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RESEARCH PAPER

Void Effects on Plane Wave Propagation in a Nonlocal Microstretch Thermoelastic Medium with Initial Stress and Magnetic Field under Three Phase Lag Theory

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Abstract

In this paper, the reflection of plane waves at a free surface of a nonlocal microstretch thermoelastic medium is investigated within the framework of the three-phase-lag (TPL) heat conduction theory. The model incorporates the combined effects of nonlocal elasticity, voids, magnetic field, and initial stress. The governing equations are formulated and solved to obtain analytical expressions for the amplitude ratios and energy ratios of reflected longitudinal and coupled transverse waves propagating with distinct phase velocities. Numerical computations are performed to examine the graphical influence of key physical parameters on wave characteristics. The results indicate that the void parameter generally reduces the amplitude ratios, whereas the nonlocal parameter significantly influences transverse wave components. The magnetic field predominantly affects longitudinal wave reflection, while initial stress produces contrasting variations in amplitude ratios. It is further observed that transverse wave amplitudes vanish at limiting angles of incidence, and the primary longitudinal wave remains dominant. Energy analysis confirms that the sum of energy ratios is unity for all angles of incidence, ensuring conservation of energy. The present study provides useful insights into wave propagation phenomena in complex thermoelastic materials with microstructural effects and external fields.

Keywords: Void parameter; Magnetic field; Initial stress; Energy ratio; Wave reflection; Nonlocal elasticity; Thermoelasticity; Microstretch medium; Three-phase-lag theory.

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1. Introduction

The nonlocal elasticity model provides a unique perspective on material behavior. According to this approach, the strain at a given location across an elastic material is not the only factor that determines the stress there. Rather, the strains are found throughout the entire substance at every other location. Because of this unique characteristic, the nonlocal continuum approach takes into account data regarding interactions between atoms or molecules inside the material. Edelen et al. [1] gave the idea of continuum nonlocal mechanics. Eringen extended the idea of nonlocality to elasticity in a series of articles [2-4]. Later on, McCay and Narsimhan [5] came up with a theory about materials that can conduct electricity but also have special properties when you consider long-distance effects. Some important researchers have made significant contributions to nonlocal elasticity, including Altan [6] and Craciun [7]. Lebon and Grmela [8] also worked on how heat spreads in solid materials but in cases where these effects are not forceful. Narendr [9] studied how waves twist through tiny rods using a special mathematical approach called spectral finite element and ideas from nonlocal continuum mechanics. Khurana and Tomar [10] also examined how waves travel in materials that have tiny spaces (voids) in them. By using a temperature-dependent elastic model in a nonlocal rotating micropolar medium.

Advanced technologies such as particle accelerators, rapid-burst nuclear reactors, and pulsating laser have led to increased interest in generalized models of thermoelasticity among researchers and engineers. These models are important because they deal with materials that respond to changes in temperature and mechanical stress, and they involve equations that describe how these responses happen over time. Several well-known generalized thermoelastic theories have got the attention of researchers. Lord and Shulman [11] proposed Fourier's law with a thermal relaxation time parameter, called the single-phase-lag theory. Later on Green and Lindsay [12] proposed a new model with two relaxation times. These relaxation times describe how stress and temperature interact over time. These models are vital for analyzing and designing systems involving extreme conditions like those originating in nuclear reactors and lasers. In addition to the previously mentioned models, Tzou [13] and Chandrasekharaiah [14] developed an alternative theory about thermoelasticity. They introduced two-phase lags τ_T and τ_q, for heat flux vector $\vec{q}$. τ_T and τ_q are used to explain how temperature changes and how heat moves in a material. He explained how temperature and heat changes occur in materials with these three different time delays. These improvements help us understand more intricate situations of how materials respond to heat flow and temperature changes in numerous real-world problems. Along with τ_T and τ_q, phase lag τ_v is introduced in the thermal displacement gradient ∇v. Quintanilla and Racke [15] examined the heat equation with three different phase-lag parameters, satisfying the condition $0 \leq \tau_v < \tau_T < \tau_q$.

Eringen established a theory called microstretch elastic bodies [16], which is an extension of micropolar theory. The simplified version of the micromorphic theory is also introduced by Eringen [17]. Micropolar theory is widely acknowledged and has several applications. Microstretch theory is simpler than micromorphic theory but not as versatile as micropolar theory. It's like a basic mathematical model that can describe certain materials but doesn't cover as many different situations as micropolar theory does. So, it's used for materials that don't quite fit into the micropolar framework. Microstretch solids are materials where the tiny particles can not only move around (translate) and rotate but also expand and contract. So, the movement of these particles is divided into seven different ways: three for moving around, three for translation and rotation, and one for stretching. These seven degrees of freedom help us to understand how these materials behave and respond to forces. In a microstretch medium, Khan and Zafar [18] investigated the behavior of laser on plane waves. Alhejaili [19] discussed the nonlocal wave propagation in photo-thermally excited magneto-microstretch semiconductors. Few core studies on three phase leg theory, thermoelastic media and MHD flow can be seen in [20-27].

The phenomenon of wave reflection in a nonlocal microstretch thermoelastic medium is examined in this work using the three-phase-lag theory. The effects of magnetic fields, voids, and initial strains are also included. In the specified medium, longitudinal and coupled transverse waves are reflected at different velocities. The amplitude ratios and energy ratios of several reflected waves are computed numerically. The amplitude ratios are also presented graphically.

2. Governing Equations

For a nonlocal microstretch thermoelastic media, the governing equations are taken from Eringen [28] and Khurana and Tomar [29].

