.

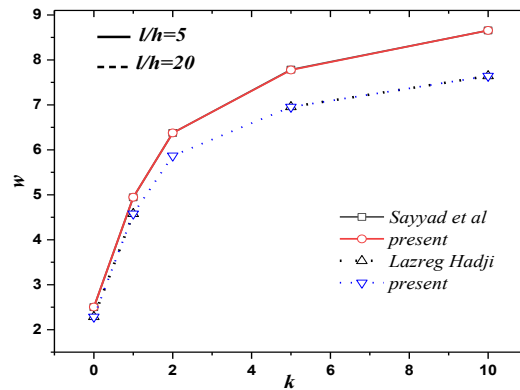


Figure 3 : Difference of non-dimensional center deflections w between deferents methods as a function of the power law index k of FG S-S beams under sinusoidal load, ($l/h=5, 20$)

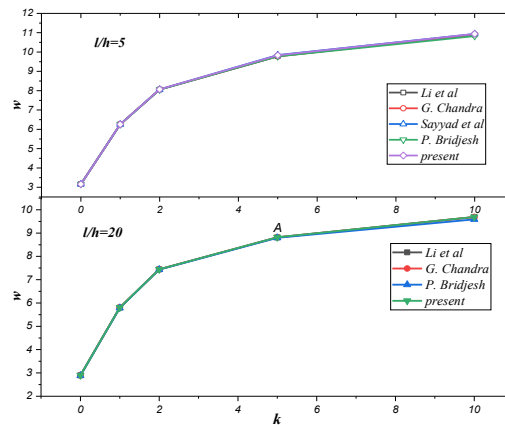


Figure 4 : Difference of axial stress between different methods as a function of the power law index k of FG S-S beams under uniform load : $l/h=5, 20$

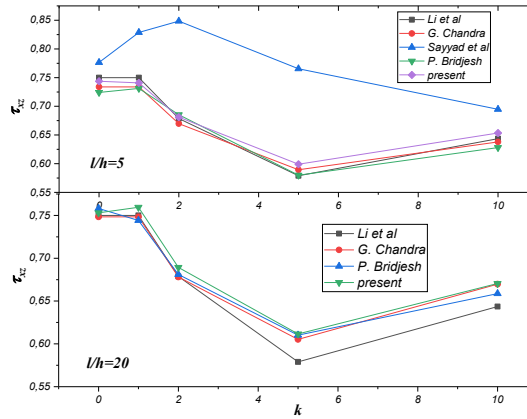


Fig.5 : Difference of tangential stress between different methods as a function of the power law index k of FG S-S beams under uniform load : (a) $l/h=5$, (b). $l/h=20$)

Figures 3–5 present a comparison of different analytical methods used to evaluate the mechanical behaviour of simply supported S–S functionally graded FGP porous beams. The analysis highlights the differences in the results obtained from these methods as a function of the power-law index k , considering two slenderness ratios: $l/h=5$, corresponding to thick beams, and $l/h=20$, corresponding to slender beams. Figure 6 illustrates the variation of the non-dimensional central deflection as a function of the slenderness ratio l/h for the two considered porosity distributions, $P1$ and $P2$, under different transverse loading conditions. The results indicate a strong dependence of the structural non-dimensional central deflection on l/h , particularly for relatively thick beams, where significant variations are observed as the slenderness ratio increases. This dependence gradually diminishes with increasing l/h , and the response tends to stabilize beyond approximately ($l/h = 15-20$). The porosity distribution also plays an important role, with $P2$ generally yielding higher values than $P1$, especially at low slenderness ratios. In addition, the transverse loading configuration has a pronounced influence on the response magnitude. The concentrated central load produces the highest response, whereas the distributed loading cases result in comparatively lower values. Overall, the results demonstrate that the mechanical response of FGP porous beams is governed by the combined effects of beam slenderness, porosity distribution, and transverse loading configuration.

