



Thermal Behavior and Flow Characteristics inside a Square Cavity with Variable Horizontal Obstacles

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Abstract

This study presents a comprehensive numerical investigation of unsteady natural convection, heat transfer, and entropy generation in a water-filled square cavity containing a centrally positioned horizontal adiabatic obstacle. The problem is relevant to buoyancy-driven transport in enclosed thermal environments, where internal obstructions can modify flow organization, thermal stratification, irreversibility, and overall energy performance. The enclosure is heated uniformly from the bottom, cooled from the top, and insulated along the vertical sidewalls. Two obstacle lengths, corresponding to 25% and 50% of the cavity width, are examined to clarify how internal blockage affects convective transport, flow stability, transition behaviour, and thermodynamic efficiency. The governing equations are solved using the finite volume method for water with a fixed Prandtl number 7.0, over a Rayleigh number range of $10^2 \leq Ra \leq 10^6$. The flow and thermal responses are analysed using streamlines, isotherms, temperature time series, phase-space trajectories, power spectra, the largest Lyapunov exponent, entropy generation, Bejan number, and the ecological coefficient of thermal performance. The results show that obstacle size has a strong influence on both flow stability and thermal performance. At high Rayleigh numbers, the influence of the 25% obstacle becomes relatively weak, producing only about 1.5% reduction in heat transfer compared with the unobstructed cavity, whereas the 50% obstacle causes a more substantial reduction of approximately 9.5%. The cavity with the 50% obstacle generates slightly higher entropy than the 25% obstacle case at the largest Rayleigh number. For all cases, the average Bejan number remains greater than 0.50, indicating that heat-transfer irreversibility is the dominant source of total entropy generation. The 25% obstacle case achieves the highest ecological coefficient of thermal performance, with a maximum value of 0.1103, compared with 0.0968 for the 50% obstacle and 0.1023 for the unobstructed cavity. These findings demonstrate that a moderate internal obstacle can provide a favourable balance between heat-transfer enhancement and entropy generation, leading to improved thermodynamic performance in buoyancy-driven enclosed flows.

Keywords: Natural convection; Unsteady flow; Effect of obstacle; Entropy generation; Heat transfer.

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1. Introduction

Natural convection (NC) in confined enclosures has received significant attention because of its central role in an inclusive range of engineering applications, including electronic cooling, solar energy systems, thermal insulation in buildings, and heat energy storage technologies [1, 2]. In such systems, fluid motion is predominantly driven by buoyancy forces generated from temperature differences along the cavity boundaries, leading to intricate flow patterns and HT characteristics. These patterns are strongly influenced by the enclosure geometry, imposed boundary conditions, and thermophysical characteristics of the working fluid. As a result, substantial research has been undertaken on NC in rectangular, triangular, square, and cylindrical cavities subjected to diverse thermal configurations. For example, Oztop and Bilgen [3] numerically investigated NC inside an unevenly heated square enclosure containing a heat-generating fluid and a partially dividing isothermal partition attached to the bottom wall. The study reported important insights into the control of NC and thermal management in enclosures with interior heat generation and partial obstructions.

A comprehensive understanding of NC in enclosures of varying geometrical configurations, such as square, rectangular, trapezoidal, and cylindrical cavities is essential, as cavity shape plays a decisive role in determining flow structures, temperature fields, and overall HT performance. Geometric variations significantly influence boundary layer development, circulation structures, and the initiation of flow instabilities, thus governing the effectiveness of convective HT. Several studies have addressed these aspects in different configurations. For instance, Bouafia and Daube [4] examined NC around a square solid body placed within a rectangular enclosure under immense temperature gradients, demonstrating that robust thermal gradients markedly alter flow structures and augment HT in the vicinity of the solid body. Bhowmick et al. [5] investigated NC in a V-shaped triangular cavity and reported that increasing the Ra leads to a changeover from steady to chaotic flow, with geometry exerting a strong influence on flow instability and HT characteristics. In a similar investigation, Bhowmick et al. [6] analyzed UNC in a valley-shaped (triangular) enclosure, highlighting the dependence of HT behavior on the Ra. Furthermore, Karataş and Derbentli [7] explored NC in differentially heated rectangular cavities subjected to time-varying periodic boundary conditions, resulting that temporal temperature discrepancies significantly change flow patterns and thermal fields, revealing in oscillatory convection. Saha et al. [8] analyzed HT characteristics through variations in the Nusselt number under different thermal conditions, providing further insight into convective transport behavior.

Charles et al. [9] investigated turbulent NC within a rectangular enclosure and reported that turbulence significantly augments HT compared to laminar regimes, whereas producing highly complex and time dependent temperature and velocity fields. In a related study, Berdnikov et al. [10] analyzed the transient expansion of NC in a rectangular enclosure acted upon by sudden heating of a side wall, showing that intense thermal forcing generates evolving flow patterns as well as dynamic temperature circulations, resulting in intricate circulatory flow structures. Fichera et al. [11] proposed an analytical framework for NC in a rectangular cavity with unevenly heated side walls, showing that the imposed temperature difference strongly governs circulation intensity and the development of thermal boundary layers. More recently, Rahaman et al. [12] explored UNC within a thermally stratified trapezoidal enclosure heated from underneath, demonstrating that preliminary stratification plays a crucial role in initiating and shaping the development of convective flow. In a follow-up study, Rahaman et al. [13] further examined transient NC and HT inside a stratified trapezoidal enclosure, providing that the strength of stratification plays a decisive role in controlling

HT characteristics and overall thermal behavior in confined systems.

Square enclosures are widely adopted as canonical configurations within NC studies because of their geometric simplicity and easily defined boundary conditions. Such configurations provide an effective platform for detailed theoretical, numerical, and experimental investigations of fundamental thermo-fluid phenomena, including boundary layer development, hydrodynamic instabilities, thermal stratification, and the changeover between laminar and turbulent flow regimes. A seminal benchmark study by De Vahl Davis [14] presented accurate steady-state numerical solutions for air-filled square enclosures over a Ra range of 10^3 to 10^6 , which has since served as a reference for validation of numerical models. Subsequent research has extended these foundational studies to incorporate more complex physical effects. Hakeem et al. [15] examined NC within a square cavity containing thermally active plates subjected to various boundary conditions, revealing that both the position and temperature of the plates exert a strong influence on HT phenomena. Choi and Kim [16] performed an analogous assessment of different thermal lattice Boltzmann frameworks, evaluating their precision and numerical constancy over a wide range of Ra.

Experimental as well as numerical analyses have further explored the impact of internal bodies and geometric modifications. Butler et al. [17] experimentally analyzed convection from a heated horizontal cylinder placed in an unevenly heated square cavity, showing that interactions between the cylinder and enclosure walls substantially modify the flow patterns and enhance local HT. Bhattacharya and Basak [18] reported that HT rates increase with rising Ra in square cavities. Saravanan [19] conducted a numerical study of NC within a square enclosure with interior heat-generating baffles, demonstrating the effect of baffle geometry and heat production on the flow dynamics and thermal characteristics. More recently, Shaon et al. [20] explored a comparative numerical study of UNC within square and rectangular enclosures filled with stratified fluid, highlighting the critical role of geometry in shaping transient flow dynamics and HT performance. Their findings indicate that rectangular enclosures tend to exhibit stronger and more complex convective circulation patterns compared to square cavities under similar thermal and stratification conditions.

Direct numerical and experimental investigations of NC in square enclosures have yielded substantial insights into flow structures, thermal behavior, and HT mechanisms under a variety of physical conditions. Kouroudis et al. [21] conducted direct numerical simulations with uniformly imposed heat fluxes on vertical walls, revealing the formation of multiple circulation cells and a transition from steady to unsteady flow with increasing Ra, along with the identification of critical thresholds for the onset of oscillatory convection. El Moutaouakil et al. [22] explored the combined effects of NC and surface radiation in a square enclosure containing an internal wavy body, demonstrating that cavity inclination, geometric waviness, and radiative HT significantly influence both flow patterns and temperature distributions. Wang et al. [23] examined UNC in a square enclosure with a centrally heated circular cylinder and showed that the vertical position of the cylinder plays a key role in controlling the initiation and intensity of periodic flow and thermal oscillations, thereby affecting overall HT performance. Similarly, Oyewola et al. [24] investigated convective HT in a square cavity with a circular internal heat source, highlighting the strong dependence of flow circulation, temperature fields, and HT efficiency on the location of the heat-generating body. El and Lafdaili [25] analyzed turbulent NC in a nanofluid-filled square enclosure subjected to an inclined magnetic field, revealing that nanoparticle concentration, Ra, and magnetic field orientation collectively govern flow dynamics and HT. Zemani et al. [26] investigated NC inside a square enclosure with a partially heated curved wall, demonstrating that both wall

geometry and heating configuration significantly influence flow organization and HT characteristics. Bawazeer and Alsoufi [27] further examined the effects of Ra and Pr, showing that increasing Ra intensifies buoyancy-driven circulation and thermal gradients, while variations in Pr markedly affect the development of thermal and velocity boundary layers.

Internal obstacles play a crucial role in modifying NC within enclosed systems by altering flow structures and HT characteristics. Their influence depends on key parameters such as size, shape, position, and material properties, and they are widely utilized in engineering applications, including cooling systems and heat exchangers, to enhance thermal performance. Raji et al. [28] conducted a numerical investigation on the effect of obstacle subdivision in a square cavity and demonstrated that geometric modification of internal obstructions significantly alters flow patterns and HT behavior. In particular, subdividing a solid obstacle promotes the interaction of multiple recirculating cells, enhancing fluid mixing and improving overall heat transfer compared to a single, continuous block. Chorin et al. [29] analyzed the impact of a localized internal obstacle in a differentially heated enclosure and showed that it significantly disturbs the primary circulation cell, leading to the formation of secondary vortices and offering a potential passive mechanism for controlling HT. Nouni et al. [30] employed the lattice Boltzmann method to study the effects of internal obstacles in a square enclosure, highlighting its capability to capture complex flow dynamics and emphasizing the importance of obstacle design in regulating convective transport. Hamid et al. [31] investigated NC in a square cavity with curvilinear corners and a centrally located circular obstacle, demonstrating that geometric modifications, particularly rounded boundaries affect boundary layer development and vortex formation. Kumar and Mahapatra [32] further explored the role of obstacle shape in a partially open square enclosure, concluding that its influence becomes increasingly significant at higher Ra due to intensified buoyancy-driven flow.

Although NC in enclosed cavities has been widely investigated, studies on square cavities containing centrally located horizontal adiabatic obstacles remain relatively limited, particularly those providing a comprehensive analysis of the transition from steady to chaotic flow, E_{gen} , and thermal performance over a wide range of Ra. To address this gap, the present study numerically investigates NC, HT, E_{gen} , and thermal performance in water-filled square cavities containing a centrally positioned horizontal adiabatic obstacle with lengths corresponding to 25% and 50% of the cavity width. The governing equations are solved using the finite volume method (FVM) for Ra ranging from 10^0 to 10^6 at a constant Pr of 7.0. Particular attention is devoted to the evolution of flow structures, temperature distributions, and the transition from steady to chaotic flow as the buoyancy force increases. In addition, the effects of obstacle size on the thermal and hydrodynamic characteristics of the cavity are systematically examined through detailed analyses of HT, E_{gen} , and thermal performance indicators. A comprehensive comparison between the obstructed and unobstructed configurations is also conducted to quantify the influence of the internal obstacle on convection characteristics and overall thermal performance within enclosed cavities.

2. Nomenclature

C_p	specific heat (J/kg.K)
$d(u_i)$	geometric distance between u_i and u_{i-1}
E_f	entropy generation due to fluid friction
E_{gen}	entropy generation
E_{avg}	average entropy generation
E_θ	entropy generation due to heat transfer

g	gravitational acceleration (m/s^2)
H, L	length and height of the cavity (m)
K	thermal conductivity ($\text{W}/(\text{m}\cdot\text{K})$)
Nu	average Nusselt number
P	pressure (N/m^2)
p	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number, $g\beta(T_h - T_c)H^3/\nu\kappa$
t	time (s)
T	temperature (K)
T_c	temperature of the cooled vertical wall (K)
T_h	temperature of the heated vertical wall (K)
T_∞	environmental temperature (K)
U, V	velocity components (m/s)
u, v	dimensionless velocity components
X, Y	coordinates
x, y	dimensionless coordinates
Greek symbols	
τ	dimensionless time
$\Delta\tau$	dimensionless time step
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
κ	thermal diffusivity (m^2/s)
θ	dimensionless temperature
θ_c	dimensionless temperature of the cooled right vertical wall
θ_h	dimensionless temperature of the heated left vertical wall
λ_L	largest Lyapunov exponent

3. Computational domain

The present study investigates two-dimensional UNC and HT in a square cavity filled with water ($\text{Pr} = 7.0$). The enclosure is subjected to a constant thermal gradient with the bottom wall maintained at a uniform hot temperature (T_h), the top wall kept at a lower cold temperature (T_c) and the vertical sidewalls assumed adiabatic. In the definition of the dimensionless temperature (θ), its value depends on choice of the reference temperature (T_∞). If $T_\infty = T_c$, dimensionless temperature is defined as $\theta = (T - T_c) / (T_h - T_c)$. Accordingly, the dimensionless temperature is $\theta_c = 0$ at the cold wall, where $T = T_c$ and $\theta_h = 1$ at the hot wall, where $T = T_h$. To examine the effect of obstacle size on the flow and thermal behavior centrally positioned horizontal adiabatic obstacles with different lengths are introduced within the cavity. All cavity walls and obstacle surfaces are considered stationary and satisfy the no-slip boundary condition. The schematic of the physical domain shown in [Fig 1](#).