2.1: Constitutive Relations:

$$\sigma_{ij}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{ij} = (\lambda u_{r,r} + \lambda_0\phi^* - \beta_1\theta - P + b\Phi)\delta_{ij} + \mu(u_{i,j}+u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\phi_r) - P\omega_{ij}, \quad (1)$$

$$m_{ij}^{Local} = (1-\epsilon^2\nabla^2)m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} + b_0\varepsilon_{mji}\phi^*_{,m}, \quad (2)$$

$$m_i^{Local} = (1-\epsilon^2\nabla^2)m_i = \alpha_0\phi^*_{,m} + b_0\varepsilon_{ijk}\phi_{j,k}, \quad (3)$$

where m_i components of nonlocal microstretch vector, α_0, λ_0, b_0 microstretch constants, λ, μ are lame's constant, $\vec{\phi}$ micro-rotation vector, σ_{ij} components of nonlocal stress tensor, $\vec{u}$ displacement vector, ϕ^* scalar microstretch, P initial stress, δ_{ij} Kronecker delta function, ω_{ij} rotation vector, and ε_{ijr} is an alternating tensor, α, β, K, γ are micropolar constants. $\beta_1 = (3\lambda + 2\mu + K)\alpha_{t_1}$, α_{t_1} linear thermal expansion coefficient, the quantities $\sigma_{ij}^{Local}, m_{ij}^{Local}, m_i^{Local}$ represent the local microstretch thermoelastic solid, $\theta = T - T_0$ where T_0, T shows reference and absolute temperature are explain as $|\frac{T-T_0}{T_0}| \ll 1$, $\epsilon = e_0 a_{cl}$ the nonlocal parameter, a_{cl} e_0 a material constant.

2.2: Stress equation of motion:

$$\nabla(\nabla.\vec{u})(\lambda+\mu) - \beta_1\nabla T + (K+\mu)\nabla^2\vec{u} + \lambda_0\nabla\phi^* + K\nabla\times\vec{\phi} = (1-\epsilon^2\nabla^2)\rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (4)$$

where ρ density.

2.3: Couple stress equation of motion:

$$-\gamma\nabla\times(\nabla\times\vec{\phi}) + \nabla\times\vec{u}K + (\alpha+\beta+\gamma)\nabla(\nabla.\vec{\phi}) - 2K\vec{\phi} = (1-\epsilon^2\nabla^2)\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \quad (5)$$

where j the microinertia.

2.4: Equation of balance stress moments:

$$\alpha_0\nabla^2\phi^* - \lambda_0\nabla.\vec{u} + \beta_0\theta - \lambda_1\phi^* = (1-\epsilon^2\nabla^2)\frac{\rho j_0}{2}\frac{\partial^2\phi^*}{\partial t^2}, \quad (6)$$

where j_0 is the microstretch inertia, λ_1 is the microstretch constant.

2.5: Heat equation:

The nonlocal generalization heat conduction equation is specified as [30]:

$$(1-\epsilon^2\nabla^2)\left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\vec{q}=-\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla\nu K_1^*-K^*\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla T, \quad (7)$$

For nonlocal thermoelastic media's linear theory, heat conduction equation is given as:

$$\rho T_0\dot{\eta}=-\nabla.\vec{q}, \quad (8)$$

$$(1-\epsilon^2\nabla^2)\rho T_0\eta=e\beta_1T_0+\rho C_E\theta+\beta_0T_0\phi^*, \quad (9)$$

where specific entropy is η and heat flux vector is $\vec{q}$. Using Eqs. (7) – (9), one can obtain:

$$\left[K^*\left(1+\tau_T\frac{\partial}{\partial t}\right)+K_1^*\left(\tau_v\frac{\partial}{\partial t}+1\right)\right]\nabla^2T=\left(\tau_q\frac{\partial}{\partial t}+1+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\ddot{e}\beta_1T_0+\rho C_E\ddot{T}+\beta_0T_0\ddot{\phi}^*+mT_0\dot{\Phi}\right), \quad (10)$$

where C_E specific heat, $K_1^*=C_E(\lambda+2\mu)/4$ material characteristic, K^* thermal conductivity, e cubical dilatation and $\beta_0=(3\lambda+2\mu+K)\alpha_{t_2}$, α_{t_2} linear thermal coefficient.

2.6: Volume fractional field equation:

The voids equation (see ref [35]) is given by

$$h_{i,i}+g=\rho\chi(1-\epsilon^2\nabla^2)\frac{\partial^2\Phi}{\partial t^2}, \quad (11)$$

where $h_i=\alpha\Phi_{,i}$, and $g=-(be_{kk}+\xi\Phi-mT)$, α,ξ,b the void and, m the thermo-void parameter.

3. Formulation of the Problem

For two-dimensional space, a homogeneous, nonlocal, thermoelastic microstretch medium is considered under three-phase lag model. Cartesian system is presented with z-axis directed vertically downward. The wave is propagating along x-axis so that the particles are equally displaced parallel to y-axis as shown in Fig. 1.

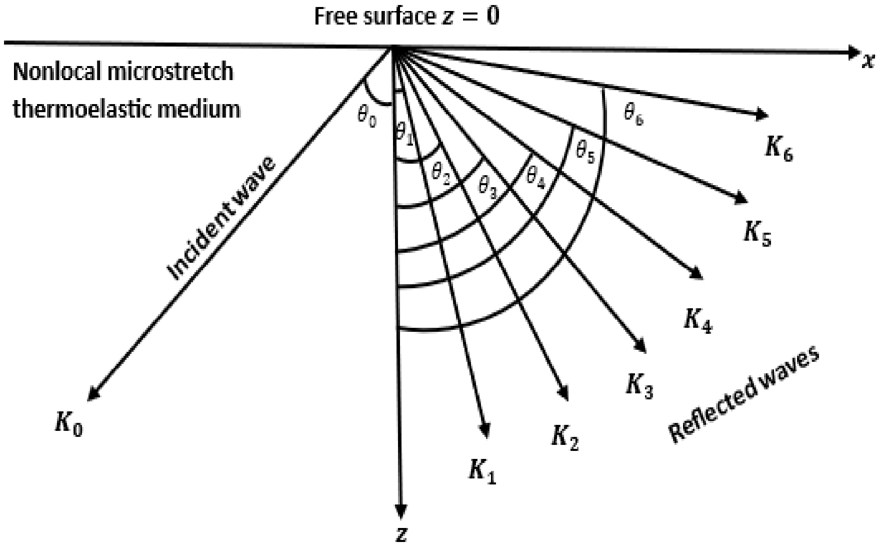

Figure 1: Geometry of the problem.