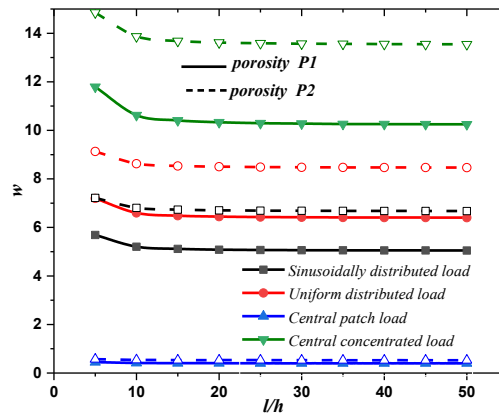


Fig. 6: Variation of the non-dimensional central deflection w versus the side-to-thickness ratio l/h of FGP beams ($k=1$) subjected to various types of mechanical load $e_0=0.4$

The analysis of the variation in the dimensionless deflection w along the beam length x/l (Figure 7) shows that the maximum deformation of the functionally graded FGP porous beam occurs at the mid-span ($x/l = 0.5$) for all loading configurations, reflecting the symmetric response of a simply supported structure, with $w(0)=w(1)=0$. The centered point load produces the highest deflection, followed by the uniformly distributed load, the sinusoidal load, and finally the central patch load, which results in the smallest deflection amplitude. Furthermore, the $P2$ porosity model consistently yields a larger deflection than the $P1$ model under the same mechanical loading conditions, demonstrating that the $P2$ porosity distribution causes a more pronounced reduction in the overall bending stiffness of the structure.

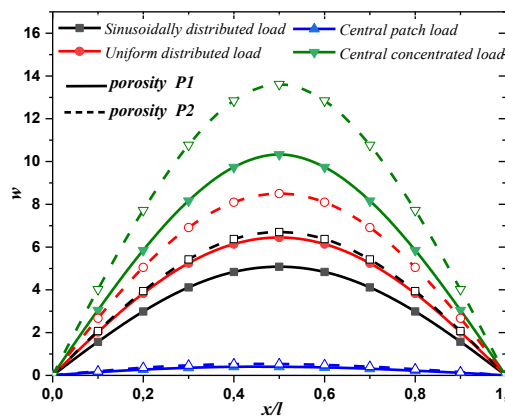


Fig. 7. Variation of the non-dimensional deflection along the length direction ($l/h=20$) of FGP beams subjected to various types of mechanical load $e_0=0.4$

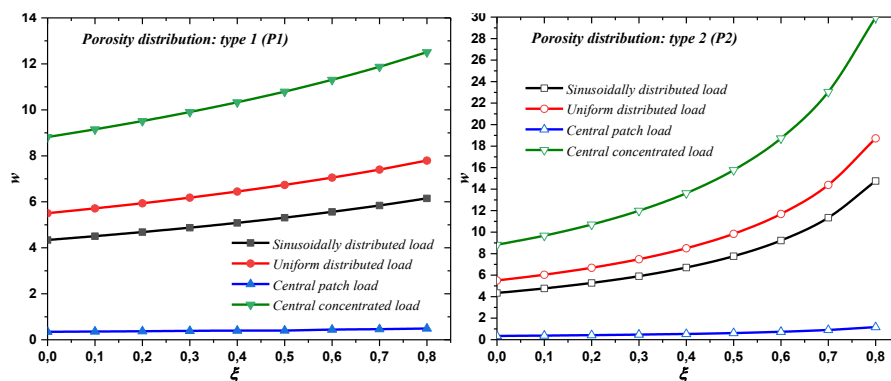


Fig. 8. Effect of porosity coefficient on variation of the non-dimensional deflection ($l/h=20$) of FGP beams subjected to various types of mechanical load with various porosity distribution types

In Figure.8 the comparative analysis of the non-dimensional deflection w with respect to the porosity parameter ξ demonstrates that increasing the pore volume reduces the overall flexural rigidity of the FGP beam, leading to a systematic increase in transverse deflections under all mechanical loading conditions. However, porosity distribution $P2$ induces a significantly more severe reduction in bending stiffness than distribution $P1$: while $P1$ results in a quasi-linear, moderate rise in deflection (reaching $w \approx 12.5$ under a central concentrated load at $\xi=0.8$), $P2$ exhibits a strongly non-linear, exponential increase that exceeds $w = 30$ for the same porosity parameter. Additionally, the loading hierarchy is preserved across both figures, where the central concentrated load generates the highest deflections due to localized stress concentration, followed in order by the uniformly distributed load, sinusoidally distributed load, and central patch load.