Natural convection within a square enclosure containing a centrally positioned horizontal adiabatic obstacle is investigated by using the coupled Navier–Stokes and energy equations. The flow is assumed to be two-dimensional, transient, laminar, and incompressible. The working fluid is considered Newtonian with constant thermophysical properties, except for the density variation in the buoyancy term, which is incorporated through the Boussinesq approximation. Accordingly, the non-dimensional governing equations adopted in the present analysis are formulated under the Boussinesq assumption and are expressed as follows [33]:

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\ \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\text{Pr}}{\sqrt{\text{Ra}}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \\ \frac{\partial v}{\partial \tau} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\text{Pr}}{\sqrt{\text{Ra}}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \text{Pr} \theta, \\ \frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{\text{Ra}}} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right). \end{cases} \quad (1)$$

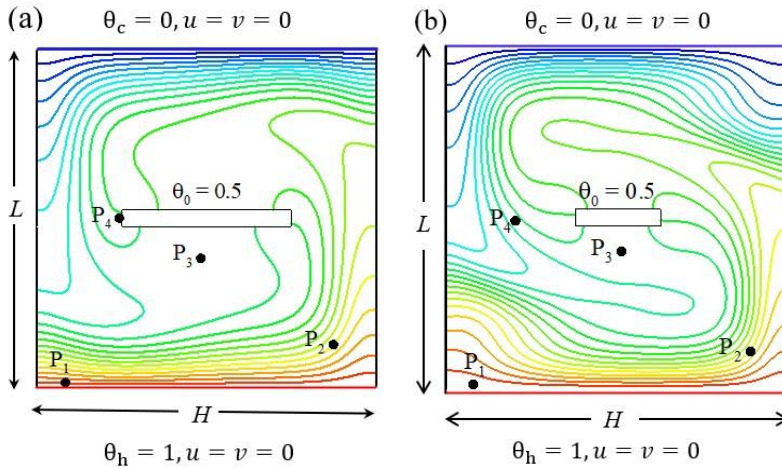


Fig 1: Computational geometry and non-dimensional boundary conditions square-shaped cavity with horizontal obstacles of 50% and 25% of cavity's length, where P_1 (-0.38, 0.01), P_2 (0.38, 0.15), P_3 (0, 0.25), P_4 (-0.26, 0.50) are results monitoring points and used in the subsequent figures.

The dimensionless boundary conditions are:

$$\begin{cases} \text{Cooled top wall : } \theta_c = u = v = 0, \\ \text{Heated bottom wall : } \theta_h = 1, u = v = 0, \\ \text{Thermally insulated vertical walls : } \frac{\partial \theta}{\partial x} = 0, u = v = 0, \\ \text{Adiabatic horizontal obstacle : } \frac{\partial \theta}{\partial n} = 0, u = v = 0. \end{cases} \quad (2)$$

The non-dimensional variables that were utilized are as follows:

$$x = \frac{X}{H}, y = \frac{Y}{H}, u = \frac{UH}{\alpha \sqrt{\text{Ra}}}, v = \frac{VH}{\alpha \sqrt{\text{Ra}}}, p = \frac{PH^2}{\rho \alpha^2 \text{Ra}}, \theta = \frac{T - T_c}{T_h - T_c}, \tau = \frac{t \alpha \sqrt{\text{Ra}}}{H^2}. \quad (3)$$

Here, x, y, u, v, p, τ and θ denote the non-dimensional counterparts of X, Y, U, V, P, t and T , respectively. Two results monitoring parameters in the enclosure: Ra and Pr defined as:

$$\text{Ra} = \frac{g \beta (T_h - T_c) H^3}{\nu \alpha} \text{ and } \text{Pr} = \frac{\nu}{\alpha}. \quad (4)$$

4. Numerical model, grid test and validation

4.1. Methodology

In the present study, the governing Eq. (1) along with the boundary conditions given in Eq. (2), are solved using the FVM within the ANSYS Fluent (more details of the numerical procedures are available in the FLUENT User's Guide). The pressure-velocity coupling is resolved using the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm. The diffusion terms are discretized employing a second order central differencing scheme whereas the convective terms are approximated using the third-order Quadratic Upstream Interpolation for Convective Kinematics (QUICK) scheme [33]. To improve spatial accuracy, a non-uniform structured rectangular mesh is adopted throughout the computational domain. Numerical stability and iterative convergence are ensured through the application of appropriate under-relaxation factors, as recommended by [12, 13]. Furthermore, the transient terms are discretized using a second-order implicit time-integration scheme. Convergence of the numerical solution is achieved when the residuals of the continuity, momentum, and energy equations fall below 10^{-5} .

4.2. Grid dependent test

To ensure the accuracy and numerical reliability of the simulations, a grid- and time-step independence study was conducted for the most demanding flow condition considered in the present work, namely $Ra = 10^6$ with the 50% obstacle configuration. Since higher Ra generate stronger thermal gradients and more complex flow structures, a discretization scheme that provides convergent solutions at this condition is expected to remain adequate for lower Rayleigh numbers.

For this purpose, three symmetric non-uniform meshes (150×150 , 200×200 and 250×250) together with two dimensionless time steps (0.05 and 0.025) were examined. Fig 2 depicts the temporal variation of the average temperature at monitoring point P_3 for the 50% obstacle case at $Ra = 10^6$, obtained using different grid resolutions and time-step sizes. The computed temperature histories exhibit excellent agreement during the transient stage, while only negligible deviations are observed in the fully developed regime. These findings demonstrate that the numerical results are essentially independent of both spatial and temporal discretization, confirming that the selected grid and time-step provide sufficient accuracy and numerical stability for all simulations performed in this study.

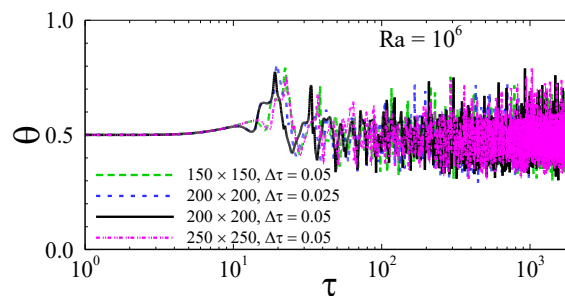


Fig 2: Temperature time series at the point $P_3(0, 0.25)$ for $Ra = 10^6$ and 50% obstacle using different grids and time steps.

Table 1 presents the average temperature values in the fully developed state (FDS) obtained using different grid resolutions and time-step sizes for the 50% obstacle configuration. The results show that the variation in the average temperature decreases progressively with mesh refinement, indicating convergence of the numerical solution. The largest deviation is observed between the 150×150 and 200×200 grids, whereas only a marginal difference exists between the 200×200 and 250×250 grids. Similarly, the influence of the time-step size on the computed temperature remains negligible. Considering the trade-off between computational cost and solution accuracy, the 200×200 non-uniform grids combined with a dimensionless time step of $\Delta\tau = 0.05$ is selected for all subsequent simulations, as it provides sufficiently accurate and computationally efficient results.

Table 1: Temperatures at $P_3(0, 0.25)$ in the fully developed stage using different meshes and time steps.

Mesh and time step	Average Temperature	Difference
150×150 and $\Delta\tau = 0.05$	0.568	1.73%
200×200 and $\Delta\tau = 0.025$	0.572	1.04%
200×200 and $\Delta\tau = 0.05$	0.578	--
250×250 and $\Delta\tau = 0.05$	0.583	0.86%

In addition, the non-dimensional dissipative time scale ($\lambda_k = A_k / H$) is evaluated using the following relation (see, ref. [34]):

$$\lambda_k = \left(32\pi\sqrt{2} \right)^{\frac{1}{4}} \left(\frac{Ra}{Pr} \right)^{-\frac{3}{8}}. \quad (5)$$

For $Ra = 10^6$, the corresponding value of λ_k is 0.04028, which justifies the selection of non-dimensional time steps of 0.05 and 0.025 for the present analysis

4.3. Model validation

To ensure the accuracy and reliability of the present numerical methodology, the results were first validated against the well-established benchmark problem of NC in a differentially heated square cavity without an internal obstacle. This configuration has been extensively investigated in the literature and is widely recognized as a standard validation case for buoyancy-driven flow and HT simulations. The present results were compared with the benchmark study of Xin and Quéré [35], who examined NC in a square cavity with adiabatic horizontal walls, shown in Fig 3. Their study reported the base-flow structure and fully developed thermal field at a Ra of 1.6×10^8 .

For the present validation, a non-uniform 200×200 computational grid was employed to accurately resolve the thermal and hydrodynamic BL near to the heated and cooled walls. The computed streamline and isotherm contours at $Ra = 1.6 \times 10^8$ and $\tau = 2000$ exhibit excellent agreement with the benchmark results of Xin and Quéré [35]. The close correspondence between the present and benchmark results confirms the capability and accuracy of the adopted numerical scheme in predicting high-Rayleigh-number NC flows within enclosed cavities.

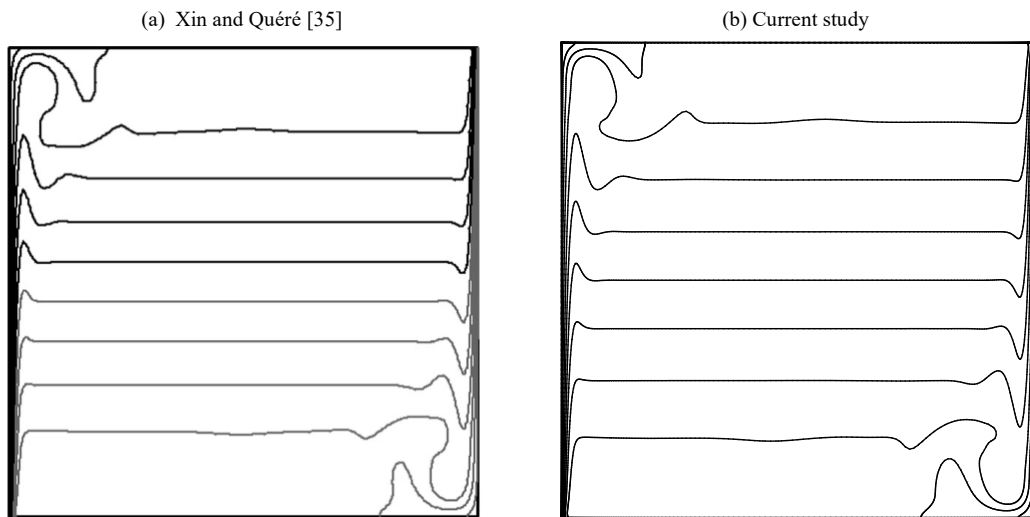


Fig 3: Comparison of isotherms obtained in the present study with those reported by of Xin and Quéré [35] for $Ra = 1.6 \times 10^8$.

A quantitative validation is further performed by comparing the Nu from the present simulations with the benchmark results of Bawazeer and Alsoufi [27] over the range $Ra = 10^2$ to 10^6 and a fixed Pr of 0.71. As summarized in Table 2, the comparison shows very good consistency over the full range of Ra , with deviations remaining within 2%. This close correspondence further substantiates the accuracy and robustness of the present model.

Table 2: Comparison of averaged Nu between the present study and Bawazeer and Alsoufi [27].