For the current problem, $\vec{u}$ displacement vector, the microstretch parameter ϕ^* and micro-rotation vector $\vec{\phi}$ are given as:

$$u(x,z,t) = u = u_1,\ w(x,z,t) = w = u_3, v = u_2 = 0, (0,\phi_2,0) = \vec{\phi}, \phi^* = \phi^*(x,z,t), \quad (12)$$

By using Eqs. $(1)-(3)$ along with Eq. (12), the stress components are given as:

$$\sigma_{xx}{}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{xx} = \lambda\frac{\partial w}{\partial z} + \lambda_0\phi^* - \beta_1\theta - P + b\Phi + (\lambda+2\mu+K)\frac{\partial u}{\partial x}, \quad (13)$$

$$\sigma_{zz}{}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{zz} = \lambda\frac{\partial u}{\partial x} + \lambda_0\phi^* - \beta_1\theta - P + b\Phi + (\lambda+2\mu+K)\frac{\partial w}{\partial x}, \quad (14)$$

$$\sigma_{zx}{}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{zx} = \left(\mu+\frac{P}{2}\right)\frac{\partial w}{\partial z} - K\phi_2 + \left(\mu+K-\frac{P}{2}\right)\frac{\partial u}{\partial z}, \quad (15)$$

$$m_{xy}{}^{Local} = (1-\epsilon^2\nabla^2)m_{xy} = \gamma\frac{\partial\phi_2}{\partial x} - b_0\frac{\partial\phi^*}{\partial z}, \quad (16)$$

$$m_{zy}{}^{Local} = (1-\epsilon^2\nabla^2)m_{zy} = \gamma\frac{\partial\phi_2}{\partial z} + b_0\frac{\partial\phi^*}{\partial x}, \quad (17)$$

$$m_x{}^{Local} = (1-\epsilon^2\nabla^2)m_x = \alpha_0\frac{\partial\phi^*}{\partial x} + b_0\frac{\partial\phi_2}{\partial z}, \quad (18)$$

$$m_z{}^{Local} = (1-\epsilon^2\nabla^2)m_z = \alpha_0\frac{\partial\phi^*}{\partial z} - b_0\frac{\partial\phi_2}{\partial x}, \quad (19)$$

Using Eq. (12), the Eqs. (4)–(6) and (10) are as follow:

$$\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial e}{\partial x} - \beta_1\frac{\partial T}{\partial x} + \left(K-\frac{P}{2}+\mu\right)\nabla^2 u + \mu_E H0^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 w}{\partial x\partial z}\right) - K\frac{\partial\phi_2}{\partial z} + \lambda_0\frac{\partial\phi^*}{\partial x} + b\frac{\partial\Phi}{\partial x} = (1-\epsilon^2\nabla^2)\rho\frac{\partial^2 u}{\partial t^2}, \quad (20)$$

$$\left(\lambda+\mu+\frac{P}{2}\right)\frac{\partial e}{\partial z} - \beta_1\frac{\partial T}{\partial z} + \left(K-\frac{P}{2}+\mu\right)\nabla^2 w + K\frac{\partial\phi_2}{\partial x} + \lambda_0\frac{\partial\phi^*}{\partial z} + b\frac{\partial\Phi}{\partial z} + \mu_E H0^2\left(\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 u}{\partial x\partial z}\right) = (1-\epsilon^2\nabla^2)\rho\frac{\partial^2 w}{\partial t^2}, \quad (21)$$

$$(1-\epsilon^2\nabla^2)\rho j\frac{\partial^2\phi_2}{\partial t^2} = K\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) + \gamma\nabla^2\phi_2 - 2K\phi_2, \quad (22)$$

$$\alpha_0\nabla^2\phi^* + \beta_0\theta - \lambda_1\phi^* - \lambda_0 e = (1-\epsilon^2\nabla^2)\rho\frac{j_0}{2}\frac{\partial^2\phi^*}{\partial t^2}, \quad (23)$$

$$\left[K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K^*\left(1+\tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t}+\tau_q^2\frac{\partial^2}{\partial t^2}\right)\left(\beta_1 T_0\frac{\partial^2 e}{\partial t^2} + mT_0\frac{\partial\Phi}{\partial t} + \rho C_E\frac{\partial^2 T}{\partial t^2} + \beta_0 T_0\frac{\partial^2\phi^*}{\partial t^2}\right), \quad (24)$$

The non-dimensional variables are introduced to facilitate the solution:

$$(\tilde{\varepsilon},\tilde{x},\tilde{z}) = (\varepsilon,x,z)\frac{\omega^*}{C_0},\ \ (\bar{t},\tilde{\tau}_v,\tilde{\tau}_T,\tilde{\tau}_q) = (t,\tau_v,\tau_T,\tau_q)\omega^*,\ \ \tilde{\theta} = \frac{T-T_0}{T_0},\ \ \tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0},(\tilde{\phi}_2,\tilde{\phi}^*) = (\phi_2,\phi^*)\frac{\rho C_0^2}{\beta_1 T_0},\ \ (\tilde{u},\widetilde{w}) = (u,w)\frac{\rho\omega^* C_0}{\beta_1 T_0},\ \ (\widetilde{m}_{ij},\widetilde{m}_i) = (m_{ij},m_i)\frac{\omega^*}{C_0\beta_1 T_0},\ \ \omega^* = \frac{\rho C_0^2 C_E}{K^*},\ \ C_0^2 = \frac{(\lambda+2\mu+K)}{\rho}. \tag{25}$$

