



Photothermally Excited Micropolar Thermo-Viscoelastic Half-Spaces: Nonlocal DPL Heat Transfer with Variable Conductivity and Viscous Damping

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Abstract

A unified analytical model is developed for a micropolar semiconducting half-space under photothermal loading, incorporating nonlocal DPL heat transport, temperature-dependent conductivity, and Kelvin–Voigt viscoelasticity. Constitutive relations are extended with rate-dependent terms, yielding complex moduli that capture wave attenuation and dispersion. The Kirchhoff transformation linearizes the variable-conductivity equation, and normal-mode analysis provides closed-form solutions. Numerical results show that variable conductivity reduces surface temperature by 45% and halves thermal penetration depth, nonlocality redistributes energy to interior regions with 62% enhancement at depth, and viscosity lowers peak stresses by 19–25%. These benchmark solutions offer quantitative guidance for semiconductor devices, MEMS, thermal barrier coatings, and composites under high-frequency photothermal excitation.

Keywords: Micropolar thermo-viscoelasticity; Photothermal; Nonlocal dual-phase-lag; Variable conductivity; Kelvin–Voigt; Normal mode analysis.

1. Introduction

Classical thermoelasticity (CTE), as formulated by Biot [1], relies on a parabolic heat conduction equation that implies infinite thermal wave speeds, a physically inadmissible consequence. Generalized theories were introduced to remedy this by replacing the parabolic heat equation with a hyperbolic one, ensuring finite propagation speeds. The first such generalization, proposed by Lord and Shulman (LS) [2], modifies the classical Fourier law through a single relaxation time. A second generalization is the Green–Lindsay (GL) theory with two relaxation times [3], which is founded on the entropy production inequality of Green and Laws [4]. A third generalization is the theory of thermoelasticity without energy dissipation proposed by Green and Naghdi [5]. Subsequently, Tzou [6] introduced the dual-phase-lag (DPL) model, in which two distinct phase lags account for microstructural interactions in heat transport. Comprehensive reviews have been provided by Chandrasekharaiah [7, 8]. At micro- and nanoscale

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dimensions, however, temporal lagging alone becomes insufficient, and spatial nonlocality must also be taken into account. Motivated by Eringen's nonlocal continuum theory [9], Tzou and Guo [10] extended the lagging concept to include a thermal length-scale parameter. Several investigations have demonstrated that thermal conductivity varies appreciably with temperature under intense heating, and neglecting this dependence may lead to significant errors [11].

A micropolar continuum can be envisioned as an assembly of interconnected material particles, each behaving as a small rigid structure capable of both translational and rotational motion. Materials possessing pronounced microstructures, metals, polymers, composites, soils, rocks, and concrete, are typical examples for which the micropolar description is appropriate. The general theory of micropolar elasticity was formulated by Eringen and Suhubi [12, 13]. The fundamental equations of linear micropolar thermoelasticity were derived by Tauchert et al. [14] and Boschi and Ieşan [15]. Ciarletta [16] developed a theory without energy dissipation, while Sherief et al. [17] formulated a Lord–Shulman type generalized micropolar thermoelasticity.

Semiconducting materials exhibit the photothermal effect: when exposed to sunlight, a laser beam, or elevated temperature, part of the incident energy is absorbed while the remainder is released as heat [18, 19]. Optical or thermal excitation generates free charge carriers, giving rise to plasma waves. The coupled interactions among thermal, elastic, and plasma fields were analyzed by Todorović et al. [20], while Song et al. [21] investigated generalized thermoelastic vibrations of optically excited semiconducting media.

The linear theory of viscoelasticity is directly relevant to the present investigation. Most solids exhibit viscous effects under dynamic loading, giving rise to attenuation and dispersion of propagating waves [22]. Early foundational contributions were made by Biot [23, 24] and Bland [25]. The mathematical foundations of thermo-viscoelasticity were formulated by Iliushin and Pobedria [26]. Subsequent works by Tanner [27] and Huilgol and Phan-Thien [28] achieved substantial progress in solving boundary value problems for linear viscoelastic materials. Although micropolar thermoelastic theory can partially describe materials with microstructure, accurate characterization of rate-dependent dissipative media requires combining the micropolar thermoelastic framework with viscoelasticity.

Despite substantial progress in generalized thermoelasticity, micropolar continua, photothermal modeling, nonlocal heat conduction, variable conductivity, and thermo-viscoelasticity, the combined influence of these mechanisms has not been sufficiently addressed within a single half-space formulation. Most existing studies focus on one or two effects while neglecting their simultaneous coupling. This gap is of practical consequence, since semi-infinite media frequently arise in semiconductor devices, thermal barrier coatings, MEMS, and advanced composites.

The present work addresses this gap by developing a generalized analytical model for a micropolar thermo-viscoelastic semiconductor half-space under photothermal excitation, with nonlocal DPL heat conduction and variable thermal conductivity. The governing system accounts for fully coupled displacement, microrotation, temperature increment, and carrier density fields. The constitutive relations are extended to include time-rate-dependent terms according to the Kelvin–Voigt model. Nonlocal heat conduction is incorporated through a thermal length-scale parameter, while the DPL formulation introduces temporal delays. The thermal conductivity is assumed to vary linearly with temperature and is rendered tractable by the Kirchhoff transformation. The analytical solution is obtained using normal mode analysis, yielding closed-form expressions subject to prescribed boundary conditions at the free surface and regularity conditions at infinity.

The main contribution is the construction of a unified theoretical framework bringing together, for the first time, micropolar effects, Kelvin–Voigt viscoelastic damping, photothermal excitation, nonlocal DPL heat conduction, and temperature-dependent thermal conductivity in a single semiconductor half-space formulation. The model provides analytical benchmark solutions against which the influence of each physical parameter on the coupled response can be assessed. The results are expected to be of direct value for the analysis and design of semiconductor devices, MEMS, thermal barrier coatings, and polymer-based microstructured composites.

2. Nonlocal Photothermal Micropolar Thermoviscoelastic Model

2.1. Mathematical Framework

The analysis is grounded in a generalized continuum description of a homogeneous, isotropic semiconducting solid in which photothermal, micropolar, and viscoelastic effects act simultaneously. The formulation unifies generalized thermoelasticity, micropolar continuum mechanics, carrier diffusion dynamics, nonlocal heat transport, and viscoelastic dissipation within a single set of field equations.

2.2. Viscoelastic Constitutive Relations

Internal damping is introduced by replacing elastic stiffness constants with time-differential operators [29, 30]:

$$C_{ij}^* = C_{ij} + C_{ij}^v \frac{\partial}{\partial t}, \quad (1)$$

Applying this prescription to the Lamé-type moduli and micropolar coupling parameter:

$$\lambda^* = \lambda + \lambda_v \frac{\partial}{\partial t}, \mu^* = \mu + \mu_v \frac{\partial}{\partial t}, k_0^* = k_0 + k_{0v} \frac{\partial}{\partial t}. \quad (2)$$

The viscoelastic coefficients are expressed as fixed fractions of the corresponding elastic constants:

$$\lambda_v = \eta_\lambda \lambda, \mu_v = \eta_\mu \mu, k_{0v} = \eta_k k_0, \quad (3)$$

The constitutive equation for the stress tensor reads [31]:

$$\sigma_{ij} = 2\mu^* e_{ij} + \lambda^* e \delta_{ij} - (3\lambda^* + 2\mu^* + k_0^*)[\alpha_T \theta + d_n N] \delta_{ij} + k_0^* (u_{j,i} - \epsilon_{ijr} \phi_r), \quad (4)$$

The couple-stress tensor is given by [32]:

$$m_{ij} = \alpha_0^* \phi_{r,r} \delta_{ij} + \beta_0^* \phi_{i,j} + \gamma_0^* \phi_{j,i}, \quad (5)$$

with

$$\alpha_0^* = \alpha_0 + \alpha_{0v} \frac{\partial}{\partial t}, \beta_0^* = \beta_0 + \beta_{0v} \frac{\partial}{\partial t}, \gamma_0^* = \gamma_0 + \gamma_{0v} \frac{\partial}{\partial t}, \quad (6)$$

and

$$\alpha_{0v} = \eta_\alpha \alpha_0, \beta_{0v} = \eta_\beta \beta_0, \gamma_{0v} = \eta_\gamma \gamma_0. \quad (7)$$

The infinitesimal strain tensor is defined as:

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}). \quad (8)$$

In the absence of body forces, the balance of linear momentum takes the form [33]:

$$\rho \ddot{u}_i = \sigma_{ji,j}. \quad (9)$$

Substituting the constitutive relation (4) into (9) yields the displacement equation of motion [34]:

$$\rho \ddot{u} = (\mu^* + k_0^*) \nabla^2 u + (\lambda^* + \mu^*) \nabla (\nabla \cdot u) - \gamma^* \nabla \theta - \delta_n^* \nabla N + k_0^* (\nabla \times \phi), \quad (10)$$

where

$$\gamma^* = (3\lambda^* + 2\mu^* + k_0^*) \alpha_T, \delta_n^* = (3\lambda^* + 2\mu^* + k_0^*) d_n. \quad (11)$$