Ra	Present study	Bawazeer and Alsoufi [27]	% of deviation
10^2	1.0127	1.0125	0.02
10^3	1.1287	1.1178	0.98
10^4	2.2197	2.2449	1.12
10^5	4.6088	4.5217	1.93
10^6	8.9189	8.8255	1.06

5. Results and discussion

Using the validated numerical framework, the present study examines the transient characteristics of UNC, HT, entropy generation (E_{gen}) within a water-filled square cavity. The thermal boundary conditions consist of a uniformly heated bottom wall, a uniformly cooled top wall, and adiabatic vertical sidewalls. To examine the influence of internal geometric obstruction on flow dynamics and thermal performance, centrally positioned horizontal adiabatic obstacles with widths of 25% and 50% of the cavity length are considered. The simulations are conducted for a fixed Pr of 7.0 over a wide range of Ra ($10^0 - 10^6$) to elucidate the flow evolution and thermal transport mechanisms. At low Ra, HT is primarily governed by conduction. As Ra rises buoyancy effects become more pronounced, leading to the development of stronger convective circulation. At higher Ra, the flow becomes inherently unsteady and exhibits complex temporal behavior. The presence of centrally located adiabatic obstacles modifies the flow topology by obstructing the primary circulation and altering the temperature distribution within the enclosure. The extent of these modifications depends strongly on the obstacle size. The 25% obstacle permits relatively stronger circulation and more effective thermal mixing, whereas the 50% obstacle imposes greater resistance to fluid motion and reducing the overall thermal performance.

5.1. Development of symmetric steady flow

Fig 4 presents the temporal evolution of the streamline and isotherm distributions at selected dimensionless times ($\tau = 0, 5$, and 100) for the cavity containing a 25% centrally positioned horizontal adiabatic obstacle at low Ra of 10^0 to 10^2 . These results illustrate the development of the flow and thermal fields from the initial condition to the final steady state under conduction-dominated natural convection. At the initial stage ($\tau = 0$), the fluid is quiescent, and the temperature field is nearly uniform throughout the cavity. Consequently, streamlines are absent or extremely weak because buoyancy-driven motion has not yet developed. At $\tau = 5$, the flow begins to develop in response to the imposed thermal gradient. For all Ra considered, a pair of weak counter-rotating vortices is formed around the centrally located obstacle. The obstacle locally disturbs the flow, forcing the fluid to circulate around its upper and lower surfaces. The isotherms remain nearly horizontal, indicating that HT is still predominantly governed by thermal conduction. By $\tau = 100$, the flow reaches a steady-state configuration for all three Ra values. At $Ra = 10^3$, buoyancy effects remain negligible, resulting in very weak circulation and nearly parallel isotherms characteristic of pure conduction.

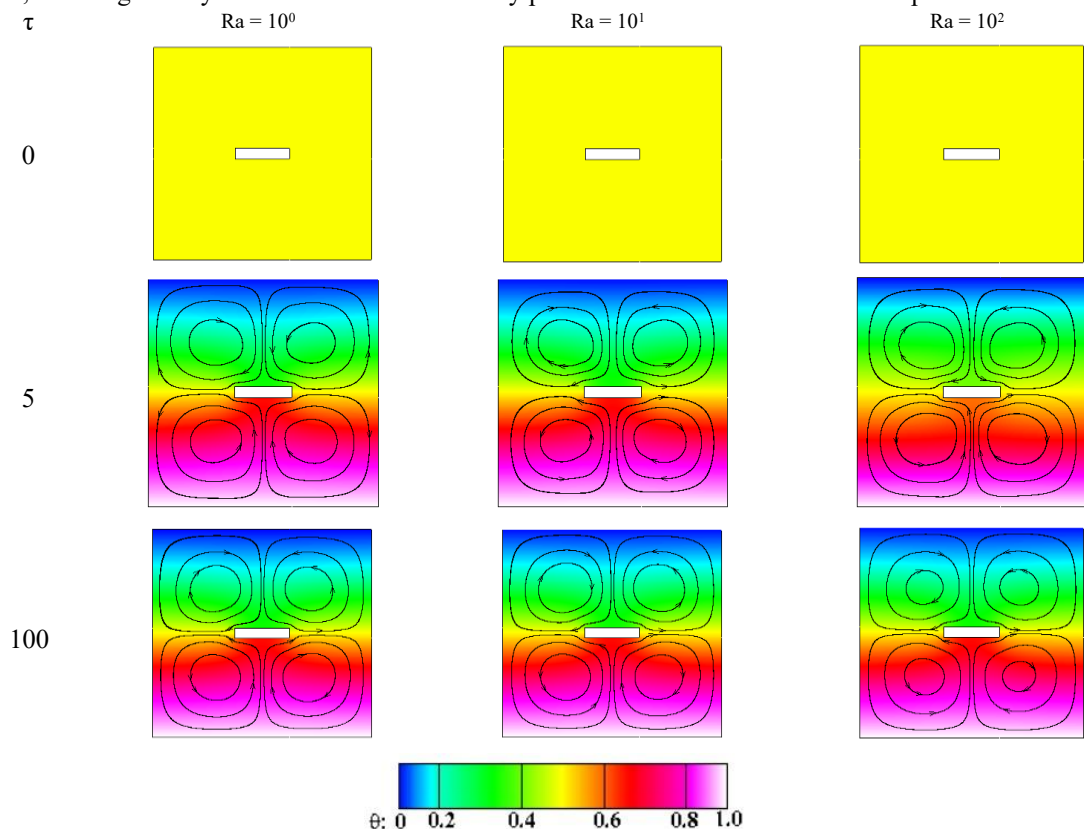


Fig 4: Temporal evolution of streamline and isotherm distributions at selected dimensionless times (τ) for $Ra = 10^0 - 10^2$ with a 25% obstacle.

Fig 5 illustrates the temporal evolution of the streamline and isotherm distributions at selected dimensionless times ($\tau = 0, 5$, and 100) for the cavity containing a centrally positioned horizontal adiabatic obstacle with a width equal to 50% of the cavity length. The flow development is examined for low Ra of 10^0 to 10^2 , where buoyancy effects are relatively weak and HT is primarily governed by thermal conduction. At the initial instant ($\tau = 0$), the fluid is stationary and the temperature field is nearly uniform throughout the enclosure. Consequently, no appreciable circulation is observed, and the streamlines are absent. At $\tau = 5$, a pair of symmetric counter-rotating vortices develops on either side of the obstacle. By $\tau = 100$, the flow reaches a steady-state configuration for all Ra considered. Owing to the larger obstacle size, the available flow passage is significantly reduced, forcing the fluid to move through the narrow gaps above and below the obstacle. As a result, the circulation remains weaker than that observed for the 25% obstacle under the same Ra.

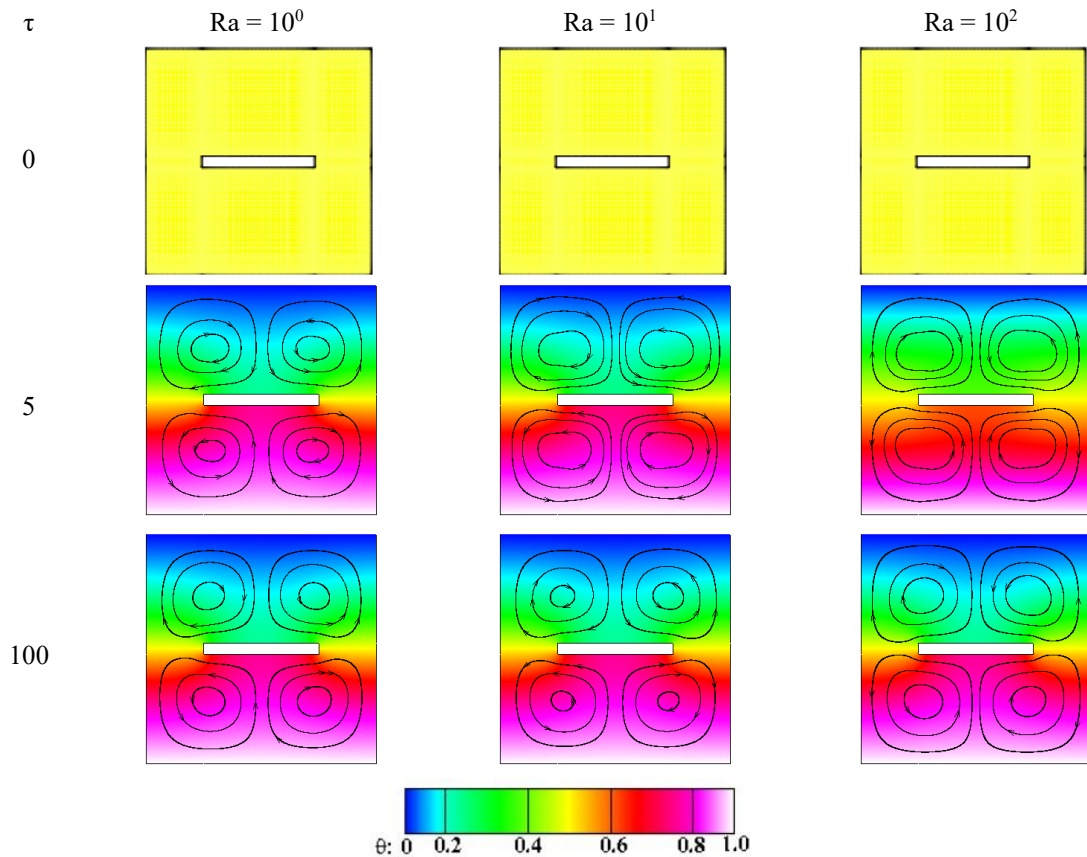


Fig 5: Temporal evolution of streamline and isotherm distributions at selected τ for $Ra = 10^0 - 10^2$ with a 50% obstacle.

5.2. Development of asymmetrical steady flow

Fig 6 illustrates the streamline and isotherm distributions comparing the steady convective regimes of the 25% and 50% horizontal obstacle configurations across intermediate Rayleigh numbers ($Ra = 10^3, 10^4, 10^5$) mapping the structural transition from geometric symmetry to steady asymmetry. At $Ra = 10^3$, both configurations exhibit a perfectly symmetric, four-cell steady-state flow field composed of mirror-image vortex pairs above and below the horizontal plates, matched by horizontally uniform thermal stratification lines that signify a highly balanced conduction-dominated state. However, elevating the buoyancy force to $Ra = 10^4$ triggers a clear structural divergence where geometric confinement plays a major role; the 25% obstacle configuration experiences a pitchfork bifurcation, breaking macro-symmetry as the upper right and lower left convective cells deform and expand unevenly, which forces the isotherms to twist diagonally across the central plate. In contrast, the elongated 50% obstacle acts as a passive stabilizer at $Ra = 10^4$, successfully suppressing these initial asymmetric disturbances to maintain an ordered, symmetric four-cell distribution. When the thermal driving force is pushed further to $Ra = 10^5$, both systems ultimately

converge into strongly advective, asymmetric steady-state topologies where severe distortions and plume-like plumes in the isotherms indicate that vigorous natural convection has completely overcome the viscous damping and structural resistance of both the short and long internal partitions.

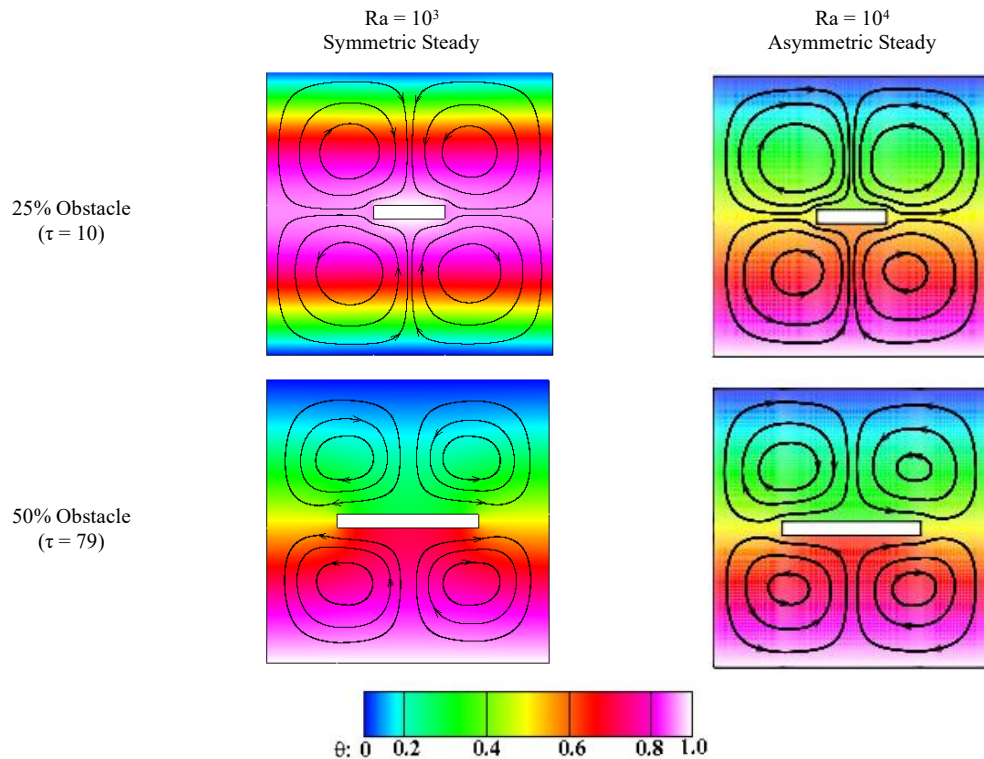


Fig 6: Streamlines and isotherms for $Ra = 10^3, 10^4$ with a 25% and 50% obstacle depicting asymmetrical steady flow.