In the form of potential functions, u and w are:

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \tag{26}$$

The potential functions q show the dilatational and ψ show displacement vector's transverse part. Using non-dimensional variables in Eq. (25) and the potential functions (26), the stresses in Eq. (14), (15), (17) and (19) and the governing Eq. $(20) - (24)$, provide the following relations (for convenience neglecting the tildes)

$$\sigma_{zz}{}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{zz} = C_1\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2} + (C_1-1)\frac{\partial^2 q}{\partial x\partial z} + a_1\phi^* - \theta - P + b\Phi, \tag{27}$$

$$\sigma_{zx}{}^{Local} = (1-\epsilon^2\nabla^2)\sigma_{zx} = C_4\frac{\partial^2 q}{\partial z^2} - C_2\frac{\partial^2 q}{\partial x^2} + (1-C_1)\frac{\partial^2\psi}{\partial x\partial z} - C_3\phi_2, \tag{28}$$

$$m_{zy}{}^{Local} = (1-\epsilon^2\nabla^2)m_{zy} = C_5\frac{\partial\phi_2}{\partial z} + C_6\frac{\partial\phi^*}{\partial x}, \tag{29}$$

$$m_z{}^{Local} = (1-\epsilon^2\nabla^2)m_z = C_7\frac{\partial\phi^*}{\partial z} - C_6\frac{\partial\phi_2}{\partial x}, \tag{30}$$

$$\left[\nabla^2 - R_H\nabla^2 - (1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\psi + a_1\phi^* - \theta - P + A_1\Phi = 0, \tag{31}$$

$$\left[\nabla^2 - a_2(1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]q - a_3\phi_2 = 0, \tag{32}$$

$$\left[\nabla^2 - 2a_4 - a_5(1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\phi_2 + a_4\nabla^2 q = 0, \tag{33}$$

$$\left[\nabla^2 - a_6 - a_7(1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\phi^* + a_8\theta - a_9\nabla^2\psi = 0, \tag{34}$$

$$\alpha^*\nabla^2\Phi + m\theta - \xi\Phi - b\nabla^2\psi = \rho\chi(1-\epsilon^2\nabla^2)\frac{\partial^2\Phi}{\partial t^2}, \tag{35}$$

Where

$$C_1 = \frac{\lambda}{\rho C_0^2}, C_3 = \frac{K}{\rho C_0^2}, C_4 = \frac{1}{a_2}, C_5 = \frac{\gamma\omega^{*2}}{\rho C_0^4},\ C_6 = \frac{b_0\omega^{*2}}{\rho C_0^4}, C_7 = \frac{\omega^{*2}}{\rho C_0^4},\ C_2 = \frac{\mu}{\rho C_0^2},$$
$$a_1 = \frac{\lambda}{(\lambda+2\mu+K)},\ a_2 = \frac{\rho C_0^2}{\mu+K-\frac{P}{2}},\ a_3 = \frac{K}{\mu+K+\frac{P}{2}},\ a_4 = \frac{KC_0^2}{\gamma_0\omega^{*2}},\ a_5 = \frac{\rho j C_0^2}{\gamma},\ a_6 = \frac{\rho j C_0^2}{\gamma}, a_7 = \frac{\rho j_0 C_0^2}{2},$$

$$a_8 = \frac{\rho\beta_0 C_0^4}{\beta_1 \omega^{*2}},\ a_9 = \frac{\lambda_0 C_0^2}{\omega^{*2}},\ a_{10} = \frac{K_1^*}{K^* \omega^*},\ a_{11} = \delta_0 = \frac{\beta_1 T_0}{C_E \rho^2 C_0^2}, \delta_1 = \frac{T_0 \beta_0' \beta_1'}{C_E \rho^2 C_0^2}, A_1 = \frac{b}{\beta_1 T_0}, C_\alpha = \frac{\mu_E H0^2}{\rho}.$$

4. Solution of the Problem

The xz-plane is experiencing positive propagation of the harmonic waves at a speed of v, the potentials are written as:

$$(q, \psi, \theta, \Phi, \phi^*, \phi_2) = (\overline{q}, \overline{\psi}, \overline{\theta}, \overline{\Phi}, \overline{\phi^*}, \overline{\phi_2}) exp\{ik(\vec{p}.\vec{r} - vt)\}, \tag{36}$$

where $\vec{p}$ the unit vector, ω is angular frequency, defined as $\omega = kv$, k wave number and $\vec{r}$ the position vector. Using Eq. (36) into Eqs. $(31) - (35)$, we get set of equations:

$$(k^2 d_2 + d_1)\overline{\psi} - \overline{\theta} + a_1 \overline{\phi^*} + A_1 \overline{\Phi} = 0, \tag{37}$$

$$(k^2 d_7 + d_8)\overline{\phi^*} + 2a_9 k^2 \overline{\psi} + a_8 \overline{\theta} = 0, \tag{38}$$

$$(k^2 d_9 + d_{10})\overline{\theta} - k^2 d_{11} \overline{\psi} + L_2 \overline{\phi^*} - m L_3 \overline{\Phi} = 0, \tag{39}$$

$$(k^2 L_4 + L_5)\overline{\Phi} + 2bk^2 \overline{\psi} + m\overline{\theta} = 0, \tag{40}$$