The distribution of the non-dimensional normal stress σ_{xx} through the thickness z/h , illustrated in Figure.9, indicates a smooth transition from compressive stresses in the lower region $z/h < 0$ to tensile stresses in the upper region $z/h > 0$, with the stress vanishing at the neutral surface $z/h=0$. As the porosity coefficient ξ increases from 0.1 to 0.5, the stress gradient becomes progressively steeper, accompanied by an increase in the maximum stress magnitudes at the outer surface $z/h=0.5$. This behaviour can be attributed to the reduction in the effective material stiffness caused by porosity, which modifies the bending stress distribution and results in a greater concentration of stress within the stiffer outer regions. A clear difference is also observed between the two porosity distributions. In particular, the $P1$ model produces higher maximum normal stresses than the $P2$ model, especially at $\xi=0.5$, for which the stress reaches approximately $\sigma_{xx} \approx 18.5$ under uniformly distributed loading, compared with about 15 for the $P2$ configuration. Furthermore, the type of transverse loading has a noticeable effect on the stress magnitude. The uniformly distributed load consistently produces higher normal stresses than the sinusoidally distributed load throughout the beam thickness, highlighting the sensitivity of the stress response to both the porosity distribution and the applied loading pattern.

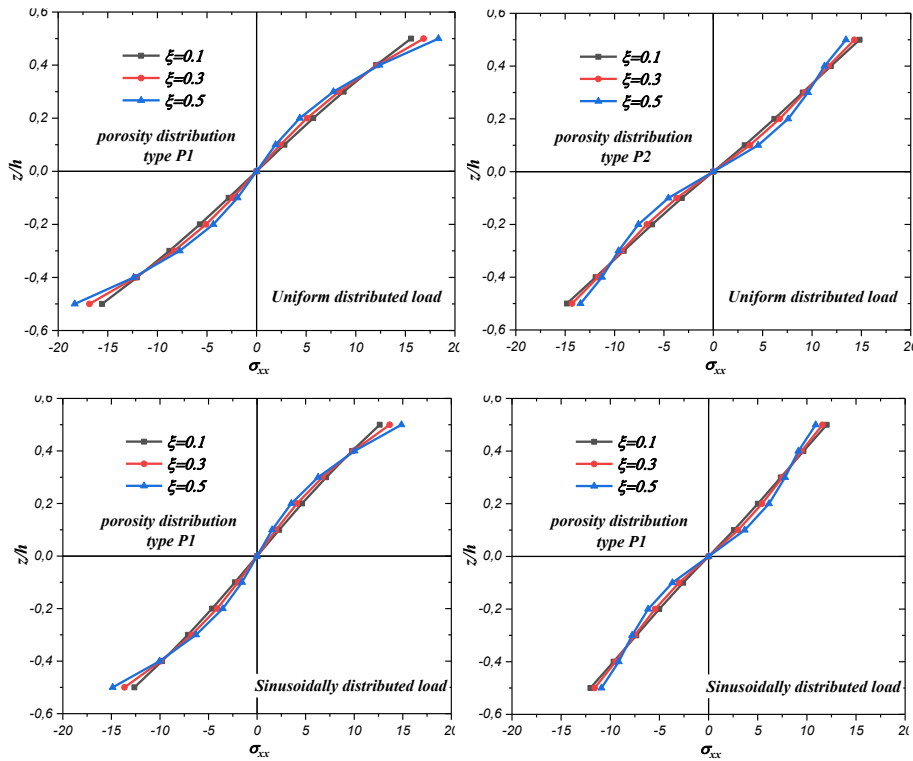
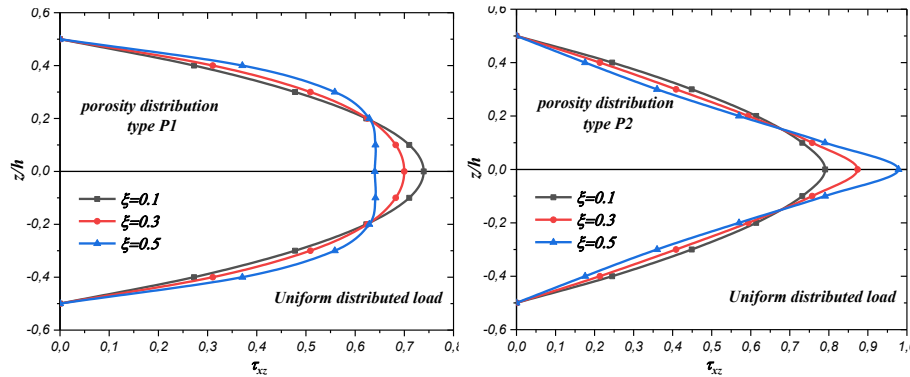