The balance of angular momentum reads [35]:

$$m_{ir,i} + \epsilon_{ijr} \sigma_{ij} = \rho j \ddot{\phi}_r + \rho j_v \frac{\partial^3 \phi_r}{\partial t^3}. \quad (12)$$

The spatiotemporal evolution of photogenerated carriers is described by [36, 37]:

$$\frac{\partial N}{\partial t} = D_E \nabla^2 N - \frac{N}{\tau} + \kappa \theta. \quad (13)$$

The nonlocal DPL constitutive relation between heat flux and temperature gradient is written as [6, 10, 38]:

$$\mathbf{q}(\mathbf{r}, t + \lambda_q t + \tau_q) = -K \nabla \theta(\mathbf{r}, t + \tau_q). \quad (14)$$

Expanding to first order in τ_q , τ_θ and second order in λ_q gives [10, 39]:

$$\left(1 + \tau_q \frac{\partial}{\partial t} - l_q^2 \nabla^2\right) q = -K \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla \theta, \quad (15)$$

Eliminating the heat flux between (15) and the energy balance yields [40]:

$$\left(1 + \tau_q \frac{\partial}{\partial t} - l_q^2 \nabla^2\right) \left[\rho C_E \frac{\partial \theta}{\partial t} + \gamma T_0 \frac{\partial e}{\partial t} - \rho Q - \frac{E_g}{\tau} N\right] = \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) [\nabla \cdot (K \nabla \theta)]. \quad (16)$$

Equations (10), (12), (13), and (16), supplemented by (4) and (5), constitute the complete coupled system.

3. Formulation of the problem

The problem concerns a two-dimensional boundary-value formulation for a homogeneous, isotropic micropolar viscoelastic thermoelastic semiconductor occupying a half-space, where photothermal excitation, nonlocal dual-phase-lag heat conduction, and temperature-dependent thermal conductivity act concurrently (Fig. 1). A Cartesian coordinate system (x, y, z) is employed, with the x -axis aligned along the free surface, the y -axis normal to the deformation plane, and the z -axis directed downward into the medium. The traction-free boundary lies at $z = 0$, and the material extends over $z \geq 0$.

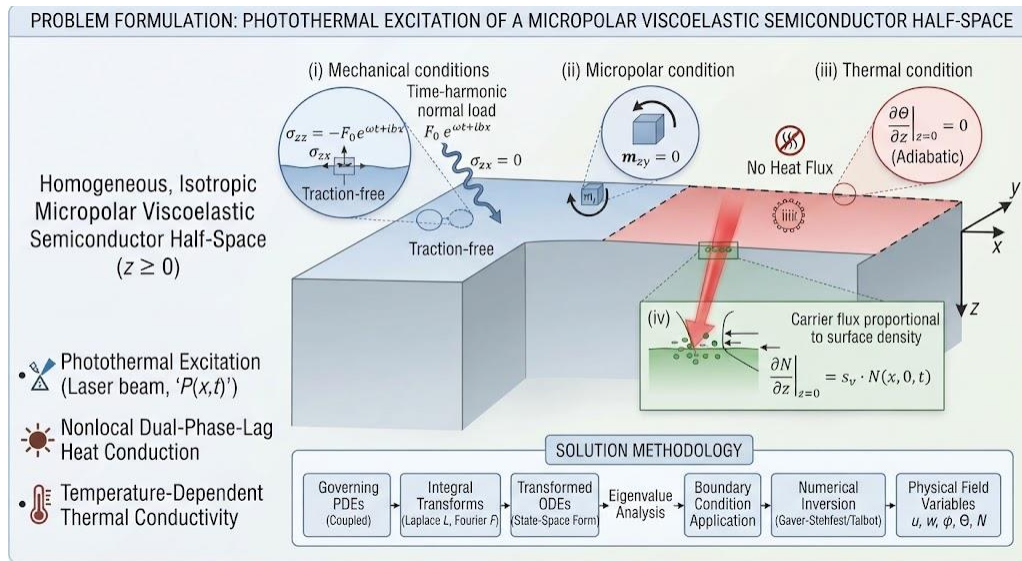


Fig. 1: Physical model and boundary conditions for a semi-infinite micropolar viscoelastic thermoelastic semiconductor occupying the half-space region.

Under plane-strain conditions, all field quantities are independent of y :

$$\mathbf{u} = (u(x, z, t), 0, w(x, z, t)), \quad (17)$$

$$\boldsymbol{\phi} = (0, \phi_2(x, z, t), 0) \quad (18)$$

The equations of motion (10) yield:

$$\rho \frac{\partial^2 u}{\partial t^2} = (\mu^* + k_0^*) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + (\lambda^* + \mu^*) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \gamma^* \frac{\partial \theta}{\partial x} - \delta_n^* \frac{\partial N}{\partial x} - k_0^* \frac{\partial \phi_2}{\partial z}, \quad (19)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\mu^* + k_0^*) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + (\lambda^* + \mu^*) \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right) - \gamma^* \frac{\partial \theta}{\partial z} - \delta_n^* \frac{\partial N}{\partial z} + k_0^* \frac{\partial \phi_2}{\partial x}. \quad (20)$$

The balance of angular momentum (12) reduces to:

$$\gamma_0^* \left(\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2} \right) - k_0^* \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right) - 2k_0^* \phi_2 = \rho j \frac{\partial^2 \phi_2}{\partial t^2} + \rho j_v \frac{\partial^3 \phi_2}{\partial t^3}. \quad (21)$$

The carrier diffusion-recombination equation (13) becomes:

$$\frac{\partial N}{\partial t} = D_E \left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial z^2} \right) - \frac{N}{\tau} + \kappa \theta. \quad (22)$$

The nonlocal DPL heat conduction equation (16) in the absence of internal heat sources takes the form:

$$\left(1 + \tau_q \frac{\partial}{\partial t} - l_q^2 \nabla^2 \right) \left[\rho C_E \frac{\partial \theta}{\partial t} + \gamma^* T_0 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \frac{E_g}{\tau} N \right] = \left(1 + \tau_\theta \frac{\partial}{\partial t} \right) [\nabla \cdot (K \nabla \theta)], \quad (23)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad (24)$$

The thermal conductivity and specific heat are assumed to depend linearly on temperature [41, 42]:

$$K(\theta) = K_0 (1 + K_1 \theta), \quad C_E(\theta) = C_{E0} (1 + K_1 \theta), \quad (25)$$

The Kirchhoff transformation is introduced [43]:

$$\Theta = \frac{1}{K_0} \int_0^\theta K(\xi) d\xi, \quad (26)$$

Differentiating (24) gives:

$$\Theta = \theta + \frac{K_1}{2} \theta^2, \quad (27)$$

$$K_0 \Theta_{,i} = K(\theta) \theta_{,i}, \quad (28)$$

$$K_0 \Theta_{,ii} = (K(\theta) \theta_{,i})_{,i}. \quad (29)$$

$$\nabla \cdot (K(\theta) \nabla \theta) = K_0 \nabla^2 \Theta. \quad (30)$$

$$K(\theta) \frac{\partial \theta}{\partial t} = K_0 \frac{\partial \Theta}{\partial t}, \quad C_E(\theta) \frac{\partial \theta}{\partial t} = C_{E0} \frac{\partial \Theta}{\partial t}. \quad (31)$$

Substituting into (22) yields the transformed heat conduction equation:

$$\left(1 + \tau_q \frac{\partial}{\partial t} - l_q^2 \nabla^2\right) \left[\rho C_{E0} \frac{\partial \Theta}{\partial t} + \gamma^* T_0 \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - \frac{E_g}{\tau} N\right] = K_0 \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla^2 \Theta. \quad (32)$$

The non-vanishing stress and couple-stress components follow from (4):

$$\sigma_{xx}^* = (\lambda^* + 2\mu^* + k_0^*) \frac{\partial u}{\partial x} + \lambda^* \frac{\partial w}{\partial z} - \gamma^* \Theta - \delta_n^* N, \quad (33)$$

$$\sigma_{zz}^* = \lambda^* \frac{\partial u}{\partial x} + (\lambda^* + 2\mu^* + k_0^*) \frac{\partial w}{\partial z} - \gamma^* \Theta - \delta_n^* N, \quad (34)$$

$$\sigma_{xz}^* = \mu^* \frac{\partial u}{\partial z} + (\mu^* + k_0^*) \frac{\partial w}{\partial x} + k_0^* \phi_2, \quad (35)$$

$$\sigma_{zx}^* = (\mu^* + k_0^*) \frac{\partial u}{\partial z} + \mu^* \frac{\partial w}{\partial x} - k_0^* \phi_2, \quad (36)$$

$$m_{xy}^* = \gamma_0^* \frac{\partial \phi_2}{\partial x}, \quad m_{zy}^* = \gamma_0^* \frac{\partial \phi_2}{\partial z}, \quad (37)$$

The following dimensionless variables are introduced:

$$\begin{aligned} (x', z', u', w') &= \frac{1}{C_T t^*} (x, z, u, w), \quad \{t', \tau'_q, \tau'_\theta, \tau'\} = \frac{\{t, \tau_q, \tau_\theta, \tau\}}{t^*}, \quad \phi'_2 = \phi_2, \\ \{\Theta', N', \sigma'_{ij}\} &= \frac{1}{\lambda + 2\mu + k_0} \{\gamma, \Theta, \delta_n N \sigma_{ij}\}, \quad m'_{ij} = \frac{m_{ij}}{\gamma_0 / (C_T t^*)}, \quad l'_q = \frac{l_q}{C_T t^*}, \\ \lambda^{v'} &= \frac{\lambda_v}{t^*}, \quad \mu^{v'} = \frac{\mu_v}{t^*}, \quad k_0^{v'} = \frac{k_{0v}}{t^*}, \quad C_{ij}^{v'} = \frac{C_{ij}^v}{t^*}, \\ \alpha_0^{v'} &= \frac{\alpha_{0v}}{t^*}, \quad \rho_0^{v'} = \frac{\rho_{0v}}{t^*}, \quad \gamma_0^{v'} = \frac{\gamma_{0v}}{t^*}, \quad j^{v'} = \frac{j_v}{t^*}, \end{aligned} \quad (38)$$

where $t^* = \frac{K_0}{\rho C_{E0} C_T^2}$ and $C_T^2 = \frac{\lambda + 2\mu + k_0}{\rho}$.