$$(k^2 d_4 + d_3)\overline{q} - a_3 \overline{\phi_2} = 0, \tag{41}$$

$$(k^2 d_5 + d_6)\overline{\phi_2} - a_4 k^2 \overline{q} = 0, \tag{42}$$

where

$d_1 = \varpi^2, d_2 = 2(-1 + d_1\varepsilon^2 + R_H), L_1 = 2d_{10}\delta_0, d_3 = a_2\varpi^2, d_4 = 2(d_3\varepsilon^2 - 1), d_5 = 2(a_5\varpi^2\varepsilon^2 - 1),\ d_6 = a_5\varpi^2 - 2a_4,\ d_7 = 2(a_7\varpi^2\varepsilon^2 - 1),\ d_9 = 2(a_{10} - \varpi^2\tau_T - \iota\varpi\tau_v a_{10} - \iota\varpi),\ d_8 = \varpi^2 a_7 - a_6,\ \ d_{10} = \left(1 - \iota\varpi\tau_q - \frac{\varpi^2\tau_q^2}{2}\right),\ \ L_2 = 2\delta_1 d_{10},\ L_3 = \rho\chi\varpi^2\varepsilon^2 - 2\alpha^*,\ L_4 = \rho\chi\varpi^2 - \xi.$

The existence condition for non-trivial solutions of Eqs. $(37) - (40)$ are:

$$v^8 + Fv^6 + Gv^4 + Hv^2 + J = 0, \tag{43}$$

$$F = \omega^2\left(\frac{Q_2}{Q_1}\right),\quad G = \omega^4\left(\frac{Q_3}{Q_1}\right),\quad H = \omega^6\left(\frac{Q_4}{Q_1}\right),\quad J = \omega^8\left(\frac{Q_5}{Q_1}\right),$$

$$Q_1 = d_1 L_5 v_{10} d_8 + d_1 L_3 m^2 d_8 - d_1 a_8 L_2 L_5,$$

$$Q_2 = \begin{pmatrix} 2mbL_3a_1a_8 + d_1d_9L_5d_8 - 2ma_9L_2A_1 - 2a_9L_5a_1d_{10} + 2bmd_8L_3 - mA_1d_8L_1 + 2bA_1a_8L_2 \\ -2d_8A_1bd_{10} + d_1d_8L_5d_{10} - 2a_9L_2L_5 + d_1d_8d_{10}L_4 - d_1L_2a_8L_4 - d_2L_2a_8L_5 \\ -L_5d_8L_1 + 2a_8A_1L_2b + m^2d_1d_7L_3 - a_1L_1a_8L_5 + m^2d_2d_8L_3 - 2m^2a_9L_3a_1 \end{pmatrix}.$$

$$Q_3 = \begin{pmatrix} -d_7L_5L_1 + d_2d_7d_{10} - d_8L_1L_4 - 2L_2a_9L_4 + 2bd_7mL_3 - 2bA_1d_8 \\ +d_2d_8d_9L_5 - 2a_9d_9a_1L_5 - 2bd_{10}d_7A_1 + d_8L_4d_{10}d_2 + d_1d_{10}d_7L_4 + d_1L_5d_9d_7 \\ -a_8L_2d_2L_4 - 2a_1d_{10}a_9L_4 + d_1L_4d_9d_8 - a_8L_1a_1L_4 \end{pmatrix},$$

$$Q_4 = d_2d_8d_9d_{14} - d_7L_1L_4 - 2bdA_1d_9 + d_7d_9L_4d_1 - 2a_1d_9L_4a_9 + d_2L_5d_9d_7 + d_{10}L_4d_2d_7, \quad Q_5 = d_2L_4d_7d_9.$$

Here, the four complex roots v_i^2 of the Eq. (43) gives the four sets of longitudinal waves. Eqs. (41) and (42) give a non-trivial solution only if the coefficient matrix's determinant equal to zero.

$$v^4 + Mv^2 + N = 0, \tag{44}$$

$$M = \frac{\varpi^2(L_4L_6 + L_3L_5 - a_3a_4)}{L_3L_6}, N = \frac{\varpi^4(L_4L_5)}{L_3L_6},$$

The roots of Eq. (44) can be found using

$$v_{5,6}^2 = \frac{-M \pm \sqrt{M^2 - 4N}}{2}, \tag{45}$$

which is related to the speed of the transverse waves. v_5^2 and v_6^2 are real and positive. It is noticed that the speeds v_5 and v_6 do not depend upon microstretch, thermal, and void factors and are impacted by the existence of nonlocality and micropolarity.

5. Reflection Phenomenon

Consider the plane wave with amplitude A_0 and velocity V_0 propagating at an angle of θ_0 with the perpendicular. Regarding the incident wave, we get longitudinal waves of amplitudes A_1, A_2, A_3, A_4 propagating with the speeds V_1, V_2, V_3, V_4 and making angles $\theta_1, \theta_2, \theta_3, \theta_4$, respectively, with the normal. The transverse waves are making angles θ_5, θ_6 with normal, having amplitudes A_5, A_6 are propagating with speeds V_5, V_6, as displayed in Fig. 1.