Fig 7 presents the temporal evolution of the velocity at monitoring point $P_3 (0, 0.25)$ for cavities containing 25% and 50% horizontal adiabatic obstacles, illustrating the onset of flow asymmetry as the Ra increases. At low Ra , the velocity remains zero with time, indicating a steady and symmetric flow field dominated by conductive HT. As the buoyancy force intensifies, the velocity begins to deviate from its constant value, marking the loss of flow symmetry and the emergence of an asymmetric state. This transition is characterized by the appearance of temporal fluctuations in the velocity, reflecting the onset of hydrodynamic instability induced by stronger NC. For the cavity with the 25% obstacle, the flow remains symmetric at $Ra = 6 \times 10^3$ and transition to asymmetric state at $Ra = 7 \times 10^3$, where the velocity exhibits non-zero value. The smaller obstacle provides less obstruction to the buoyancy-driven circulation, allowing the primary vortex to strengthen and become unstable at an earlier stage. As a result, the flow departs from the symmetric steady state more readily, indicating greater sensitivity to buoyancy-induced perturbations. In contrast, the 50% obstacle delays the onset of the asymmetric state. The larger obstacle restricts fluid circulation and suppresses the growth of convective instabilities by reducing the available flow passage and weakening the interaction between the upper and lower regions of the cavity. Consequently, the velocity remains zero for $Ra = 9 \times 10^3$, signifying the symmetric flow. At $Ra = 10^4$, the velocity becomes non-zero which indicate an asymmetric flow behaviour. The cavity with 25% obstacle a pitchfork bifurcation occurs between $6 \times 10^3 < Ra \leq 7 \times 10^3$, whereas the cavity with 50% pitchfork bifurcation occurs between $9 \times 10^3 < Ra \leq 10^4$.

5.3. Temporal evaluation of temperature

Fig 8 illustrates the temporal evolution of the TTS at monitoring point $P_1 (-0.38, 0.01)$ for cavities containing 25% and 50% centrally positioned obstacles. The temporal evolution of the TTS provides a clear indication of the successive transitions from steady to periodic and ultimately chaotic flow as the Ra increases. At relatively low Ra , the TTS rapidly approaches a constant value after a short transient period, indicating a steady-state convection regime. As the Ra exceeds the critical value, the TTS begins to exhibit regular sinusoidal oscillations, signifying the onset of periodic flow. For the cavity with a 25% obstacle, sustained periodic oscillations first appear at $Ra = 10^5$, whereas the corresponding transition occurs at the higher value of $Ra = 2 \times 10^5$ for the 50% obstacle. The oscillations possess

nearly constant amplitude and period, indicating the formation of a stable limit-cycle attractor associated with self-sustained buoyancy-driven flow. These periodic fluctuations arise from the cyclic generation, growth, and interaction of thermal plumes within the cavity, leading to repetitive variations in the local temperature field.

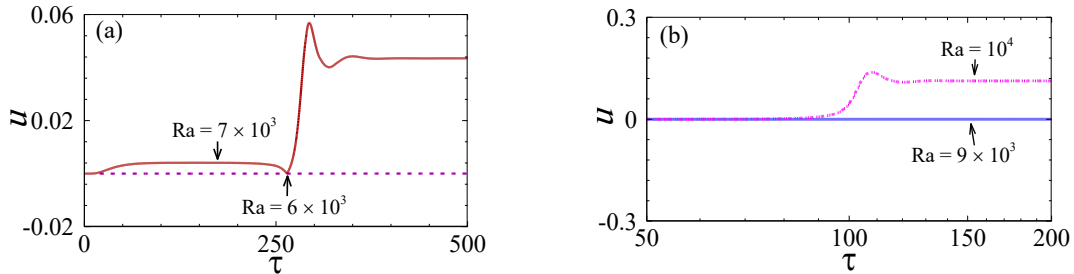


Fig 7: Velocity time series illustrating the transition to asymmetric state at the point $P_3(0, 0.25)$ with (a) 25% and (b) 50% obstacle respectively.

With a further increase in the Ra , the periodic oscillations lose their regularity and evolve into irregular fluctuations with continuously varying amplitudes and frequencies, marking the onset of chaotic flow. For the 25% obstacle, this transition is observed at $Ra = 7 \times 10^5$, whereas the 50% obstacle reaches the chaotic regime at the lower Ra of $Ra = 5 \times 10^5$. The temperature signals become increasingly complex, exhibiting aperiodic behavior without a discernible repeating pattern. Such broadband fluctuations are characteristic of deterministic chaos, resulting from strong nonlinear interactions among multiple vortical structures and thermal plumes. A comparison between the two obstacle configurations demonstrates that obstacle size significantly influences the critical Ra associated with each dynamical transition. Although the larger (50%) obstacle delays the transition from steady to periodic convection because of increased geometric confinement and suppression of the primary circulation, it promotes an earlier transition from periodic to chaotic behavior once oscillations are established.

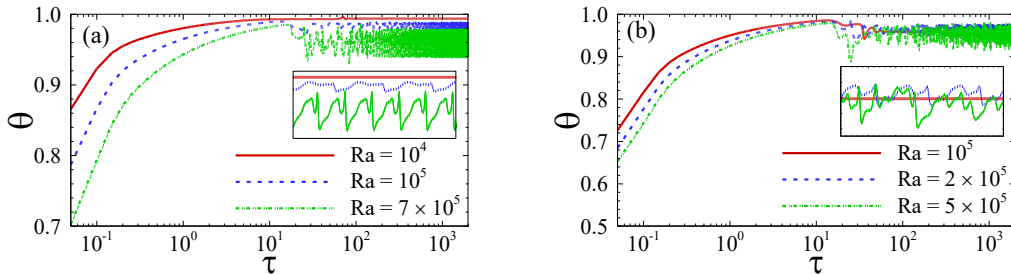


Fig 8: Temperature time series illustrating the transition from steady to chaotic state at the point $P_1(-0.38, 0.01)$ with (a) 25% and (b) 50% obstacle respectively.

5.4. Development of unsteady flow

Fig 9 presents the instantaneous streamline and isotherm distributions at the dimensionless time ($\tau = 20$), showcasing the transition of the thermal and flow fields into highly non-linear regimes under the influence of varying obstacle lengths (25% and 50%) and elevated Ra . For the 25% obstacle configuration, the flow at $Ra = 10^5$ exhibits a periodic unsteady regime characterized by coherent circulation patterns. The streamlines form well-organized recirculating cells occupying most of the cavity, while the isotherms remain continuous with moderate distortion near the heated and cooled walls. The thermal plumes rise along the hot wall and descend adjacent to the cold wall in a regular and repeatable manner, indicating stable oscillatory convection. The periodic variation in the flow field arises from the competition between buoyancy-driven acceleration and viscous damping, producing sustained oscillations without disrupting the overall flow organization. For the 50% obstacle configuration, periodic behavior is observed at the higher critical value of $Ra = 2 \times 10^5$. Although the streamline topology remains ordered, the larger obstacle significantly modifies the circulation by constricting the available flow passage and generating stronger recirculation zones around its surfaces. The isotherms exhibit greater curvature and clustering near the obstacle corners, reflecting intensified local temperature gradients and enhanced thermal interaction between the obstacle and the surrounding fluid. These features indicate that the larger blockage alters the heat transport mechanism and delays the establishment of periodic convection compared with the smaller obstacle.

As the Ra increases further, the flow undergoes a transition to a chaotic regime. For the 25% obstacle, chaotic convection is observed at $Ra = 7 \times 10^5$, where the streamline pattern becomes highly irregular with continuously evolving vortical structures. Multiple secondary eddies emerge and interact dynamically with the primary circulation, while the isotherms become increasingly distorted and fragmented due to vigorous mixing. The thermal plumes lose their regular periodicity and exhibit random oscillations, indicating a chaotic flow. A similar chaotic state is obtained for the 50% obstacle at the lower Ra of $Ra = 5 \times 10^5$. The streamlines display pronounced vortex interactions and frequent changes in flow topology caused by the intensified shear generated around the larger obstacle. Simultaneously, the isotherms become highly convoluted and densely packed in localized regions, demonstrating substantial enhancement of convective HT and thermal mixing. Compared with the 25% obstacle configuration, the larger obstacle promotes the onset of chaotic convection at a lower Ra by increasing flow confinement, strengthening plume interactions, and amplifying hydrodynamic instabilities.

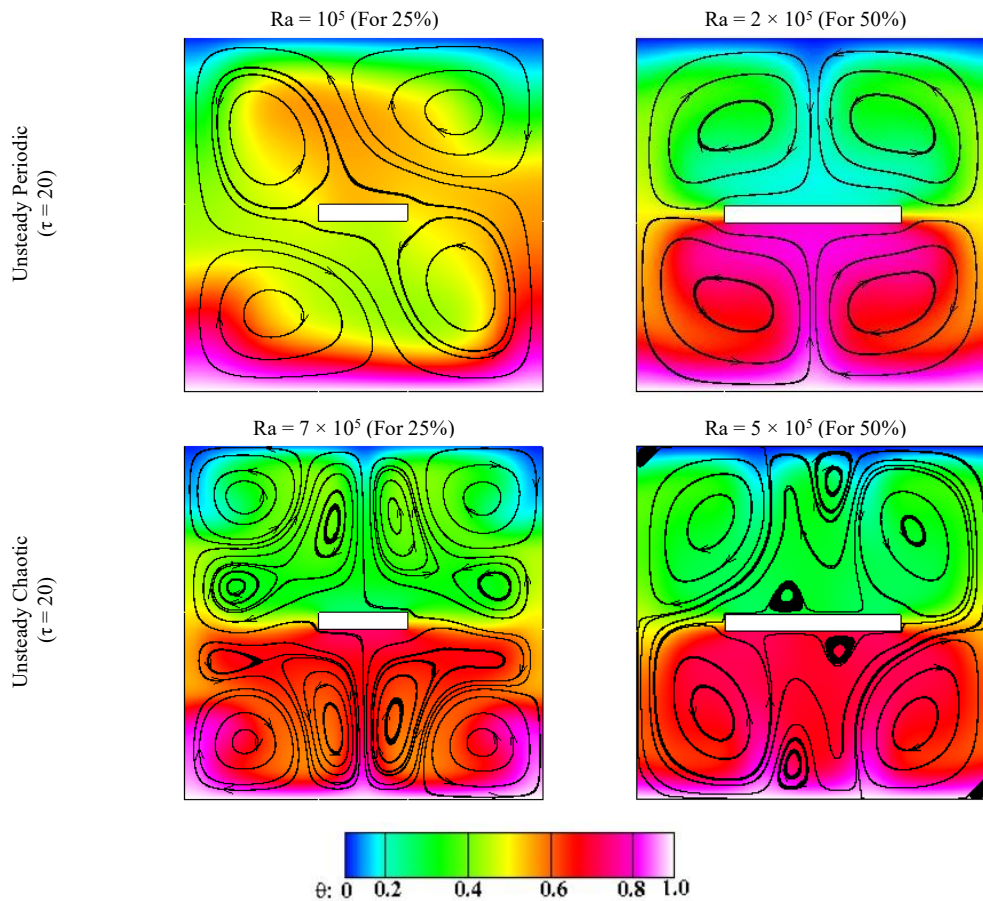


Fig 9: Streamlines and isotherms showing unsteady flow at different Ra with 25% and 50% obstacle at $\tau = 20$.

5.5. Limit point and limit cycles

Fig 10 illustrates the trajectories in the $u-\theta$ plane at the monitoring point $P_1 (-0.38, 0.01)$ for cavities containing 25% and 50% centrally positioned obstacles. For the cavity with a 25% obstacle, the trajectory at $Ra = 9 \times 10^4$ shown in Fig 10(a) converges to a single fixed point in phase space, indicating that the flow approach constant values after the initial transient period. This limit-point attractor is characteristic of a stable steady-state solution, where the flow is time-independent. As the Ra increases to $Ra = 10^5$ in Fig 10(b), the fixed point loses stability and evolves into a well-defined closed orbit. The emergence of this limit cycle signifies the onset of self-sustained periodic oscillations through a Hopf bifurcation. In this regime, the flow varies periodically with a constant amplitude and frequency, reflecting a stable oscillatory convection pattern.