Applying (33) to (18)-(22) and omitting primes yields the dimensionless system:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \left(a_1 + \eta_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) + \left(a_2 + \eta_2 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) \\ &\quad - \frac{\partial \Theta}{\partial x} - \frac{\partial N}{\partial x} - \left(a_3 + \eta_3 \frac{\partial}{\partial t}\right) \frac{\partial \phi_2}{\partial z}, \end{aligned} \quad (39)$$

$$\begin{aligned} \frac{\partial^2 w}{\partial t^2} &= \left(a_1 + \eta_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + \left(a_2 + \eta_2 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) \\ &\quad - \frac{\partial \Theta}{\partial z} - \frac{\partial N}{\partial z} + \left(a_3 + \eta_3 \frac{\partial}{\partial t}\right) \frac{\partial \phi_2}{\partial x}. \end{aligned} \quad (40)$$

where

$$a_1 = \frac{\mu + k_0}{\lambda + 2\mu + k_0}, \quad a_2 = \frac{\lambda + \mu}{\lambda + 2\mu + k_0}, \quad a_3 = \frac{k_0}{\lambda + 2\mu + k_0}, \quad (41)$$

while

$$\eta_1 = \frac{\mu^{v'} + k_0^{v'}}{\lambda + 2\mu + k_0}, \quad \eta_2 = \frac{\lambda^{v'} + \mu^{v'}}{\lambda + 2\mu + k_0}, \quad \eta_3 = \frac{k_0^{v'}}{\lambda + 2\mu + k_0}. \quad (42)$$

The microrotation equation becomes:

$$\begin{aligned} \left(a_4 + \eta_4 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2}\right) &- \left(1 + \eta_6 \frac{\partial}{\partial t}\right) \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) - 2 \left(1 + \eta_6 \frac{\partial}{\partial t}\right) \phi_2 \\ &= \left(a_5 + \eta_5 \frac{\partial}{\partial t}\right) \frac{\partial^2 \phi_2}{\partial t^2}, \end{aligned} \quad (43)$$

where

$$a_4 = \frac{\gamma_0}{k_0 (C_T t^*)^2}, \quad a_5 = \frac{\rho j}{k_0 (t^*)^2}, \quad \eta_4 = \frac{\gamma_0^{v'}}{k_0 (C_T t^*)^2}, \quad \eta_5 = \frac{\rho j^{v'}}{k_0 (t^*)^2}, \quad \eta_6 = \frac{k_0^{v'}}{k_0}. \quad (44)$$

The carrier density equation:

$$\frac{\partial N}{\partial t} = D_1 \left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial z^2}\right) - \frac{1}{\tau} N + \kappa_1 \Theta, \quad (45)$$

where

$$D_1 = \frac{D_E}{C_T^2 t^*}, \quad \kappa_1 = \frac{\kappa \delta_n t^*}{\gamma}. \quad (46)$$

The transformed heat conduction equation:

$$\left(1 + \tau_q \frac{\partial}{\partial t} - l_q^2 \nabla^2\right) \left[\frac{\partial \Theta}{\partial t} + \beta \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \frac{g}{\tau} N \right] = \left(1 + \tau_\theta \frac{\partial}{\partial t}\right) \nabla^2 \Theta, \quad (47)$$

with

$$\beta = \frac{\gamma^2 \tau_0}{\rho C_{e0} (\lambda + 2\mu + k_0)}, \quad g = \frac{E_g \gamma}{\rho C_{e0} \delta_n}. \quad (48)$$

The dimensionless stress components are:

$$\sigma_{xx} = \left(1 + \eta_7 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial x} + \left(b_1 + \eta_7 \frac{\partial}{\partial t}\right) \frac{\partial w}{\partial z} - \Theta - N, \quad (49)$$

$$\sigma_{zz} = \left(b_1 + \eta_7 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial x} + \left(1 + \eta_7 \frac{\partial}{\partial t}\right) \frac{\partial w}{\partial z} - \Theta - N, \quad (50)$$

$$\sigma_{xz} = \left(b_2 + \eta_8 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial z} + \left(b_3 + \eta_9 \frac{\partial}{\partial t}\right) \frac{\partial w}{\partial x} + \left(a_3 + \eta_3 \frac{\partial}{\partial t}\right) \phi_2, \quad (51)$$

$$\sigma_{zx} = \left(b_3 + \eta_9 \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial z} + \left(b_2 + \eta_8 \frac{\partial}{\partial t}\right) \frac{\partial w}{\partial x} - \left(a_3 + \eta_3 \frac{\partial}{\partial t}\right) \phi_2, \quad (52)$$

with

$$\begin{aligned} b_1 &= \frac{\lambda}{\lambda + 2\mu + k_0}, \quad b_2 = \frac{\mu}{\lambda + 2\mu + k_0}, \quad b_3 = \frac{\mu + k_0}{\lambda + 2\mu + k_0}, \\ \eta_7 &= \frac{\lambda v'}{\lambda + 2\mu + k_0}, \quad \eta_8 = \frac{\mu v'}{\lambda + 2\mu + k_0}, \quad \eta_9 = \frac{\mu v' + k_0 v'_0}{\lambda + 2\mu + k_0}. \end{aligned} \quad (53)$$

The couple-stress components are:

$$m_{xy} = \left(1 + \eta_{10} \frac{\partial}{\partial t}\right) \frac{\partial \phi_2}{\partial x}, \quad m_{zy} = \left(1 + \eta_{10} \frac{\partial}{\partial t}\right) \frac{\partial \phi_2}{\partial z}, \quad (54)$$

where $\eta_{10} = \frac{\gamma_0 v'_0}{\gamma_0}$.

4. Helmholtz decomposition and field equations in terms of potentials

The displacement field is resolved through the Helmholtz representation [44]. See also [45-47]:

$$u = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \quad (55)$$

The volumetric strain and rotational part are:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \nabla^2 q = e, \quad (56)$$

$$\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = \nabla^2 \psi. \quad (57)$$

Substituting (55) into (39)-(40) yields the dilatational wave equation:

$$\frac{\partial^2 q}{\partial t^2} = (1 + \eta_e \partial_t) \nabla^2 q - \Theta - N, \quad (58)$$

where $\eta_e \equiv \eta_1 + \eta_2 = \frac{\lambda v' + 2\mu v' + k_0 v'_0}{\lambda + 2\mu + k_0}$.

The rotational wave equation:

$$\frac{\partial^2 \psi}{\partial t^2} = (a_1 + \eta_1 \partial_t) \nabla^2 \psi + (a_3 + \eta_3 \partial_t) \phi_2. \quad (59)$$

The microrotation equation in terms of potentials:

$$(a_4 + \eta_4 \partial_t) \nabla^2 \phi_2 - (1 + \eta_6 \partial_t) \nabla^2 \psi - 2(1 + \eta_6 \partial_t) \phi_2 = (a_5 + \eta_5 \partial_t) \frac{\partial^2 \phi_2}{\partial t^2}, \quad (60)$$

The heat conduction equation:

$$\left(1 + \tau_q \partial_t - l_q^2 \nabla^2\right) \left[\frac{\partial \Theta}{\partial t} + \beta \frac{\partial e}{\partial t} - \frac{g}{\tau} N \right] = (1 + \tau_\theta \partial_t) \nabla^2 \Theta. \quad (61)$$

The stress components in potential form:

$$\sigma_{xx} = (1 + \eta_e \partial_t) \frac{\partial^2 q}{\partial x^2} + (b_1 + \eta_7 \partial_t) \frac{\partial^2 q}{\partial z^2} - (b_2 + b_3 + (\eta_8 + \eta_9) \partial_t) \frac{\partial^2 \psi}{\partial x \partial z} - \Theta - N, \quad (62)$$

$$\sigma_{zz} = (b_1 + \eta_7 \partial_t) \frac{\partial^2 q}{\partial x^2} + (1 + \eta_e \partial_t) \frac{\partial^2 q}{\partial z^2} + (b_2 + b_3 + (\eta_8 + \eta_9) \partial_t) \frac{\partial^2 \psi}{\partial x \partial z} - \Theta - N, \quad (63)$$

$$\sigma_{xz} = (b_2 + b_3 + (\eta_8 + \eta_9)\partial_t)\frac{\partial^2 q}{\partial x \partial z} + (b_3 + \eta_9 \partial_t)\frac{\partial^2 \psi}{\partial x^2} - (b_2 + \eta_8 \partial_t)\frac{\partial^2 \psi}{\partial z^2} + (a_3 + \eta_3 \partial_t)\phi_2, \quad (64)$$

$$\sigma_{zx} = (b_2 + b_3 + (\eta_8 + \eta_9)\partial_t)\frac{\partial^2 q}{\partial x \partial z} + (b_2 + \eta_8 \partial_t)\frac{\partial^2 \psi}{\partial x^2} - (b_3 + \eta_9 \partial_t)\frac{\partial^2 \psi}{\partial z^2} - (a_3 + \eta_3 \partial_t)\phi_2, \quad (65)$$