Fig.9. effect of various types of mechanical loads on distribution of normal stresses across the thickness of the FGP beam

Figure.10 illustrates the effect of the mechanical loading type, namely uniformly distributed and sinusoidally distributed loads, on the through-thickness distribution of the transverse shear stress τ_{xz} in the FGP beam for the two porosity distributions, *P1* and *P2*, and for different values of the porosity coefficient ξ . The results show that τ_{xz} vanishes at the upper and lower surfaces of the beam, in accordance with the prescribed boundary conditions, and reaches its maximum value near the mid-plane. Increasing the porosity coefficient ξ significantly affects the magnitude of the transverse shear stress, with a more pronounced influence observed for the *P2* distribution, whereas the *P1* distribution exhibits comparatively moderate variations. The comparison between the two loading conditions also indicates that the uniformly distributed load generally produces higher transverse shear stresses than the sinusoidal load. Overall, the continuous variation of τ_{xz} across the thickness demonstrates the ability of the proposed model to accurately capture the transverse shear-stress behaviour of porous FGP beams under different mechanical loading conditions.



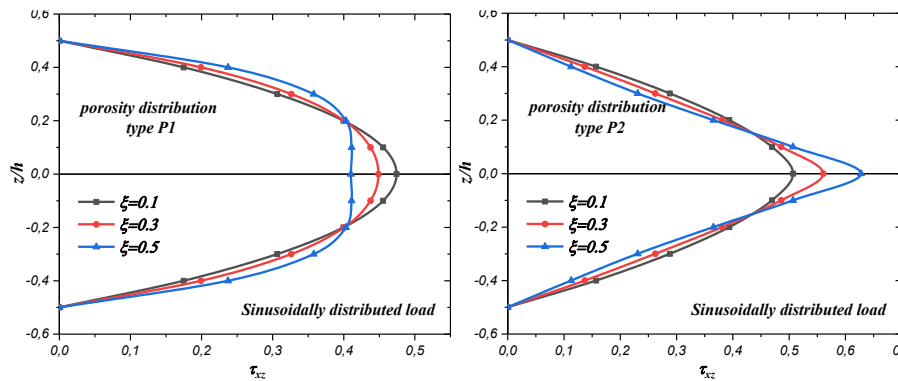


Fig.9. effect of various types of mechanical loads on distribution of transversal stresses across the thickness of the FGP beam

7. Conclusion

This study presented an analytical investigation of the static bending behaviour of functionally graded porous FGP beams using a higher-order shear deformation theory. The proposed formulation incorporates transverse shear deformation without requiring a shear correction factor and provides an efficient analytical framework for simply supported beams through Navier's method. The accuracy of the developed model was first established through extensive comparisons with available analytical solutions, showing excellent agreement for central deflections, axial stresses, and transverse shear stresses under different loading conditions, slenderness ratios, and material gradation indices. The numerical results demonstrate that increasing the power-law index generally increases the beam deformability, whereas increasing the slenderness ratio reduces the dimensionless transverse deflection, with the response becoming nearly stable for $(l/h \approx 15-20)$. Porosity was found to significantly reduce the effective flexural stiffness and consequently increase the transverse deflection and stress levels. The *P2* porosity distribution, characterized by softer surfaces, produces substantially larger deflections than the *P1* distribution, particularly at higher porosity coefficients. Among the investigated loading conditions, the central concentrated load generates the largest deflection, followed by uniformly distributed, sinusoidal, and central patch loads. The maximum deflection consistently occurs at the beam mid-span, while the transverse shear stress vanishes at the upper and lower surfaces and reaches its maximum near the mid-plane. Increasing porosity also intensifies the through-thickness stress gradients, with *P1* generally producing higher normal stresses than *P2*. Overall, the proposed formulation provides a reliable and computationally efficient tool for predicting the static response of porous FGP beams and offers useful insights into the combined effects of material gradation, porosity distribution, slenderness ratio, and loading configuration.

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