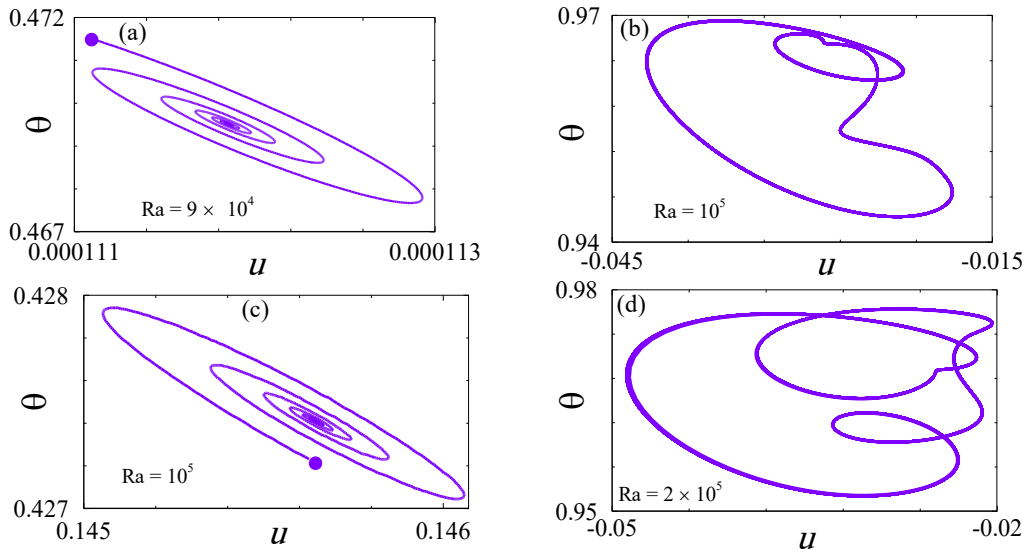


Fig 10: Trajectories in the u - θ plane at point P_1 (-0.38, 0.01): (a) and (c) convergence to a limit point at $Ra = 9 \times 10^4$ and 10^5 ; (b) and (d) formation of a limit cycle at $Ra = 10^5$ and 2×10^5 with 25% and 50% obstacle respectively.

A similar transition is observed for the cavity with a 50% obstacle, although it occurs at a lower Ra . At $Ra = 10^5$ shown in Fig 10(c), the phase trajectory still converges to a fixed point, confirming that the flow remains in a stable steady state. However, increasing the Ra to $Ra = 2 \times 10^5$ depicted in Fig 10(d) produces a distinct closed limit cycle, demonstrating the transition to periodic convection. Compared with the 25% obstacle configuration, the larger obstacle induces oscillatory behavior at a lower critical Ra , suggesting that increased blockage enhances flow instability.

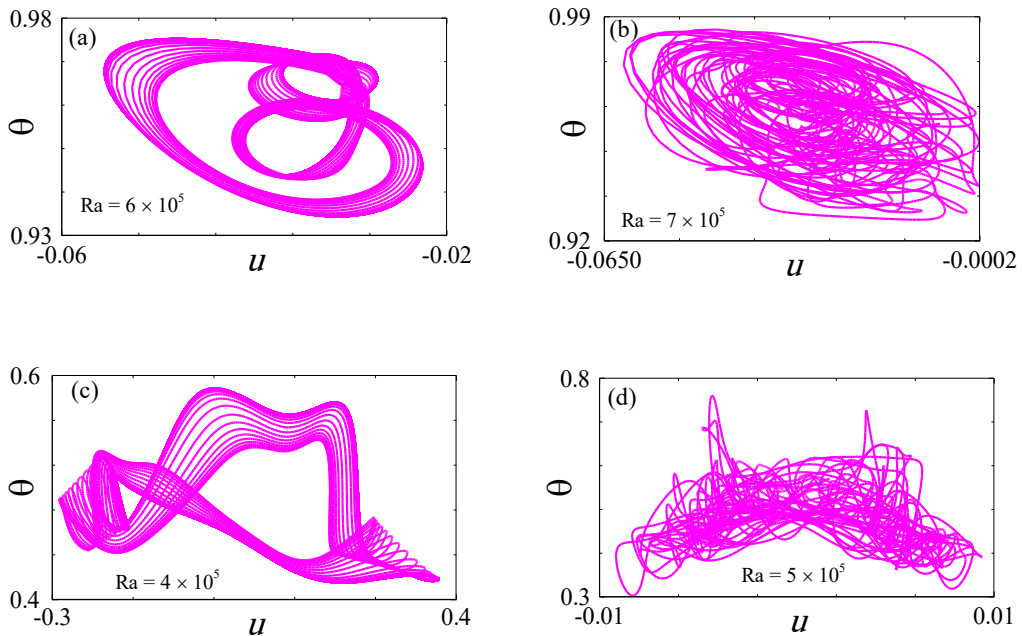


Fig 11: Phase-space trajectories in the u - θ plane at point P_1 (-0.38, 0.01): (a) and (c) closed limit cycle indicating periodic behavior at $Ra = 6 \times 10^5$ and 4×10^5 , and (b) irregular trajectory indicating chaotic behavior at $Ra = 7 \times 10^5$ and 5×10^5 with 25% and 50% obstacle respectively.

Fig 11 presents the phase-space trajectories in the u - θ plane at monitoring point P_1 (-0.38, 0.01), to illustrate the transition from periodic chaotic flow for cavities containing 25% and 50% centrally positioned obstacles. For the cavity with a 25% obstacle, the phase-space trajectory at $Ra = 6 \times 10^5$ as shown in Fig 11(a) forms a well-defined closed limit cycle. The presence of a single closed orbit indicates that the flow oscillates periodically with a unique

dominant frequency after the initial transients have decayed. Such a stable limit cycle is characteristic of a deterministic periodic state. As the Ra increases to $Ra = 7 \times 10^5$ as depicted in Fig 11(b), the closed orbit breaks down into an irregular and densely occupied trajectory. The phase-space no longer converges to a single periodic orbit but instead fills a finite region without repeating the same path. This behavior is a clear indication of chaotic flow.

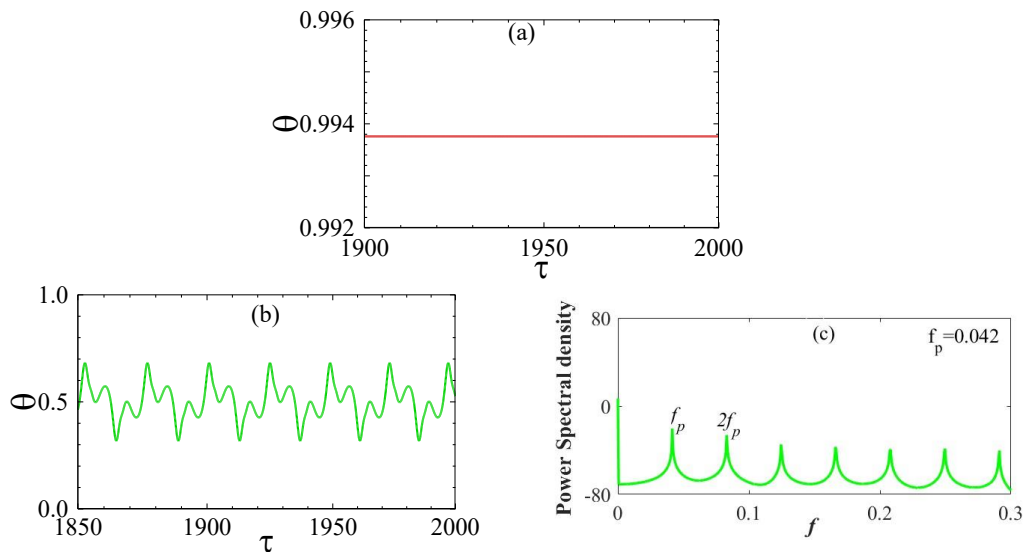
A similar trend is observed for the cavity containing a 50% obstacle, although the transition occurs at a lower Ra . At $Ra = 4 \times 10^5$ shown in Fig 11(c), the phase portrait exhibits a closed limit cycle, confirming periodic flow behavior. However, when the Ra is increased to $Ra = 5 \times 10^5$ illustrated in Fig 11(d), the trajectory becomes irregular and scattered, signifying the onset of chaotic flow. Compared with the 25% obstacle configuration, the larger obstacle promotes an earlier transition to chaos by altering the flow passage, intensifying shear layers around the obstacle, and increasing the interaction between ascending and descending thermal plumes. These effects amplify flow instability and accelerate the breakdown of periodic flow.

5.6. Power spectral density to analysis transition to chaos

Fig 12 presents the temperature time series (TTS) and the corresponding power spectral density (PSD) at the monitoring points P_1 (-0.38, 0.01) for the cavity containing a 25% centrally positioned obstacle. The temporal and spectral responses clearly demonstrate the progressive transition of the buoyancy-driven flow from a steady regime to periodic oscillations, and finally deterministic chaos as the Ra increases.

At $Ra = 10^4$ in Fig 12(a), the temperature remains constant with time, producing a horizontal time-history profile that indicates the establishment of a stable steady-state solution. As the Ra increases to $Ra = 10^5$ in Fig 12(b), the flow loses its steady nature through a Hopf bifurcation and develops periodic oscillations. The TTS exhibits regular sinusoidal fluctuations with nearly constant amplitude and period, while the PSD in Fig 12(c) is characterized by a sharp fundamental frequency together with several harmonics. Further increase in the Ra to $Ra = 6 \times 10^5$ the TTS remains periodic and the corresponding PSD with fundamental peak frequency confirm the periodic behaviour as shown in Fig 12(d, e).

At $Ra = 6 \times 10^5$ shown in Fig 12(f, g), the flow undergoes a complete transition to chaos. The TTS becomes irregular and aperiodic, exhibiting large-amplitude, non-repeating fluctuations over the entire observation period. Correspondingly, the PSD transforms from a discrete line spectrum into a broadband continuous distribution extending over a wide range of frequencies. The disappearance of isolated dominant peaks and the development of broadband energy indicate that thermal energy is distributed over numerous interacting modes, providing clear evidence of deterministic chaotic convection.



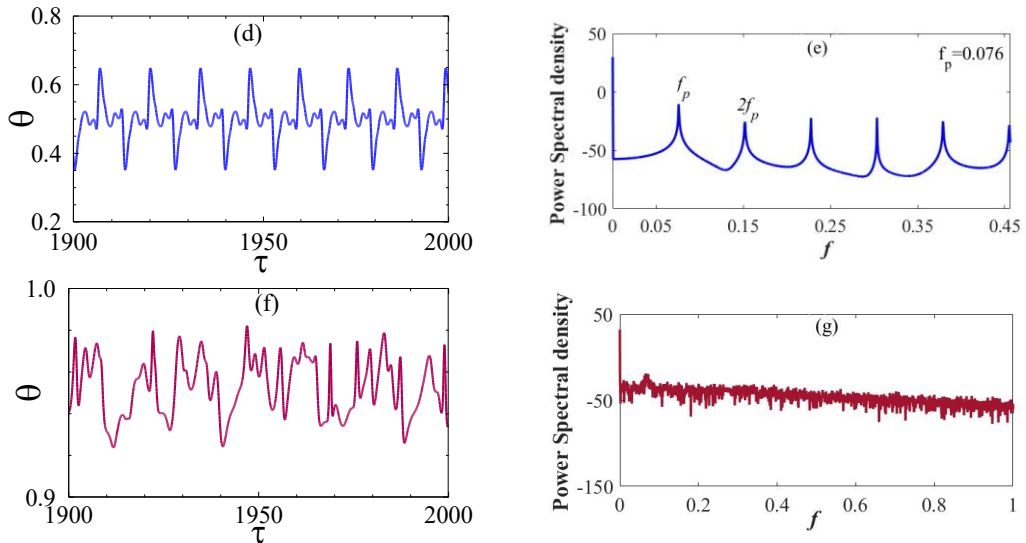
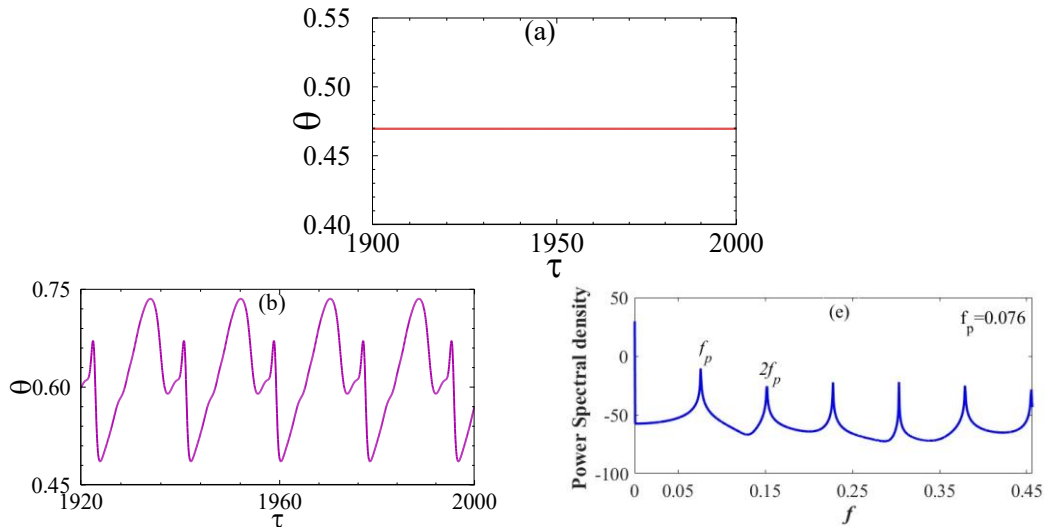


Fig 12: Temperature time series at the fully developed stage and power spectral density at point P_1 (-0.38,0.01) for (a) $Ra = 10^4$, for (b) and (c) $Ra = 10^5$, for (d) and (e) $Ra = 6 \times 10^5$, for (f) and (g) $Ra = 7 \times 10^5$ with 25% obstacle.