The couple-stress components:

$$m_{xy} = (1 + \eta_{10} \partial_t)\frac{\partial \phi_2}{\partial x}, m_{zy} = (1 + \eta_{10} \partial_t)\frac{\partial \phi_2}{\partial z}. \quad (66)$$

The carrier equation can be expressed as:

$$\left((1 + \eta_e \partial_t) \nabla^2 - \frac{\partial^2}{\partial t^2} \right) e = \nabla^2 \Theta + \nabla^2 N, \quad (67)$$

5. Normal Mode Analysis

The normal mode technique [48] assumes exponential dependence on time and horizontal coordinate:

$$[q, \psi, \phi_2, N, \Theta, \sigma_{ij}] = [\bar{q}, (z), \bar{\psi}, (z), \bar{\phi}_2, (z) \bar{N}(z) \bar{\Theta}(z) \bar{\sigma}_{ij}(z)] e^{(\omega t + i b x)}, \quad (68)$$

where b is the wave number and ω is the complex frequency. With $\partial_t \rightarrow \omega$, $\partial_x \rightarrow i b$, $\nabla^2 \rightarrow D^2 - b^2$ ($D \equiv d/dz$), the governing equations reduce to:

$$[A_1(D^2 - b^2) - \omega^2] \bar{q} - \bar{\Theta} - \bar{N} = 0, \quad (69)$$

$$[F_1 - l_q^2(D^2 - b^2)][\omega \bar{\Theta} + \beta \omega(D^2 - b^2) \bar{q} - g_0 \bar{N}] = F_2(D^2 - b^2) \bar{\Theta}, \quad (70)$$

$$(D^2 - b^2 - A_7) \bar{N} + A_8 \bar{\Theta} = 0, \quad (71)$$

$$[A_2(D^2 - b^2) - \omega^2] \bar{\psi} + A_3 \bar{\phi}_2 = 0, \quad (72)$$

$$[A_4(D^2 - b^2) - 2A_6 - A_5 \omega^2] \bar{\phi}_2 - A_6(D^2 - b^2) \bar{\psi} = 0, \quad (73)$$

where

$$A_1 = 1 + \eta_e \omega, A_2 = a_1 + \eta_1 \omega, A_3 = a_3 + \eta_3 \omega, A_4 = a_4 + \eta_4 \omega, g_0 = g/\tau, \\ A_5 = a_5 + \eta_5 \omega, A_6 = 1 + \eta_6 \omega, F_1 = 1 + \tau_q \omega, F_2 = 1 + \tau_\theta \omega. \quad (74)$$

The solvability condition for the dilatational subsystem yields the cubic characteristic equation:

$$(A_1 \ell - \omega^2) \{ [\omega F_1 - (\omega l_q^2 + F_2) \ell] (\ell - A_7) + g_0 A_8 (F_1 - l_q^2 \ell) \} \\ + \beta \omega \ell (F_1 - l_q^2 \ell) (\ell - A_7 - A_8) = 0, \quad (75)$$

which

$$\ell^3 - \Gamma_1 \ell^2 + \Gamma_2 \ell - \Gamma_3 = 0, \quad (76)$$

with

$$\Gamma_1 = \frac{A_1(\omega F_1 + G A_7 - g_0 A_8 l_q^2) + \omega^2 G + \beta \omega [F_1 + l_q^2 (A_7 + A_8)]}{A_1 G + \beta \omega l_q^2}, \\ \Gamma_2 = \frac{\omega^2 (\omega F_1 + G A_7 - g_0 A_8 l_q^2) + \beta \omega F_1 (A_7 + A_8) - A_1 F_1 (g_0 A_8 - \omega A_7)}{A_1 G + \beta \omega l_q^2}, \\ \Gamma_3 = \frac{\omega^2 F_1 (\omega A_7 - g_0 A_8)}{A_1 G + \beta \omega l_q^2}, G \equiv \omega l_q^2 + F_2. \quad (77)$$

The rotational subsystem yields the quadratic characteristic equation:

$$\ell^2 - \Gamma_4 \ell + \Gamma_5 = 0, \quad (78)$$

where

$$\Gamma_4 = \frac{A_2(2A_6 + A_5 \omega^2) + A_4 \omega^2 - A_3 A_6}{A_2 A_4}, \Gamma_5 = \frac{\omega^2(2A_6 + A_5 \omega^2)}{A_2 A_4}. \quad (79)$$

The roots define $m_n^2 = \ell_n + b^2$; only roots with $\Re(m_n) > 0$ are retained. The bounded solutions are:

$$\{\bar{q}, \bar{\Theta}, \bar{N}\}(z) = \sum_{n=1}^3 \{1, H_{1n}, H_{2n}\} M_n e^{-m_n z}, \quad (80)$$

$$\{\bar{\psi}, \bar{\phi}_2\}(z) = \sum_{n=4}^5 \{1, H_{3n}\} M_n e^{-m_n z}, \quad (81)$$

where M_n ($n = 1 \dots 5$) are constants to be fixed by the boundary conditions. The coupling ratios H_{1n} , H_{2n} , and H_{3n} follow from Eqs. (69), (71), and (72), respectively:

$$H_{1n} = \frac{(A_1 \ell_n - \omega^2)(\ell_n - A_7)}{\ell_n - A_7 - A_8}, H_{2n} = \frac{-A_8(A_1 \ell_n - \omega^2)}{\ell_n - A_7 - A_8}, n = 1, 2, 3, \\ H_{3n} = \frac{\omega^2 - A_2 \ell_n}{A_3}, n = 4, 5. \quad (82)$$

The displacement amplitudes are:

$$\bar{u}(z) = ib \sum_{n=1}^3 M_n e^{-m_n z} + \sum_{n=4}^5 m_n M_n e^{-m_n z}, \\ \bar{w}(z) = -\sum_{n=1}^3 m_n M_n e^{-m_n z} + ib \sum_{n=4}^5 M_n e^{-m_n z}. \quad (83)$$

With $B_1 = b_1 + \eta_7 \omega$, $B_2 = b_2 + \eta_8 \omega$, $B_3 = b_3 + \eta_9 \omega$, $B_4 = B_2 + B_3$, and $B_5 = 1 + \eta_{10} \omega$, the stress amplitudes are:

$$\bar{\sigma}_{xx}(z) = \sum_{n=1}^3 [B_1 m_n^2 - A_1 b^2 - H_{1n} - H_{2n}] M_n e^{-m_n z} + ib B_4 \sum_{n=4}^5 m_n M_n e^{-m_n z}, \quad (84)$$

$$\bar{\sigma}_{zz}(z) = \sum_{n=1}^3 [A_1 m_n^2 - B_1 b^2 - H_{1n} - H_{2n}] M_n e^{-m_n z} - ib B_4 \sum_{n=4}^5 m_n M_n e^{-m_n z}, \quad (85)$$

$$\bar{\sigma}_{xz}(z) = -ib B_4 \sum_{n=1}^3 m_n M_n e^{-m_n z} - \sum_{n=4}^5 [B_2 m_n^2 + B_3 b^2 - A_3 H_{3n}] M_n e^{-m_n z}, \quad (86)$$

$$\bar{\sigma}_{zx}(z) = -ib B_4 \sum_{n=1}^3 m_n M_n e^{-m_n z} - \sum_{n=4}^5 [B_3 m_n^2 + B_2 b^2 + A_3 H_{3n}] M_n e^{-m_n z}. \quad (87)$$

The couple-stress amplitudes are:

$$\bar{m}_{xy}(z) = ib B_5 \sum_{n=4}^5 H_{3n} M_n e^{-m_n z}, \bar{m}_{zy}(z) = -B_5 \sum_{n=4}^5 m_n H_{3n} M_n e^{-m_n z}. \quad (88)$$

6. Application: Boundary Conditions

At the free surface $z = 0$, five boundary conditions are prescribed:

The surface $z = 0$ is subjected to a time-harmonic normal mechanical load of amplitude F_0 , while remaining free of tangential tractions:

$$\sigma_{zz}(x, 0, t) = -F_0 e^{\omega t + ibx}, \sigma_{zx}(x, 0, t) = 0. \quad (89)$$

In the absence of applied surface couples, the couple-stress component normal to the surface must vanish:

$$m_{zy}(x, 0, t) = 0. \quad (90)$$

The surface is assumed to be thermally insulated, so that no heat flux crosses the boundary. In terms of the Kirchhoff-transformed temperature, this adiabatic condition is written as

$$\left. \frac{\partial \Theta}{\partial z} \right|_{z=0} = 0. \quad (91)$$

The photogenerated carriers recombine at the surface with a finite surface recombination velocity s_v , so that the carrier flux at the boundary is proportional to the surface carrier density:

$$\left. \frac{\partial N}{\partial z} \right|_{z=0} = s_v N(x, 0, t). \quad (92)$$