The potentials of reflected waves are expressed as:

$$(q, \phi^*, \theta, \Phi) = (1, \Omega_1, \Xi_1, \kappa_1)A_0G_0^- + \sum_{m=1}^{4}(1, \Omega_m, \Xi_m, \kappa_m)A_mG_m^+, \tag{46}$$

$$(\psi, \phi_2) = \sum_{m=5}^{6}(1, \eta_m)A_mG_m^+, \tag{47}$$

Where

$G_m^- = exp[\iota k_0(x\sin\theta_0 - z\cos\theta_0) - \iota\omega_1 t]$, $G_m^+ = exp[\iota k_m(z\cos\theta_m + x\sin\theta_m) - \iota\omega_m t]$, the. Ω_m, Ξ_m and κ_m are the coupling parameters among $\bar{q}, \bar{\phi}^*, \bar{\theta}, \bar{\Phi}$ and η_m the coupling parameters between $\bar{\psi}$ and $\bar{\phi}_2$. The coupling parameters are defined by

$$\Omega_m = \frac{2bA_1k_m^2a_8 - 2A_1k_m^2a_9 - 2k_m^2a_9(k_m^2L_{14} + L_{15}) - a_8(k_m^2L_1 + L_2)(k_m^2L_{14} + L_{15})}{a_1a_8(k_m^2L_{14} + L_{15}) + (k_m^2L_7 + L_8)(k_m^2L_{14} + L_{15}) + mA_1(k_m^2L_7 + L_8)},$$

$$\Xi_m = \frac{(k_m^2L_1 + L_2)\Omega_m + 2a_9k_m^2}{a_8},$$

$$\kappa_m = \frac{m(k_m^2 L_7 + L_8)\Omega_m + 2a_9 k_m^2 - 2b a_8 k_m^2}{a_8(k_m^2 L_{14} + L_{15})}, \ (m = 1,2,3,4), \tag{48}$$

$$\eta_m = \frac{(k_m^2 L_4 + L_3)}{a_3}, \ (m = 5,6). \tag{49}$$

6. **Boundary Conditions:**

At $z = 0$, boundary conditions are:

i. Neglecting of transverse and tangential component of stress

$$P = -\sigma_{zz}, \tag{50}$$

$$\sigma_{zx} - P\omega_{zx} = 0, \tag{51}$$

ii. Neglecting of couple stress

$$m_{zy}(x,0,t) = 0, \tag{52}$$

iii. Neglecting of microstress

$$m_z(x,0,t) = 0, \tag{53}$$

iv. No change in temperature

$$\theta(x,0,t) = 0, \tag{54}$$

v. Neglecting of equilibrated stress.

$$\frac{\partial \Phi}{\partial z}(x,0,t) = 0, \tag{55}$$

The Eqs. (50)-(55) are corresponding to local stress boundary conditions, are defined as:

$$P + \sigma_{zz}^{Local} = \sigma_{zx}^{Local} - P\omega_{zx} = m_{zy}^{Local} = m_z^{Local}\frac{\partial \Phi}{\partial z} == \theta = 0. \tag{56}$$

The above boundary conditions are trivially satisfied iff $\omega_1 = \omega_3 = \omega_2 = \omega_4 = \omega_5 = \omega_6$, which gives

$$k_5 \sin\theta_5 = k_1 \sin\theta_0 = k_1 \sin\theta_1 = k_4 \sin\theta_4 = k_2 \sin\theta_2 = k_3 \sin\theta_3 = k_6 \sin\theta_6$$

This can further be written as:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} = \frac{\sin\theta_5}{V_5} = \frac{\sin\theta_6}{V_6}. \tag{57}$$

Using Eqs. (46,47) to boundary conditions and using extended Snell's law in Eq. (57), we get six linear non-homogeneous equations which are explained in matrix form as:

$$\sum_{j=1}^{6} q_{mj} Z_j = X_m, \ (m = 1,2,\dots,6), \tag{58}$$

where $Z_j = \frac{K_j}{K_0}, (j = 1,2,\dots,6)$ are the amplitude ratios. $[q_{mj}]$ are the elements of the coefficient matrix are given below:

$$q_{1j} = \left(C_1 + (1 - C_1)\cos^2\theta_j + \frac{\Omega_j}{k_j^2} - \frac{a_1\Xi_j}{k_j^2}\right)k_j^2,\ q_{2j} = (C_1 - 1)k_j^2 \sin\theta_j \cos\theta_j$$

$$q_{4j} = C_7\eta_j k_j \cos\theta_j\,,\ q_{3j} = C_6\Xi_j k_j \sin\theta_j\,, q_{5j} = \Omega_j,\ q_{6j} = \kappa_j k_1 \cos\theta_j,$$

$$q_{1j} = (C_1 - 1)k_j^2 \cos\theta_j \sin\theta_j\,, (j = 4,5,6), q_{2j} = \left(C_2 \sin^2\theta_j - \frac{C_8\eta_j}{k_j^2} - C_4\cos^2\theta_j\right)k_j^2,$$

$$q_{6j} = 0, q_{3j} = C_5\eta_j k_j \cos\theta_j\,, q_{4j} = -C_6 k_j \eta_j \sin\theta_j\,, q_{5j} = 0,$$

$$X_1 = -q_{11}, X_4 = q_{41},\ X_5 = -q_{51},\ X_2 = q_{21},\ X_6 = q_{61},\ X_3 = -q_{31}.$$

7. Energy Partition

The examination of harmonic wave propagation cannot be confirmed at the boundary surface without the balance of energy. Split of energy between incident wave and six reflected waves at $z = 0$.