Fig 13 illustrates the TTS and their corresponding PSDs at the monitoring point P_2 (0.38, 0.15), located within a region of stronger convective circulation for the cavity with 50% obstacle. Similar to the behavior observed at P_1 , the temporal evolution demonstrates a sequence of dynamical transitions from steady convection to periodic and chaotic flow with increasing Ra .

At $Ra = 10^5$ in Fig 13(a), the temperature remains constant with time, producing a horizontal time-history profile that indicates the establishment of a stable steady-state solution. As the Ra increases to $Ra = 2 \times 10^5$ in Fig 13(b), the flow loses its steady nature through a Hopf bifurcation and develops periodic oscillations. The TTS exhibits regular sinusoidal fluctuations with nearly constant amplitude and period, while the PSD in Fig 13(c) is characterized by a sharp fundamental frequency together with several harmonics. Further increase in the Ra to $Ra = 4 \times 10^5$ the TTS remains periodic and the corresponding PSD with fundamental peak frequency confirm the periodic behaviour as shown in Fig 13(d, e).

At $Ra = 5 \times 10^5$ shown in Fig 13(g, f), the flow undergoes a complete transition to chaos. The TTS becomes irregular and aperiodic, exhibiting large-amplitude, non-repeating fluctuations over the entire observation period. Correspondingly, the PSD transforms from a discrete line spectrum into a broadband continuous distribution extending over a wide range of frequencies. The disappearance of isolated dominant peaks and the development of broadband energy indicate that thermal energy is distributed over numerous interacting modes, providing clear evidence of deterministic chaotic convection.



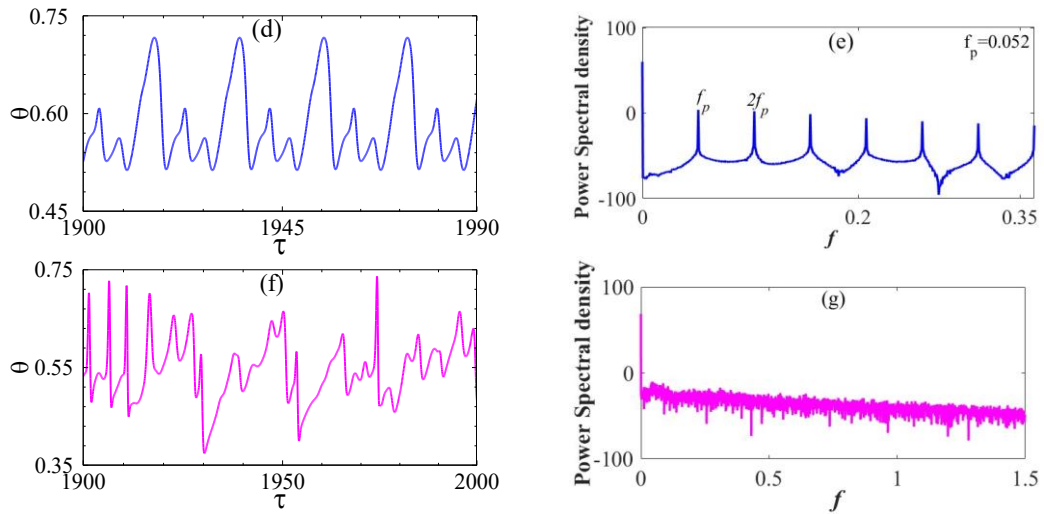


Fig 13: Temperature time series and corresponding the PSD at point $P_2(0.38, 0.15)$: (a) $Ra = 10^5$; (b–c) $Ra = 2 \times 10^5$; (d–e) $Ra = 4 \times 10^5$; and (f–g) $Ra = 5 \times 10^5$.

5.7. Lyapunov exponent analysis and onset of chaos

Previous studies [36] have demonstrated that the transition from ordered to chaotic motion in buoyancy-driven flows can be quantitatively characterized by the largest Lyapunov exponent (λ_L). This parameter measures the average exponential rate of divergence of initially nearby trajectories in phase space. Specifically, if ε_0 denotes the initial separation between two trajectories and $\varepsilon(\tau)$ represents their separation after a time interval τ , the evolution can be expressed as [37]:

$$\varepsilon(\tau) \approx \varepsilon_0 e^{\lambda_L \tau}. \quad (6)$$

where a positive value of λ_L indicates exponential divergence and hence chaotic dynamics. The largest Lyapunov exponent is computed as:

$$\lambda_L = \frac{1}{\tau_n - \tau_0} \sum_{j=1}^n \ln \left(\frac{\varepsilon(u_j)}{\varepsilon(u_{j-1})} \right). \quad (7)$$

where $\varepsilon(u_j)$ denotes the Euclidean distance between u_j and u_{j-1} , τ_0 is the initial time ($\tau_0 = 1500$), and τ_n corresponds to the final time after n iterations ($\tau_n = 2000$ with $n = 40000$ in the present study).

Fig 14 presents the variation of the largest Lyapunov exponent (λ_L) with the Ra for cavities containing centrally positioned obstacles of 25% and 50% of the cavity width. A negative or zero value of λ_L corresponds to stable steady or periodic motion, whereas a positive value signifies sensitive dependence on initial conditions and, consequently, the onset of chaotic convection.

For the cavity with a 25% obstacle (Fig 14(a)), the value of λ_L remains approximately zero over the lower Ra range $Ra < 10^5$ and for $Ra = 7 \times 10^5$ it becomes negative indicating that the flow is either steady symmetric or periodic and does not exhibit sensitivity to initial conditions. This behavior is consistent with the previously identified symmetric, asymmetric and weakly oscillatory flow regimes. In contrast, when the Ra exceeds this threshold a transition toward chaotic dynamics is observed. At $Ra = 7 \times 10^5$, λ_L becomes positive ($\lambda_L = 0.00029$) confirming the onset of chaos in the system. This finding agrees well with the earlier temporal and flow-structure analyses where irregular and aperiodic fluctuations were observed in both temperature and velocity fields.

A distinctly different transition is observed for the cavity containing the larger 50% obstacle (Fig 14(b)). In this configuration, value of the λ_L remains zero or negative up to $Ra = 2 \times 10^5$, indicating that the flow is still confined to steady or periodic states. However, the transition to chaos occurs at a significantly lower critical Rayleigh number than that of the 25% obstacle case. Specifically, the λ_L becomes strictly positive at $Ra = 5 \times 10^5$, demonstrating the earlier onset of chaotic convection.

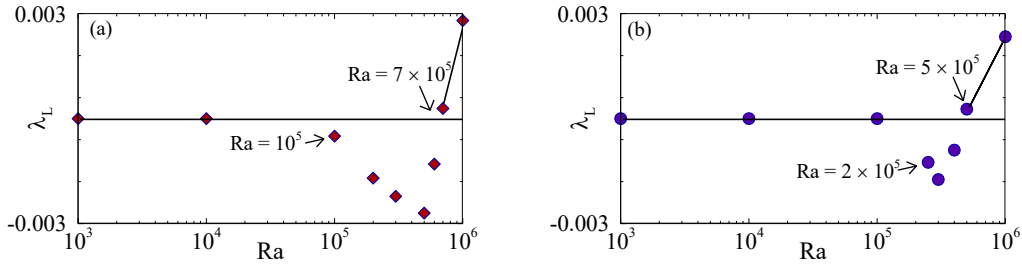


Fig 14: Variation λ_L with Ra , illustrating the shift from steady to chaotic flow with 25% and 50% Obstacle.

6. Heat transfer

The average Nusselt number (consider, Nu) at the bottom and top walls is defined as follows (see, ref. [20] for further details):

$$Nu = \int_0^1 \frac{\partial \theta}{\partial y} dx. \quad (8)$$

Fig 15 illustrates the temporal evolution of the Nu and the normalized averaged Nusselt number ($Nu Ra^{-1/5}$) at the heated bottom wall for Rayleigh numbers ranging from 10^3 to 10^6 under three geometric configurations: a cavity containing a centrally positioned 50% obstacle, a cavity with a 25% obstacle, and an obstacle-free cavity. The normalized Nusselt number is presented to facilitate comparison of the transient HT response irrespective of the absolute magnitude of the steady-state HT.

For all configurations, the Nu exhibits a pronounced transient behavior immediately after the initiation of heating. At low Ra of 10^3 and 10^4 , HT is primarily governed by thermal conduction, resulting in a smooth monotonic evolution toward the steady-state value. The absence of oscillations in both the Nu and the normalized Nu indicates that the flow remains stable throughout the transient process. As the Ra increases to 10^5 and 10^6 , buoyancy-induced convection becomes progressively dominant, producing a rapid increase in the HT rate during the early stages followed by a gradual stabilization as the thermal and hydrodynamic fields become fully developed.

The influence of the obstacle is evident from the transient characteristics of the HT rate. In the cavity containing the 50% obstacle (Fig 15(a) and Fig 15(b)), the larger blockage significantly restricts the circulation of the convective cells, delaying the establishment of the fully developed flow. Consequently, the Nu approaches its steady-state value more gradually than in the other configurations, particularly at higher Ra where NC is strongest. The normalized Nu confirms this delayed thermal response through a relatively slower convergence to unity, indicating an extended transient period caused by the obstruction of the buoyancy-driven flow.

For the cavity with the 25% obstacle (Fig 15(c) and Fig 15(d)), the reduction in blockage allows stronger circulation around the obstacle while still modifying the flow structure. Compared with the 50% obstacle case, the transient HT develops more rapidly, and the normalized Nu reaches its steady-state value within a shorter time. This behavior suggests that a moderate obstacle provides less resistance to the formation of convective vortices while maintaining the characteristic transient evolution associated with NC.

In the absence of an obstacle (Fig 15(e) and Fig 15(f)), the cavity exhibits the fastest thermal response among all configurations. The unrestricted development of the primary circulation cell enables efficient transport of thermal energy from the heated bottom wall to the colder upper surface. Consequently, the Nu increases rapidly before reaching its asymptotic steady-state value, while the normalized Nu approaches unity within the shortest dimensionless time. At the highest Ra , the enhanced buoyancy forces further accelerate the establishment of convection, producing the quickest stabilization of the HT process.

Overall, Fig 15 demonstrates that increasing the Ra accelerates the transient development of NC and increases the HT rate for all cavity configurations. However, the introduction of a central obstacle delays the evolution toward steady state by impeding fluid circulation, with the effect becoming more pronounced as the obstacle size increases. Although the obstacle modifies the rate at which the thermal field develops, the normalized Nu reveals that all cases eventually converge to stable steady-state conditions, confirming that the geometric obstruction primarily affects the transient dynamics rather than the final thermal equilibrium.