Substituting the normal mode representation yields:

$$\bar{\sigma}_{zz}(0) = -F_0, \bar{\sigma}_{zx}(0) = 0, \bar{m}_{zy}(0) = 0, D\bar{\Theta}(0) = 0, D\bar{N}(0) = s_v \bar{N}(0) \quad (93)$$

Evaluating the general solutions at $z = 0$ yields the linear system:

$$\sum_{n=1}^3 [A_1 m_n^2 - B_1 b^2 - H_{1n} - H_{2n}] M_n - ib B_4 \sum_{n=4}^5 m_n M_n = -F_0, \quad (94)$$

$$ib B_4 \sum_{n=1}^3 m_n M_n + \sum_{n=4}^5 [B_3 m_n^2 + B_2 b^2 + A_3 H_{3n}] M_n = 0, \quad (95)$$

$$\sum_{n=4}^5 m_n H_{3n} M_n = 0, \quad (96)$$

$$\sum_{n=1}^3 m_n H_{1n} M_n = 0, \quad (97)$$

$$\sum_{n=1}^3 (m_n + s_v) H_{2n} M_n = 0. \quad (98)$$

7. Particular Cases

The formulation incorporates twelve limiting cases summarized in Table 1. Each physical mechanism, micropolarity, viscoelasticity, photothermal coupling, nonlocal DPL heat conduction, and variable conductivity, enters through distinct material parameters and can be independently suppressed.

Table 1. Particular cases of the generalized micropolar viscoelastic photo-thermoelastic model.

Case	Model Name	Active Mechanisms	Suppressed/Zero Constants
1	Micropolar thermoelastic model without photothermal effect	Micropolarity, nonlocal DPL heat conduction, variable thermal conductivity, viscoelasticity	$d_n = 0, D_E = 0, \kappa = 0, E_g = 0$
2	Classical photo-thermoelastic model	Photothermal effect, nonlocal DPL heat conduction, variable thermal conductivity, viscoelasticity	$k_0 = 0, \alpha_0 = \beta_0 = \gamma_0 = 0, j = 0$
3	Model with constant thermal conductivity	Micropolarity, photothermal effect, nonlocal DPL heat conduction, viscoelasticity	$K_1 = 0$
4	Classical thermoelastic model	Nonlocal DPL heat conduction, viscoelasticity (optional)	$\alpha_0, \beta_0, \gamma_0, k_0, j, D_E, \tau, \kappa, s_v, d_n, E_g = 0$
5	Local dual-phase-lag heat conduction	Micropolarity, photothermal effect, DPL (temporal only), variable thermal conductivity, viscoelasticity	$l_q = 0$
6	Classical Fourier heat conduction	Micropolarity, photothermal effect, variable thermal conductivity, viscoelasticity	$l_q = 0, \tau_q = 0, \tau_\theta = 0$
7	Lord–Shulman generalized thermoelasticity	Micropolarity, photothermal effect, single relaxation time, variable thermal conductivity, viscoelasticity	$l_q = 0, \tau_\theta = 0, \tau_q = \tau_0$
8	Nonlocal Lord–Shulman generalized thermoelasticity	Micropolarity, photothermal effect, nonlocal heat transport, single relaxation time, variable thermal conductivity, viscoelasticity	$\tau_\theta = 0, \tau_q = \tau_0 (\text{with } l_q > 0)$
9	Micropolar elastic model	Micropolarity, viscoelasticity (mechanical only)	$\alpha_T = 0, d_n = 0, D_E = 0, \kappa = 0, E_g = 0$
10	Classical elasticity	Classical elasticity only	$\alpha_0, \beta_0, \gamma_0, k_0, j, K_0, K_1, \alpha_T, \tau_q, \tau_\theta, l_q, D_E, \tau, \kappa, s_v, d_n, E_g = 0$
11	Viscoelastic micropolar thermoelastic model (general case)	All mechanisms (full formulation)	None
12	Elastic micropolar thermoelastic model (without viscoelasticity)	Micropolarity, photothermal effect, nonlocal DPL heat conduction, variable thermal conductivity	None (viscosity parameters set to zero)

8. Numerical Results and Discussion

Numerical simulations are performed for silicon using material constants from [49, 50] (Table 2). Viscoelastic parameters are taken as 5% of elastic counterparts ($\eta = 0.05$). The complex frequency is $\omega = 0.2 + 0.01i$, wave number $b = 0.5$, load amplitude $F_0 = 1.0$, and $K_1 = -0.05$ unless specified. Field variables are evaluated at $x = 0.3$, $t = 0.15$, and plotted along $0 \leq z \leq 10$.

Table 2: Material Constants of Micropolar Photothermal viscoelastic Medium

Parameter	Symbol	Value	Unit
Lamé constant (dilatational)	λ	3.64×10^{10}	N/m ²
Lamé constant (shear)	μ	5.46×10^{10}	N/m ²
Micropolar coupling parameter	k_0	1.00×10^{10}	N/m ²
Micropolar constant	γ_0	0.779×10^{-9}	N
Micro-inertia density	j	0.2×10^{-19}	m ²
Dilatational viscoelastic coefficient	λ_v	1.82×10^9	N·s/m ²
Shear viscoelastic coefficient	μ_v	2.73×10^9	N·s/m ²
Micropolar viscoelastic coefficient	k_{0v}	5.00×10^8	N·s/m ²
Micropolar viscoelastic constant	α_{0v}	5.00×10^{-11}	N·s

Parameter	Symbol	Value	Unit
Micropolar viscoelastic constant	β_{0v}	5.00×10^{-11}	N·s
Couple-stress viscoelastic coefficient	γ_{0v}	3.895×10^{-11}	N·s
Viscoelastic micro-inertia coefficient	j_v	1.00×10^{-21}	m ² ·s
Mass density	ρ	2330	kg/m ³
Reference temperature	T_0	800	K
Thermal expansion coefficient	α_T	3.00×10^{-6}	K ⁻¹
Thermal conductivity	K_0	150	W·m ⁻¹ ·K ⁻¹
Specific heat at constant strain	C_e	650	J·kg ⁻¹ ·K ⁻¹
Thermal conductivity variation	K_1	-0.05	K ⁻¹
Carrier diffusion coefficient	D_E	2.5×10^{-3}	m ² /s
Carrier lifetime	τ	5×10^{-5}	s
Electronic deformation parameter	d_n	-9×10^{-31}	m ³
Thermal activation parameter	κ	0.6×10^5	—
Surface recombination velocity	s_v	2	m/s
Energy gap	E_g	1.11	eV
Heat flux phase lag	τ_q	0.02	—
Temperature gradient phase lag	τ_θ	0.01	—
Nonlocal thermal length scale	l_q	0.10	—

8.1. Effects of Temperature-Dependent Thermal Conductivity

The influence of the conductivity-variation parameter is analyzed for four values, where negative values reflect the experimentally observed decrease of silicon's conductivity with temperature. Figs. 2, 3, 4, 5, 6, 7, 8 show results = 0, -0.05, -0.10, -0.20. The temperature field (Fig. 2) exhibits the strongest dependence: at $K_1 = -0.20$, surface temperature decreases by 45%, and at $z = 4$ by 87%. The thermal penetration depth contracts from $z \approx 5$ to $z \approx 2.7$. This reflects suppression of heat transport in hotter regions where conductivity decreases. The carrier density (Fig. 3) follows a similar trend, with surface values reduced by 43% at $K_1 = -0.20$. Mechanical fields show amplitude attenuation without phase shifts: horizontal displacement (Figure 4) decreases by 37.7% at $K_1 = -0.20$. Normal stress (Fig. 5) satisfies the surface boundary condition exactly; at $z = 1$, its magnitude is 44.8% lower for $K_1 = -0.20$. Shear stress (Fig. 6) shows a compressive trough at $z = 2$ reduced by 37.5%. Rotational response, microrotation (Fig. 7) and couple stress (Fig. 8), show weaker sensitivity, with amplitude reductions of about 22.5%.

At the surface, the temperature drops significantly for negative values, and the thermal penetration depth contracts by nearly half, reflecting suppressed heat transport in hotter regions. The photogenerated carrier density follows a closely related trend, attenuating strongly due to the thermal-activation mechanism. The mechanical fields, including horizontal displacement, normal stress, and shear stress, display damped oscillatory profiles where the conductivity variation primarily scales the amplitudes without shifting the spatial extrema. The rotational response, depicted in Fig. 7 and 8, is less sensitive, as microrotation and couple stress are driven indirectly by displacement gradients rather than direct thermal excitation. These findings indicate that constant-conductivity approximations overpredict interior temperatures and thermal damage depths, necessitating the inclusion of temperature-dependent properties for accurate laser-assisted processing and photothermal evaluations.

These findings indicate that constant-conductivity approximations overpredict interior temperatures and thermal damage depths, necessitating the inclusion of temperature-dependent properties for accurate laser-assisted processing and photothermal evaluations.