Per unit area, the instant rate of transfer of energy [31] is define as:

$$Y = \sigma_{zz}\dot{w} + \sigma_{zx}\dot{u} + m_{zy}\dot{\phi} + m_z\dot{\phi}^*, \tag{59}$$

Let $< Y_i >$ and $< Y_0 >$ $(i = 1,2,\dots,6)$ show the mean energy transmitted by incident and reflected waves, respectively. $E_i (i = 1,2,3,4)$ is the ratio of the incident wave's energy to reflected longitudinal waves. Also, $E_i (i = 5,6)$ for the ratios of incident wave energy to energy transferred by reflected transverse waves. $E_i (i = 1,2,\dots,6)$ are as follow:

$$E_i = \frac{<Y_i>}{<Y_0>},\quad (i = 1,2,3,4,5,6), \tag{60}$$

$$Y_i = \left[\left(1 + \frac{\tau_i}{k_i^2} - \frac{a_1\eta_i}{k_i^2} - \frac{d_6\kappa_i}{k_i^2}\right) - \frac{\iota\eta_i^2 C_7}{k_i^2}\right] Z_i^2 k_i^3 \omega_i \cos\theta_i\,,\quad (i = 1,2,3,4),$$

$$Y_i = Z_i^2 k_i^3 \omega_i \left[\cos\theta_i \left(C_2 \sin^2\theta_i - \frac{C_8\xi_i}{k_i^2} - C_4\cos^2\theta_i\right) + \sin\theta_i \left(\frac{\iota C_6 \xi_i^{\,2}}{k_i^2} - (C_1 - 1)\cos^2\theta_i\right)\right], (i = 5,6)$$

$$Y_0 = \left[\left(1 + \frac{\tau_1}{k_1^2} - \frac{a_1\eta_1}{k_1^2} - \frac{d_6\kappa_1}{k_1^2}\right) - \frac{\iota\eta_1^2 C_7}{k_1^2}\right] Z_0^2 k_1^3 \omega_1 \cos\theta_0.$$

By using the following numerical values, the amplitude and energy ratios are computed. Figs. (2)-(4) display physical behavior of initial stress, void effect, magnetic field's effect and nonlocal parameter.

$C_E = 1.04 \times 10^3 JKg^{-1}K^{-1},\ \lambda = 9.4 \times 10^{10} kgm^{-1}s^{-2}, \alpha_{t_1} = 0.5 \times 10^{-3} K^{-1} \rho = 1.74 \times 10^3 kgm^{-3}, \mu = 4.0 \times 10^{10} kgm^{-3}, K^* = 1.7 \times 10^2 Jm^{-1}s^{-1}K^{-1},\ \gamma = 0.779 \times 10^{-8} kgm^{-1}s^{-2}, \tau_q = 0.3s, \tau_v = 0.1s, \tau_T = 0.2s, \varepsilon = 0.195 \times 10^{-9} m, K = 1.0 \times 10^2 kgm^{-1}s^{-2}, j = 0.2 \times 10^{-6} m^2,\ \alpha_{t_2} = 0.4 \times 10^{-3} K^{-1}, T_0 = 298K, e_0 = 0.39, a_{cl} = 0.5 \times 10^{-9} m. j_0 = 0.196 \times 10^{-6} m^2, \lambda_0 = 5.7702 \times 10^{-8} kgms^{-2}, \alpha_0 = 1.5947 \times 10^{-5} kgms^{-2}, \lambda_1 = 3.4650 \times 10^{-8} kgms^{-2}, b_0 = 0.9 \times 10^{-9} kgms^{-2}, m = 2 \times 10^6 Nm^{-2},\ \chi = 1.753 \times 10^{-15} m^2,\ \xi = 1.475 \times 10^{10} Nm^{-2},\ \alpha^* = 3.688 \times 10^{-5} N, b = 1.13849 \times 10^{10} Nm^{-2}$

Fig. (2) explored the behavior of non-local parameter on amplitude ratios. The amplitude ratio $|Z_1|$ decreases as void parameter increases. The amplitude ratios $|Z_2|$ to $|Z_6|$ rise as nonlocal parameter increases while Fig. (2) demonstrates that $|Z_4|$ decreases from $\theta = 30° \text{ to } 60°$ as the nonlocal parameter ϵ decreases. Fig. (3) shows void effect on the amplitude ratios under three-phase lag theory. Fig. (3) deliberated that the amplitude ratios decrease as void parameter values are increasing. The amplitude ratios show decreasing effect in all profiles as increasing the value of void parameter b because voids in the medium are increasing as a result waves faces more hurdles and displacement of particles in the medium decreases. Fig. (4) is portrayed to observe the impact of magnetic field on amplitude ratios. It is examined that $|Z_1|$ to $|Z_4|$ rises because magnetic field is perpendicular to the wave's propagation in longitudinal waves while the $|Z_5|$ and $|Z_6|$ trim down by increasing magnetic field because the magnetic field in transverse waves is parallel to the wave's propagation. The consequence of initial stress on amplitude ratios is depicted in Fig. (5). It shows variation of $|Z_i|$ on initial stress parameter against angle of incidence. It is investigated that the amplitude ratio $|Z_1|$ to $|Z_4|$ decreases as the value of initial stress increases while the amplitude ratios $|Z_5|$ and $|Z_6|$ increases by increasing initial stress.

Fig (6) illustrates the impact of $|E_i|$ (i=1,2,3,4,5,6) and sum of energy ratios against incidence P wave. It is examined that $|E_1|$ and total of energy ratios are almost equivalent and equals to one because amplitude ratio of $|E_1|$ is greater as compared to others. The values of $|E_6|$ and $|E_7|$ are quite minimal. The energy carried by $|E_3|$, $|E_4|$ and $|E_5|$ are greater as compared with $|E_6|$ and $|E_7|$. Furthermore, it is also observed that during the reflection, there is no energy loss.