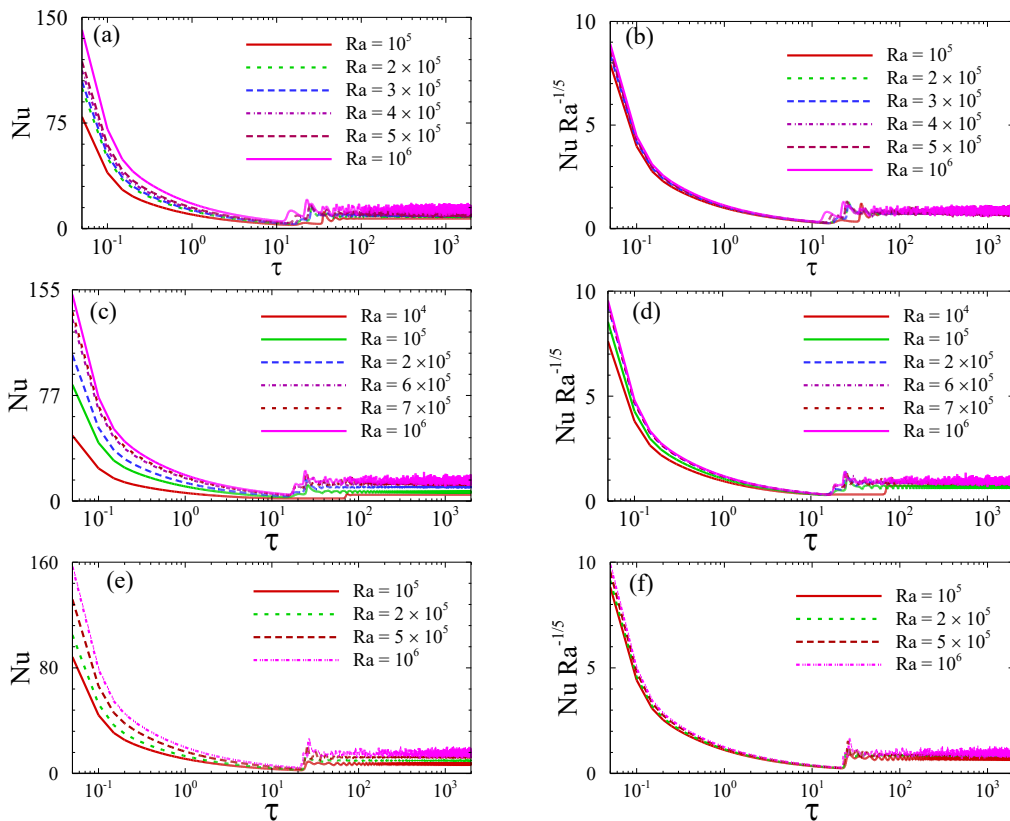


Fig 15: Time evolution of (a, c, e) the Nu and (b, d, f) the normalized Nusselt number at the heated bottom wall for various Ra corresponding to (a, b) 50% obstacle, (c, d) 25% obstacle, and (e, f) obstacle-free configurations, respectively.

7. Entropy generation analysis

In this study, E_{gen} is employed as a key criterion for evaluating the thermal performance of the mentioned square cavity with horizontal obstacle of half and one-fourth length. The contributions of both HT and fluid friction to E_{gen} are considered. Based on the dimensionless parameters defined in Eq. (1) and the local thermodynamic equilibrium of linear transport theory, the dimensionless local E_{gen} for two-dimensional heat and fluid flow in a Cartesian coordinate system can be expressed explicitly as [38]:

Entropy due to heat transfer:

$$E_{\theta} = \left(\frac{\partial \theta}{\partial x} \right)^2 + \left(\frac{\partial \theta}{\partial y} \right)^2, \quad (9)$$

Entropy due to fluid friction:

$$E_f = \psi \left[2 \left\{ \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right\} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right], \quad (10)$$

where ψ is the irreversibility distribution ratio and defined as:

$$\psi = \frac{\mu T_{\infty}}{\alpha} \left(\frac{k}{L \Delta T} \right)^2, \quad (11)$$

Local entropy generation (E_l) is the summation of E_{θ} and E_f , so that:

$$E_l = E_\theta + E_f, \quad (12)$$

The local Bejan number (Be_l) defined by the ratio of the E_θ and E_l irreversibilities:

$$Be_l = \frac{E_\theta}{E_l}. \quad (13)$$

Fig 16 to Fig 18 illustrate the distributions of E_θ , E_f , E_l and Be_l for the cavities containing a 50% obstacle, a 25% obstacle, and without an obstacle at different Ra. The results demonstrate that the obstacle size strongly influences the thermal and hydrodynamic fields by modifying the available flow passage and the structure of the circulation cells. As the obstacle size increases, the flow becomes increasingly constrained, altering the distributions of thermal gradients, velocity gradients, and consequently the local E_{gen} characteristics.

For the cavity with a 50% obstacle shown in Fig 16, at $Ra = 10^5$, the distributions of E_l closely resemble those of E_θ , indicate that the total E_{gen} is almost entirely governed by HT irreversibility. The contribution of E_f is negligible because the water-filled cavity produces relatively low viscous dissipation despite the development of NC. Accordingly, the Be_l contours exhibit predominantly high values throughout the cavity, indicating that E_{gen} is governed mainly by HT irreversibility while the contribution of fluid-friction irreversibility remains less significant.

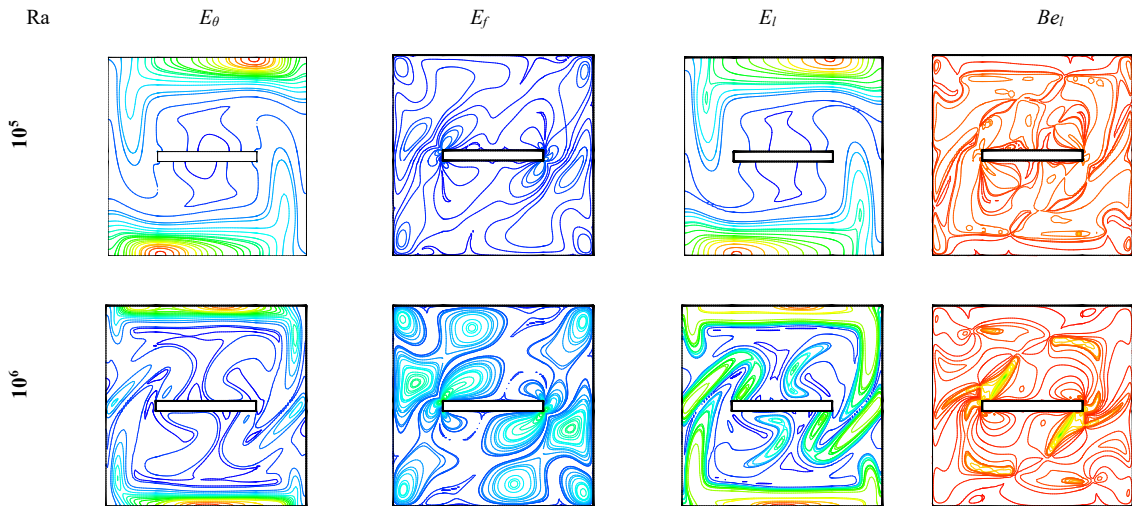


Fig 16: Distribution of E_{gen} due to HT, FF, local E_{gen} and local Be_l for different Ra with an obstacle of 50%.

As the Ra increases to $Ra = 10^6$, the flow transitions into a chaotic regime, as discussed in the previous section. The intensified buoyancy forces promote stronger vortex interactions and enhance convective heat transport, resulting in a more complex spatial distribution of E_θ , E_f , and E_l . Although the magnitude of E_f increases because of stronger velocity gradients and higher shear stresses associated with the chaotic flow, its contribution to the total E_{gen} remains considerably smaller than that of E_θ . Consequently, the overall distribution of E_l continues to resemble that of E_θ , demonstrating that HT irreversibility remains the dominant mechanism of E_{gen} . The corresponding Be_l contours exhibit slightly lower values in localized regions near the obstacle surfaces and zones of intense vortex interaction, reflecting the increased contribution of fluid-friction irreversibility. Nevertheless, Be_l remains greater than 0.50 throughout most of the enclosure, confirming that thermal irreversibility continues to dominate the E_{gen} process in the water-filled cavity even under chaotic flow conditions. However, in this section, observe that the total E_{gen} is still dominated by E_θ . Though, with the rise of the Ra viscous dissipation increases but due to the water filled cavity the E_f is still less significant contributor to the total E_{gen} . Consequently, the Be_l remains high and E_{gen} is governed mainly by HT irreversibility while the contribution of fluid-friction irreversibility remains less significant.

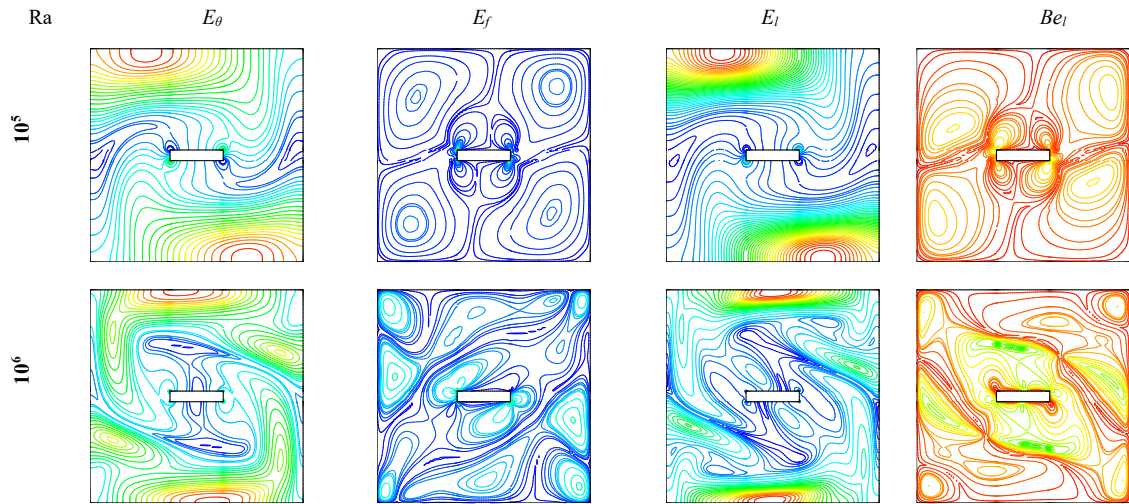


Fig 17: Distribution of E_{gen} due to HT, FF, local E_{gen} and local Be_l for different Ra with an obstacle of 25%.

Fig 17 and Fig 18 illustrate the distributions of E_{gen} due to HT (E_θ), E_{gen} due to FF (E_f), local E_{gen} (E_l), and the local Bejan number (Be_l) or the cavities containing a 25% obstacle and without an obstacle, respectively. Although the spatial distributions of these quantities differ from those observed for the 50% obstacle configuration because of the changes in flow passage and circulation intensity, the underlying physical mechanisms remain essentially unchanged.

In all three cavity configurations, E_{gen} is controlled predominantly by HT irreversibility over the entire range of Ra considered, while fluid-friction irreversibility becomes increasingly important only at higher Ra .

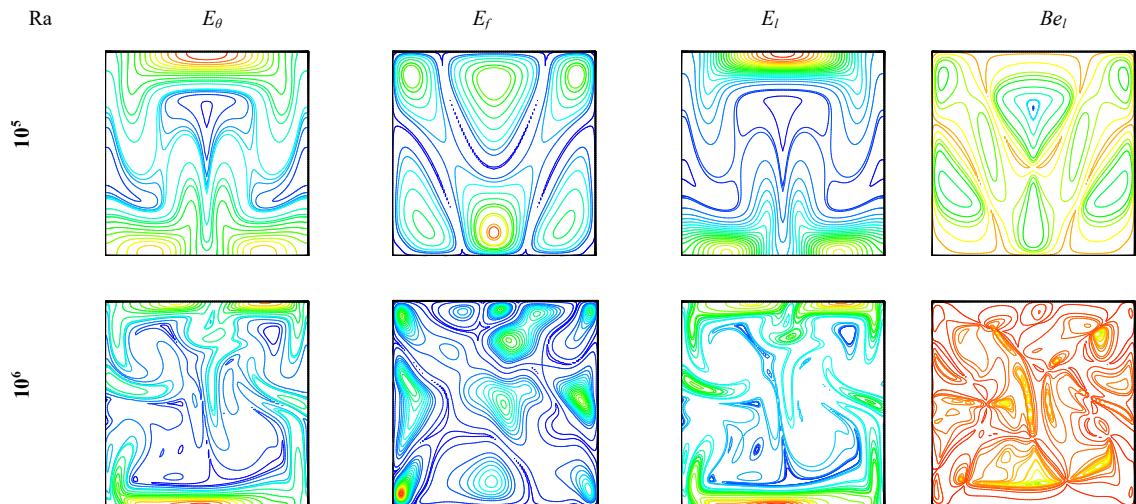


Fig 18: Distribution of E_{gen} due to HT, FF, local E_{gen} and local Be_l for different Ra without obstacle.