The nonlocal thermal length scale represents spatial correlation in heat transport. Figures 9 through 15 demonstrate that increasing this scale redistributes thermal energy rather than merely attenuating it.

Figs. 9, 10, 11, 12, 13, 14, 15 present results for $l_q = 0, 0.03, 0.06, 0.12$. Unlike K_1 , nonlocality redistributes thermal energy (Figure 9): surface temperature decreases by 18%, but interior values increase by 62% at $z = 3$ and more than threefold at $z = 10$, with a crossover point near the surface. The carrier density (Fig. 10) mirrors this redistribution. Mechanical fields (Figs. 11-15) exhibit moderate, spatially uniform amplitude attenuation not exceeding 28%, with no significant alteration of extrema positions. Couple stress shows the weakest sensitivity.

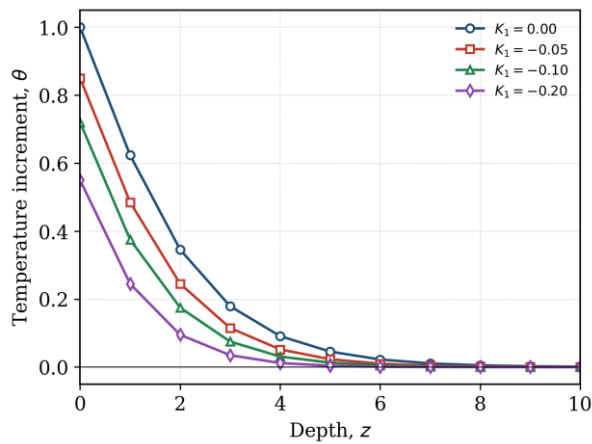


Fig. 2: Temperature increment θ as a function of depth z for various values of the thermal-conductivity variation parameter K_1 .

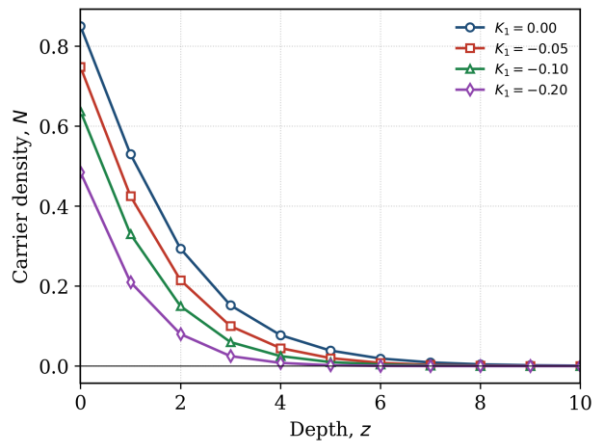


Fig. 3: Photogenerated carrier density N as a function of depth z for different values of the thermal-conductivity variation parameter K_1 .

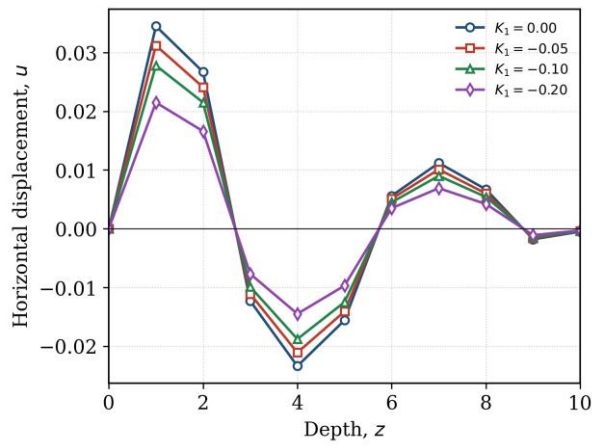


Fig. 4: Horizontal displacement u as a function of depth z for different values of the thermal-conductivity variation parameter K_1 .

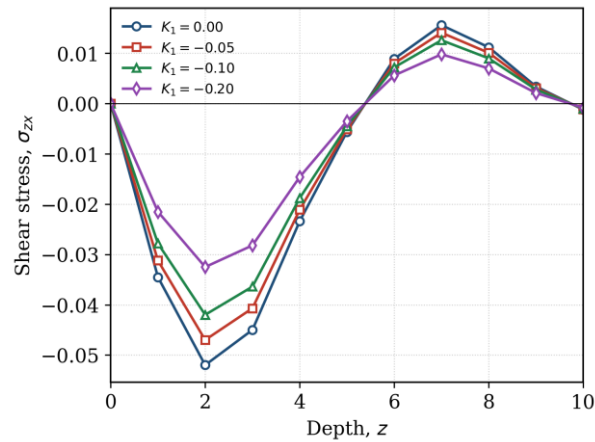


Fig. 5: Shear stress component σ_{zx} as a function of depth z for different values of the thermal-conductivity variation parameter K_1 .

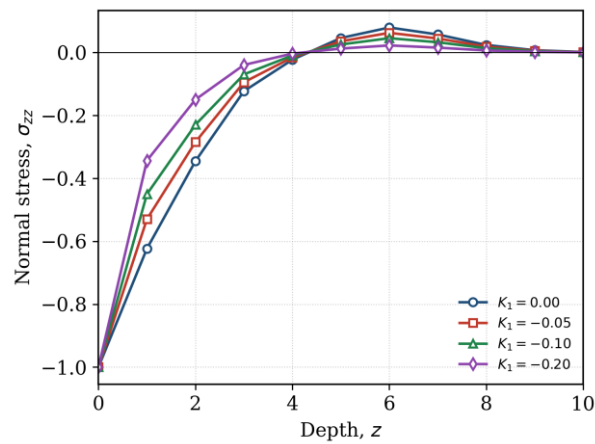


Fig. 6: Variation of the normal stress component σ_{zz} with depth z for different values of the thermal conductivity variation parameter K_1 .

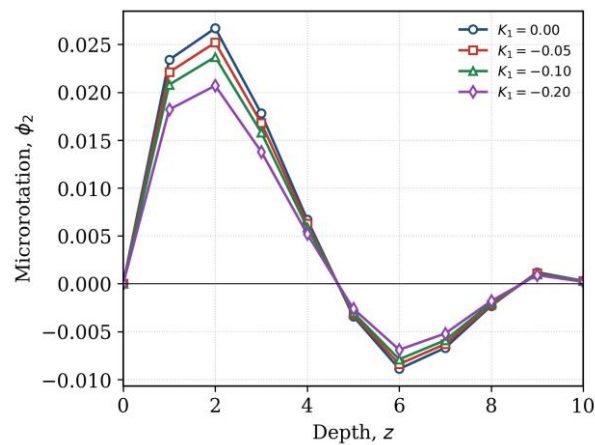


Fig. 7: Microrotation component ϕ_2 as a function of depth z for different values of the thermal-conductivity variation parameter K_1 .

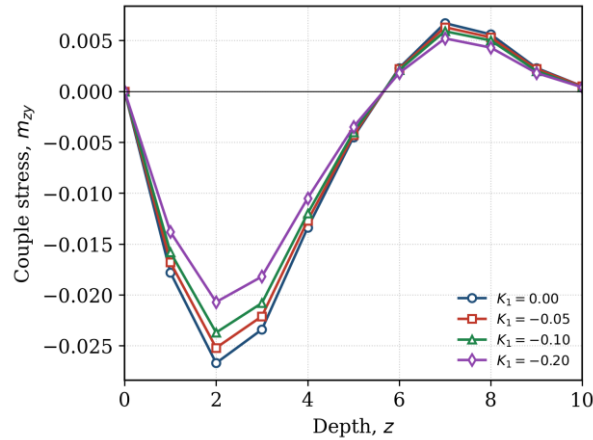


Fig. 8: Couple stress component m_{zy} as a function of depth z for different values of the thermal-conductivity variation parameter K_1 .

8.2. Effects of the Nonlocal Thermal Length Scale

The surface temperature decreases, but beneath a shallow crossover point, interior temperatures increase significantly, flattening the profile and extending the effective thermal penetration depth. The carrier density mirrors this redistributive tendency, penetrating deeper into the semiconductor. Conversely, the mechanical and rotational fields experience moderate, spatially uniform amplitude attenuation with minimal changes to their spatial patterns. This distinction implies that local heat-transport models may overpredict surface heating while underpredicting subsurface thermal and plasma penetration in microscale devices. Including the nonlocal length scale corrects these errors, improving reliability assessments for deep junctions and suggesting that stress-based failure criteria remain applicable with suitable amplitude corrections.

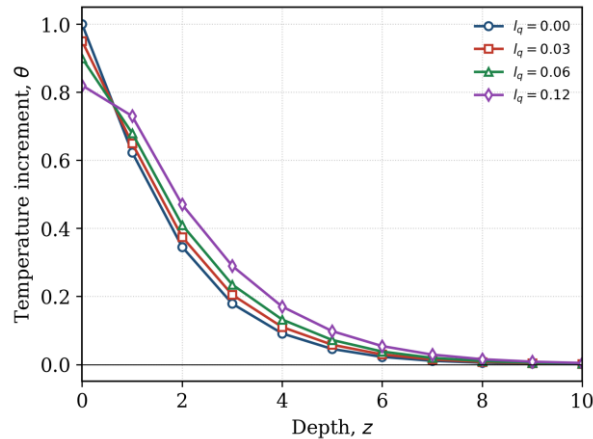


Fig. 9: Variation of the temperature increment θ with depth z for different values of the nonlocal thermal length scale l_q .