8. Thermal performance analysis

The average entropy generation (E_{avg}) and the average Bejan number (Be_{avg}) are defined by (refer to [34], for details):

$$E_{avg} = \frac{\int_0^1 \int_0^1 E_l \, dx \, dy}{\int_0^1 \int_0^1 dx \, dy}, \quad (14)$$

$$Be_{avg} = \frac{\int_0^1 \int_0^1 Be_i dx dy}{\int_0^1 \int_0^1 dx dy}. \quad (15)$$

The ecological coefficient of performance (ECOP) is described as (for additional details, see [38]):

$$ECOP = \frac{Nu}{E_{avg}}. \quad (16)$$

Table 3 presents the variation of the Nu , E_{avg} , and Be_{avg} with increasing Ra for three enclosure configurations: without an obstacle, with a centrally positioned 25% obstacle, and with a centrally positioned 50% obstacle. Overall, increasing Ra significantly enhances HT and E_{gen} , while reducing the Bejan number, indicating a transition from HT dominated irreversibility to fluid-friction-dominated irreversibility. The presence of an internal obstacle suppresses HT and E_{gen} , with the larger obstacle producing the greatest reduction.

At the lowest Ra value, the cavity without an obstacle exhibits the highest HT performance, with $Nu = 2.75$, compared with 2.15 and 2.14 for the 25% and 50% obstacle cases, respectively. These correspond to reductions of approximately 21.8% and 22.2% relative to the unobstructed cavity, demonstrating that the obstacle restricts the buoyancy-induced circulation and weakens convective heat transport. As Ra increases, Nu rises substantially for all configurations, reaching 15.13, 14.91, and 13.69 at the highest Ra . Although the influence of the 25% obstacle becomes less pronounced at high Ra (only about 1.5% lower than the unobstructed case), the 50% obstacle still causes a noticeable reduction of approximately 9.5%, indicating that a larger blockage continues to impede the development of strong convective cells.

A similar trend is observed for the average entropy generation (E_{avg}). For all enclosure configurations, E_{avg} increases rapidly with Ra , reflecting the intensification of thermal gradients and fluid motion as buoyancy forces become stronger. In the unobstructed cavity, E_{avg} increases from 3.86 to 147.85, whereas corresponding values of 3.01–135.21 and 3.05–141.38 are obtained for the 25% and 50% obstacle cases, respectively. At low Ra , the obstacle considerably reduces E_{gen} due to the weakened convective transport. However, at high Ra , the difference between configurations becomes smaller because stronger buoyancy forces partially overcome the flow resistance introduced by the obstacle. Interestingly, the 50% obstacle produces slightly higher E_{gen} than the 25% obstacle at the largest Ra , suggesting that the larger obstruction enhances local velocity and temperature gradients around its surfaces, thereby increasing irreversible losses despite reducing the overall HT rate.

Table 3. The changes of Nu , E_{avg} and Be_{avg} with different Ra values for the enclosures with different conditions.

Ra	Cavity without obstacle			Cavity with 25% obstacle			Cavity with 50% obstacle		
	Nu	E_{avg}	Be_{avg}	Nu	E_{avg}	Be_{avg}	Nu	E_{avg}	Be_{avg}
10^3	2.75	3.86	0.98	2.15	3.01	0.99	2.14	3.05	0.99
10^4	5.12	9.42	0.87	4.54	7.68	0.89	3.98	8.16	0.91
10^5	8.21	41.64	0.70	8.01	39.29	0.72	7.61	41.36	0.74
10^6	15.13	147.85	0.61	14.91	135.21	0.63	13.69	141.38	0.65

The average Bejan number (Be_{avg}) exhibits an opposite trend, decreasing monotonically with increasing Ra for all cases. Initially Be_{avg} is close to unity (0.98–0.99), indicating that E_{gen} is primarily governed by HT irreversibility under weak natural convection. As Ra increases, Be_{avg} decreases sharply to 0.61, 0.63, and 0.65 for the unobstructed, 25% obstacle, and 50% obstacle configurations, respectively. This decline signifies the increasing contribution of fluid friction irreversibility due to stronger buoyancy-driven circulation. Moreover, the obstacle consistently produces higher Be_{avg} than the unobstructed cavity, particularly for the 50% obstacle, implying that the suppression of fluid motion reduces the relative importance of viscous dissipation and maintains HT irreversibility as the dominant source of E_{gen} over a wider range of Ra . Furthermore, because the enclosure is filled with water, the Be_{avg} remains greater than 0.50 for all cases considered. This indicates that thermal irreversibility continues to be the primary contributor to the total E_{gen} , even at the highest Ra investigated. Although fluid friction irreversibility becomes increasingly significant with stronger NC, it does not exceed the contribution of HT irreversibility within the present range of operating conditions.

The presence of a centrally positioned obstacle within a square cavity significantly affects the Ecological Coefficient of Performance (ECOP) by altering the flow field, HT rate, and E_{gen} . The obstacle interrupts the primary

buoyancy-induced circulation and forces the fluid to flow around its surfaces, resulting in the formation of secondary vortices and enhanced mixing. These changes directly influence both the thermal performance and thermodynamic irreversibility of the enclosure. At low Ra , where conduction dominates the HT process, the centrally positioned obstacle has only a marginal effect on ECOP because the buoyancy-driven flow is relatively weak. As the Ra increases, NC becomes stronger, and the influence of the central obstacle becomes more pronounced. However, the central obstacle also introduces additional flow resistance. At sufficiently high Ra , viscous dissipation and thermal irreversibility around the obstacle increases. If these irreversibilities become dominant, the gain in HT is offset by increased E_{gen} , causing ECOP to decline. Thus, the ecological performance depends on the balance between HT enhancement and the additional thermodynamic losses introduced by the obstacle. Overall, a centrally positioned obstacle can improve the ecological performance of a square cavity when its size and shape are properly optimized.

Table 4: Variation of Ecological Coefficient of Performance (ECOP) with different Ra for the cavities with different conditions.

Ra	ECOP		
	Cavity without obstacle	Cavity with 25% obstacle	Cavity with 50% obstacle
10^3	0.71243523	0.71428571	0.70163934
10^4	0.54352442	0.59114583	0.48774509
10^5	0.19716619	0.20386867	0.18399419
10^6	0.10233344	0.11027291	0.09683124

Table 4 presents the variation of the ECOP with Ra for three cavity configurations: without an obstacle, with a centrally positioned 25% obstacle, and with a centrally positioned 50% obstacle. The ECOP combines the effects of HT and E_{gen} and is therefore a useful indicator for evaluating the overall thermodynamic performance of NC systems. A higher ECOP indicates superior ecological performance, reflecting an improved balance between useful HT and irreversible energy losses.

For all cavity configurations, the ECOP decreases markedly with increasing Ra . In the unobstructed cavity, the ECOP decreases from 0.7124 at $Ra = 10^3$ to 0.1023 at $Ra = 10^6$, representing an overall reduction of approximately 85.6%. Similar reductions are observed for the cavities with 25% and 50% obstacles, where the ECOP decreases from 0.7143 to 0.1103 and from 0.7016 to 0.0968, respectively. This monotonic decline indicates that although higher Ra enhance convective HT, the accompanying increase in E_{gen} is proportionally greater, thereby reducing the overall ecological performance of the enclosure.

At the lowest Ra of 10^3 , the cavity with the 25% obstacle achieves the highest ECOP of 0.7143, followed closely by the unobstructed cavity with an ECOP of 0.7124, whereas the cavity with the 50% obstacle exhibits the lowest ECOP of 0.7016. Compared with the unobstructed cavity, the 25% obstacle improves the ECOP by approximately 0.26%, suggesting that a moderate obstruction can effectively regulate the weak buoyancy-driven circulation, thereby reducing irreversible losses without significantly impairing HT. In contrast, the larger 50% obstacle decreases the ECOP by approximately 1.5%, indicating that excessive blockage suppresses convective transport and deteriorates the thermodynamic performance.

As the Ra increases to 10^4 , the beneficial effect of the 25% obstacle becomes more pronounced. The ECOP decreases to 0.5911, which is approximately 8.8% higher than that of the unobstructed cavity (0.5435), whereas the 50% obstacle reduces the ECOP to 0.4877, about 10.3% lower than the unobstructed configuration. This behavior suggests that a moderately sized obstacle optimizes the interaction between buoyancy-driven flow and thermal transport by suppressing excessive mixing and limiting E_{gen} , while a larger obstacle imposes excessive flow resistance that outweighs any thermodynamic benefits.

At higher Ra of 10^5 and 10^6 , the ECOP continues to decline for all configurations as NC becomes increasingly vigorous. Nevertheless, the cavity with the 25% obstacle consistently maintains the highest ECOP, achieving values of 0.2039 and 0.1103, compared with 0.1972 and 0.1023 for the unobstructed cavity. These improvements correspond to approximately 3.4% and 7.8%, respectively. The sustained enhancement indicates that the cavity with 25% obstacle effectively controls the development of strong circulation cells, reducing the growth of E_{gen} while preserving satisfactory HT. Conversely, the 50% obstacle consistently yields the lowest ECOP, demonstrating that excessive blockage significantly weakens thermal transport and cannot compensate for the associated irreversible losses.

9. Conclusions

A detailed numerical investigation was performed to examine the influence of obstacle size on unsteady NC and HT in the square cavities filled with water ($Pr = 7.0$). Three cavity configurations were considered: an unobstructed cavity, a cavity with a horizontal obstacle occupying 25% of the cavity width, and a cavity with a horizontal obstacle

occupying 50% of the cavity width. The flow and thermal characteristics were analyzed over a wide range of Ra from $10^0 - 10^6$. The results demonstrate that both the Ra and obstacle size significantly affect flow stability, bifurcation characteristics, HT, and E_{gen} behavior. Based on the obtained results, the key findings of the study can be summarized as follows:

- The flow undergoes a sequence of transitions from symmetric steady state to asymmetric steady state, periodic oscillations, and finally chaotic behavior as the Ra increases.
- For the cavity with a 25% obstacle, a pitchfork bifurcation occurs in the range $6 \times 10^3 < Ra \leq 7 \times 10^3$, whereas for the cavity with 50% obstacle, pitchfork bifurcation occurs in the range $9 \times 10^3 < Ra \leq 10^4$.
- Hopf bifurcation occurs between the $Ra = 9 \times 10^4$ and 10^5 for the 25% obstacle configuration, whereas for the 50% obstacle configuration it occurs between $Ra = 10^5$ and 2×10^5 .
- The flow transition from the periodic to the chaotic flow takes place between $Ra = 6 \times 10^5$ and 7×10^5 for the 25% obstacle cavity, while for the 50% obstacle cavity this transition occurs earlier, between $Ra = 4 \times 10^5$ and 5×10^5 .
- Owing to the use of water as the working fluid, E_{gen} within the cavity is governed primarily by HT irreversibility, whereas the contribution of fluid-friction irreversibility remains comparatively insignificant for all three cavity configurations considered.
- At $Ra = 10^6$, Nu rises substantially for all configurations, reaching values 15.13, 14.91, and 13.69 for the unobstructed cavity, the cavity with a 25% obstacle, and the cavity with a 50% obstacle, respectively. Although the influence of the 25% obstacle becomes less pronounced at high Ra , resulting in only about a 1.5% reduction compared with the unobstructed case, the 50% obstacle causes a more significant reduction of approximately 9.5%.
- The cavity with 50% obstacle produces slightly higher E_{gen} than the cavity with 25% obstacle at the largest Ra investigated. This indicates that the larger obstruction intensifies local velocity and temperature gradients around its surfaces, thereby increasing irreversible losses despite reducing the overall HT rate.
- The Be_{avg} remains greater than 0.50 for all cases considered, confirming that thermal irreversibility remains the dominant contributor to total E_{gen} , even at the highest Ra examined.
- The cavity with a 25% obstacle exhibits the highest ECOP, attaining a maximum value of 0.1103 compared with 0.0968 and 0.1023 for the 50% obstacle and unobstructed cavities, respectively. This corresponds to an enhancement of approximately 12.4% over the 50% obstacle configuration and 7.8% over the unobstructed cavity. These findings demonstrate that a moderate obstacle size provides the most favourable balance between HT enhancement and E_{gen} , leading to superior thermodynamic performance. Therefore, the 25% obstacle configuration is identified as the optimal design among the cases considered for maximizing the thermal efficiency of the enclosure.

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