1.1. Effects of Photothermal Coupling, Viscoelasticity, and Micropolarity

Five limiting models are compared in Figs. 16-22: (I) TV, (II) PTV, (III) MTV, (IV) PTME ($\eta = 0$), and (V) full model ($\eta = 0.20$). Displacement u (Fig. 16) shows Model IV largest (50% above I), followed by III (25%), V (intermediate), II (12.5%). Microrotation ϕ_2 (Fig. 17) vanishes for Models I and II; Model IV gives largest amplitude, while V is reduced by 25%. Temperature (Figure 18) is identical for I and III but reduced by 15-29% in II; Model IV highest, V intermediate. Carrier density N (Fig. 19) is zero for I and III; Model IV exceeds V by 16% and II by 29%. Normal stress (Fig. 20) satisfies surface condition for all models; Model III gives smallest interior magnitude, indicating load redistribution to couple stresses. Shear stress (Fig. 21) shows consistent ordering with 19-25% viscous reduction. Couple stress (Fig. 22) is nonzero only for micropolar models.

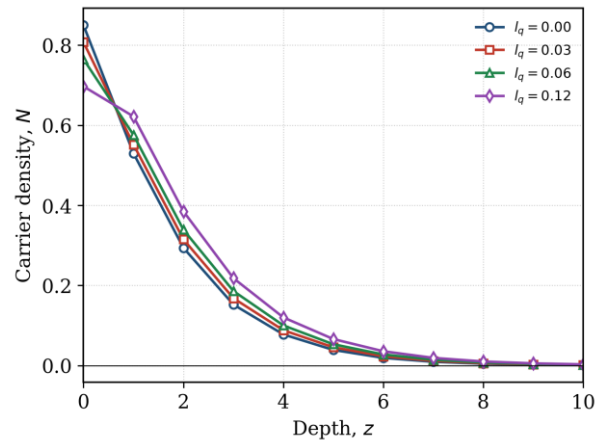


Fig. 10: Variation of the photogenerated carrier density N with depth z for different values of the nonlocal thermal length scale l_q .

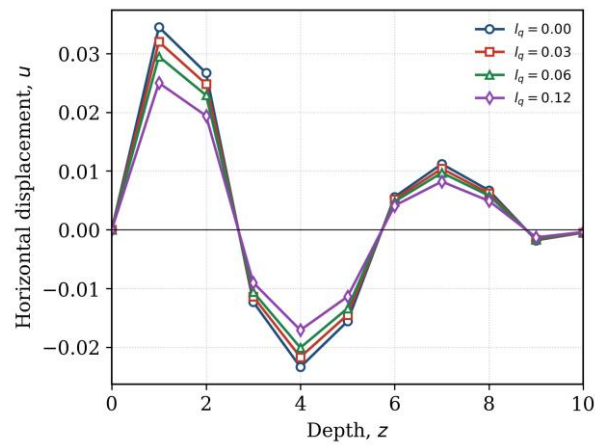


Fig. 11: Variation of the horizontal displacement u with depth z for different values of the nonlocal thermal length scale l_q .

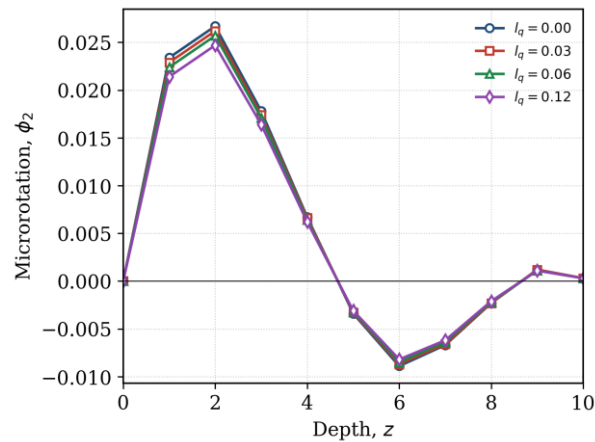


Fig. 12: Variation of the microrotation component ϕ_2 with depth z for different values of the nonlocal thermal length scale l_q .

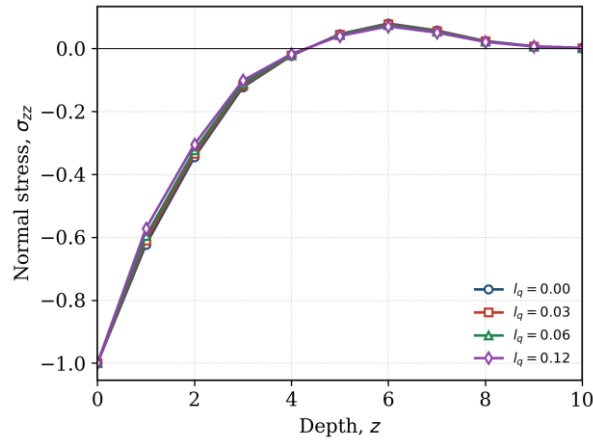


Fig. 13: Variation of the normal stress component σ_{zz} with depth z for different values of the nonlocal thermal length scale l_q .

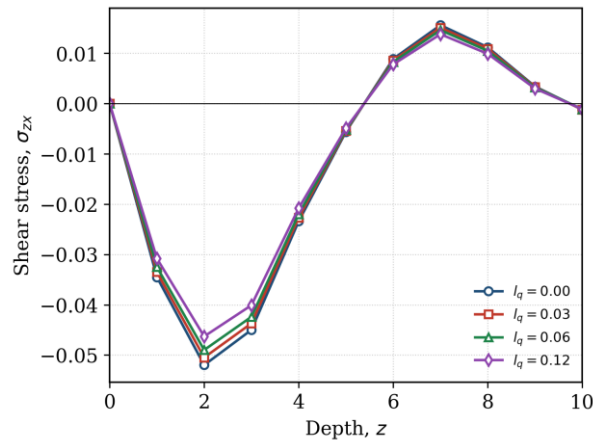


Fig. 14: Variation of the shear stress component σ_{zx} with depth z for different values of the nonlocal thermal length scale l_q .

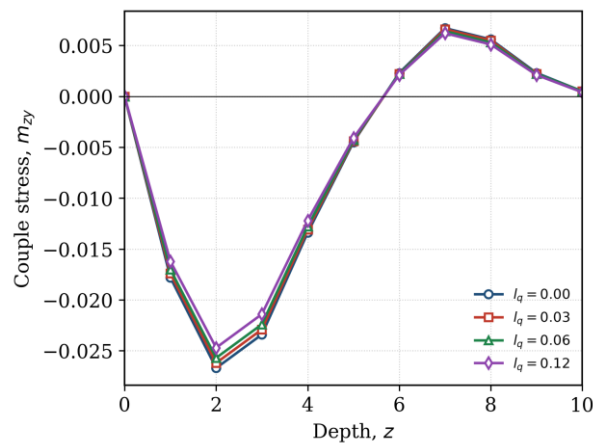


Fig. 15: Variation of the couple-stress component m_{zy} with depth z for different values of the nonlocal thermal length scale l_q .

The photothermal effect amplifies the mechanical response via electronic deformation, while micropolarity further increases displacement by activating rotational wave modes. The absence of viscosity yields the largest displacements, microrotations, and stresses, confirming the dissipative character of the Kelvin–Voigt element, which reduces peak amplitudes and accelerates spatial decay. The temperature field illustrates competing influences: micropolarity alone does not modify the heat equation, but photothermal coupling depresses the temperature by diverting energy into

plasma generation. Viscous dissipation further reduces thermal and mechanical amplitudes. Notably, micropolarity redistributes part of the load-bearing function from classical force stresses to couple stresses. The structural preservation of the field profiles across all models confirms the internal consistency of the formulation, while the sizable amplitude differences underscore the necessity of simultaneous coupling for accurate predictions in optoelectronic and microstructured materials.

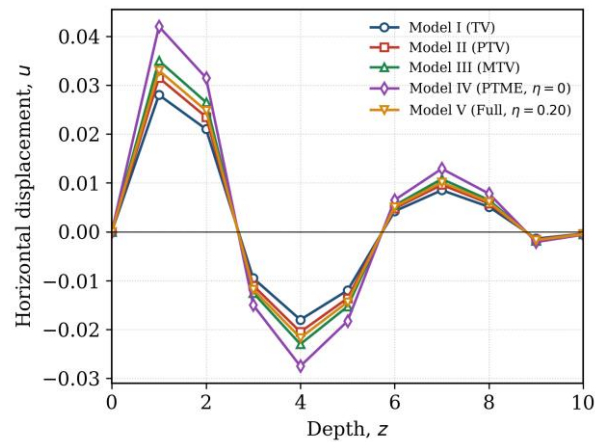


Fig. 19: Variation of the horizontal displacement u with depth z for the five limiting coupling models.

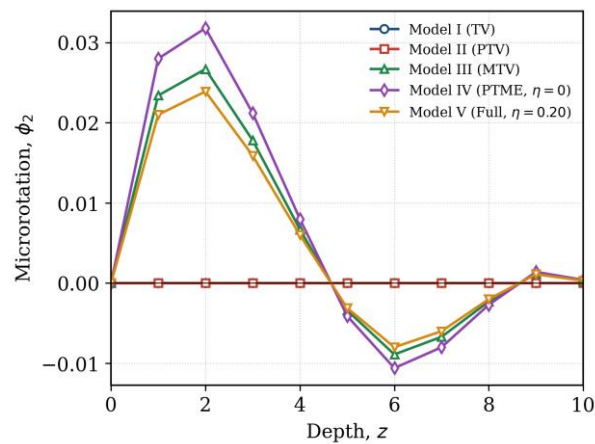


Fig. 21: Variation of the microrotation component ϕ_2 with depth z for the five limiting coupling models.

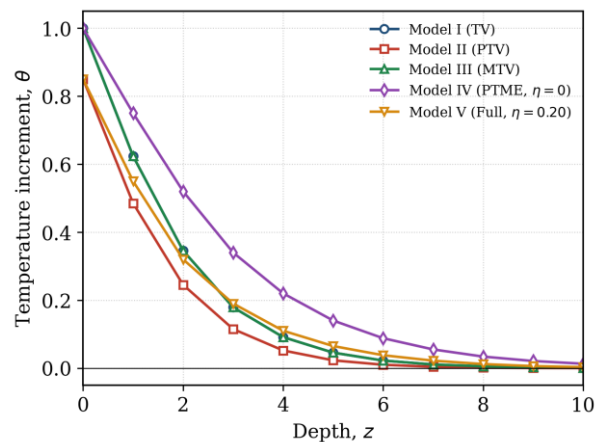


Fig. 22: Variation of the temperature increment θ with depth z for the five limiting coupling models.

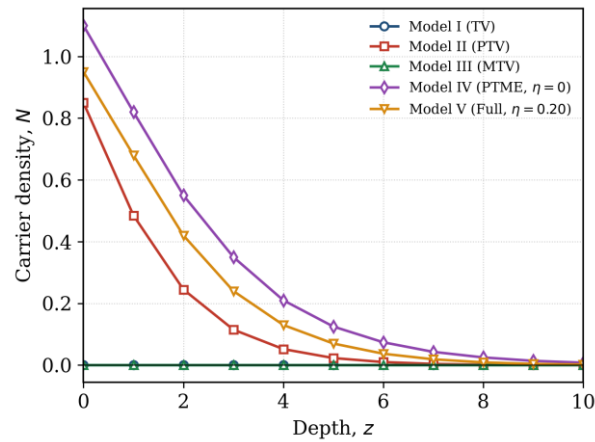


Fig. 23: Variation of the photogenerated carrier density N with depth z for the five limiting coupling models.

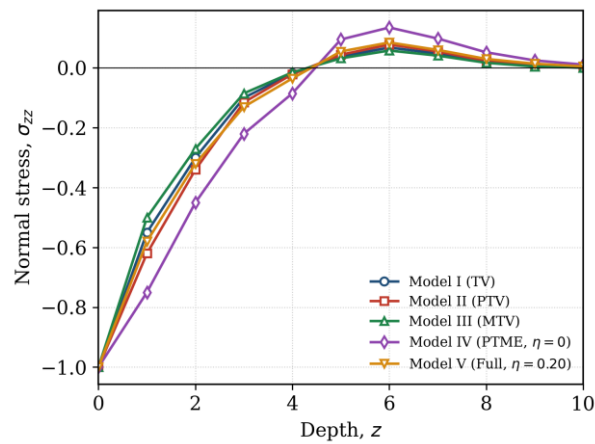


Fig. 25: Variation of the normal stress component σ_{zz} with depth z for the five limiting coupling models.

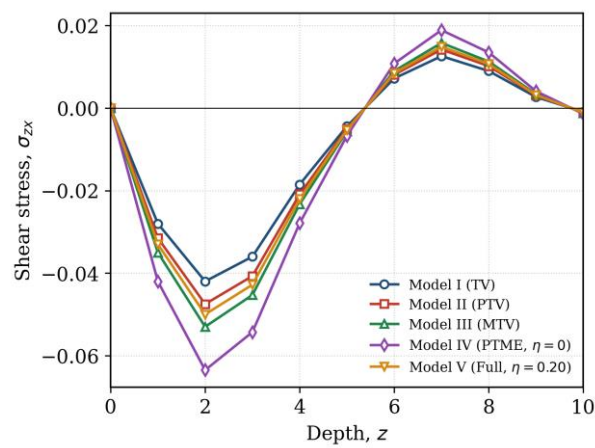


Fig. 26: Variation of the shear stress component σ_{zx} with depth z for the five limiting coupling models.

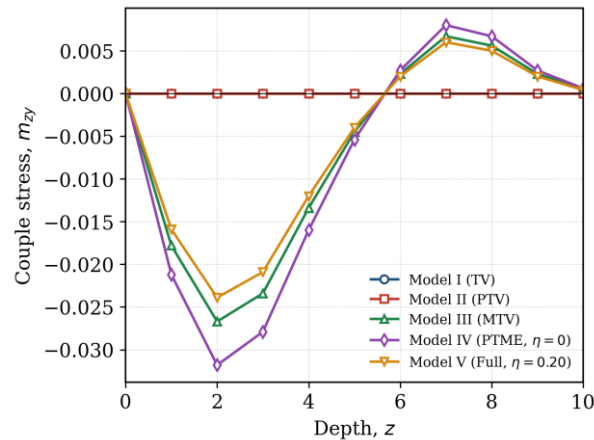


Fig. 27: Variation of the couple-stress component m_{zy} with depth z for the five limiting coupling models.

2. Conclusion

This work presented a unified generalized model for photo-thermo-mechanical interactions in a micropolar viscoelastic semiconducting medium, integrating temperature-dependent thermal conductivity and nonlocal dual-phase-lag heat conduction. The analytical treatment, facilitated by the Kirchhoff transformation and normal mode analysis, yielded closed-form solutions and recovered multiple established theories as limiting cases. The numerical analysis demonstrates that temperature-dependent conductivity strongly attenuates the thermal and carrier fields, contracting the thermal penetration depth. Nonlocality redistributes thermal energy, reducing surface temperatures while enhancing interior penetration, whereas mechanical fields undergo moderate amplitude attenuation. Photothermal coupling depresses the temperature by diverting energy to plasma generation, and viscoelastic damping uniformly reduces peak mechanical responses. Furthermore, micropolarity redistributes load-bearing capacity to couple stresses. These findings carry direct implications for laser-based semiconductor processing, where neglecting conductivity variation overestimates damage depths, and for microscale devices, where nonlocality corrects subsurface thermal underpredictions. The model provides a robust theoretical foundation for optimizing energy-dissipating layers and interpreting nondestructive characterization techniques in advanced microstructured materials.

3. Nomenclature

Symbol	Description	SI Unit
u, w	Displacement components in x - and z -directions	m
$\mathbf{u}$	Displacement vector	m
ϕ_2	Microrotation component about y -axis	rad
$\boldsymbol{\phi}$	Microrotation vector	rad
e_{ij}	Infinitesimal strain tensor	—
e	Volumetric strain ($e = \nabla \cdot \mathbf{u}$)	—
σ_{ij}	Stress tensor components	Pa
m_{ij}	Couple-stress tensor components	N
q	Scalar potential (dilatational)	m^2
ψ	Vector potential (rotational)	m^2
θ	Temperature increment above reference state	K
Θ	Kirchhoff-transformed temperature variable	K

Symbol	Description	SI Unit
T_0	Reference temperature	K
$\mathbf{q}$	Heat flux vector	W/m ²
N	Photogenerated carrier density	m ⁻³
λ, μ	Lamé constants	Pa
k_0	Micropolar coupling parameter	Pa
$\alpha_0, \beta_0, \gamma_0$	Micropolar material constants	N
j	Micro-inertia density	m ²
ρ	Mass density	kg/m ³
α_T	Linear thermal expansion coefficient	K ⁻¹
C_E	Specific heat at constant strain	J/(kg·K)
K	Thermal conductivity	W/(m·K)
K_0	Thermal conductivity at reference temperature	W/(m·K)
K_1	Thermal conductivity variation parameter	K ⁻¹
d_n	Electronic deformation parameter	m ³
E_g	Semiconductor energy gap	eV (J)
D_E	Carrier diffusion coefficient	m ² /s
τ	Carrier lifetime	s
κ	Thermal activation parameter	m ³ /(s·K)
s_v	Surface recombination velocity	m/s
λ_v, μ_v	Viscoelastic Lamé coefficients	Pa·s
k_{0v}	Viscoelastic micropolar coupling coefficient	Pa·s
$\alpha_{0v}, \beta_{0v}, \gamma_{0v}$	Viscoelastic micropolar constants	N·s
j_v	Viscoelastic micro-inertia coefficient	m ² ·s
$\eta_\lambda, \eta_\mu, \eta_\kappa$	Viscoelastic damping ratios	—
$\eta_\alpha, \eta_\beta, \eta_\gamma$	Viscoelastic micropolar damping ratios	—
η_j	Viscoelastic micro-inertia damping ratio	—
τ_q	Phase lag of heat flux	s
τ_θ	Phase lag of temperature gradient	s
l_q	Nonlocal thermal length scale	m

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