8. Conclusions

The reflection behavior of plane waves in a nonlocal microstretch thermoelastic medium under the three-phase-lag theory has been analyzed by incorporating voids, magnetic field, and initial stress effects. The analytical formulation and numerical results reveal that the physical parameters significantly alter the amplitude and energy distribution of reflected waves. The void parameter leads to a reduction in most amplitude ratios $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_5|$, due to increased discontinuities in the medium, while the nonlocal parameter mainly governs the behavior of transverse waves. The magnetic field strongly influences longitudinal wave amplitudes, whereas initial stress induces opposite trends in different wave modes. The dominant contribution of the primary longitudinal wave is clearly observed, while transverse wave amplitudes approach zero at specific angles of incidence. Moreover, the computed energy ratios satisfy the energy balance condition, confirming that no energy loss occurs during the reflection process. No energy is lost during the reflection phenomenon. As a result, the law of energy balance is satisfied for each angle of incidence.These findings highlight the importance of nonlocal and microstructural effects in accurately modeling wave propagation in advanced thermoelastic materials and may be useful in the design of engineering systems involving coupled field interactions.

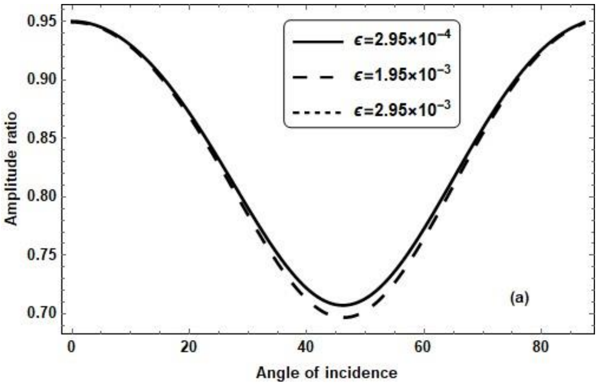

Fig. 2(a): $|Z_1|$ **against angle of incidence.**

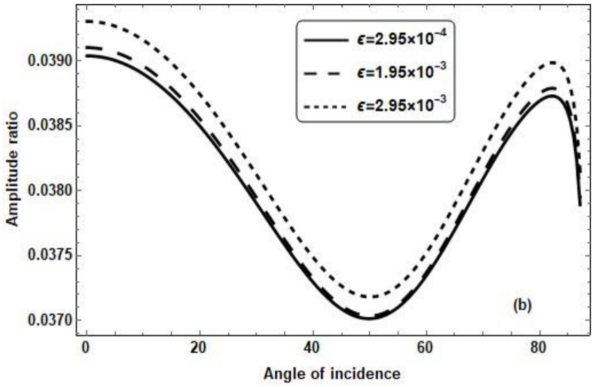

Fig. 2(b): $|Z_2|$ **against angle of incidence.**

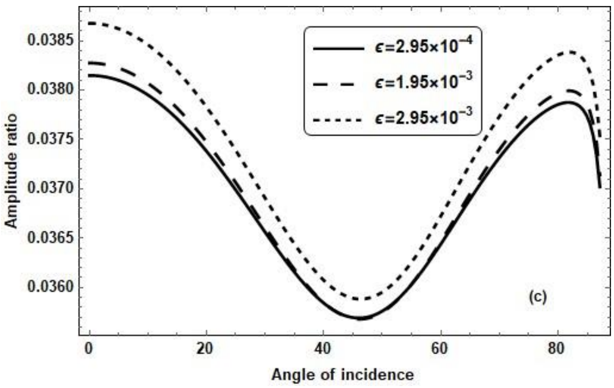

Fig. 2(c): $|Z_3|$ **against angle of incidence.**

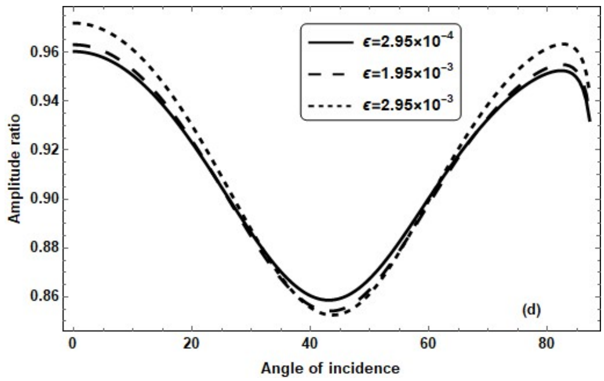

Fig. 2(d): $|Z_4|$ **against angle of incidence.**

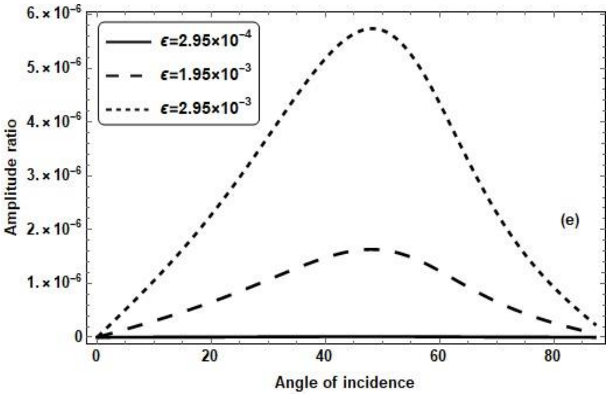

Fig. 2(e): $|Z_5|$ **against angle of incidence.**

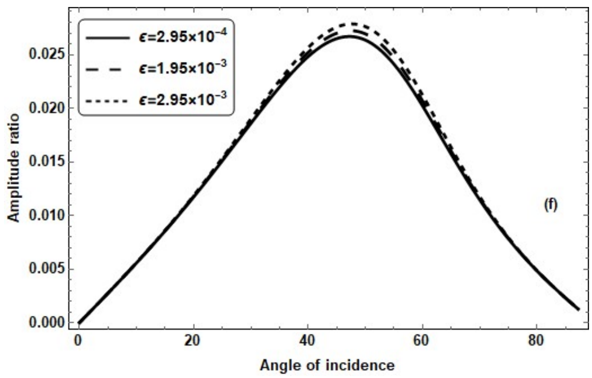

Fig. 2(f): $|Z_6|$ **against angle of incidence.**

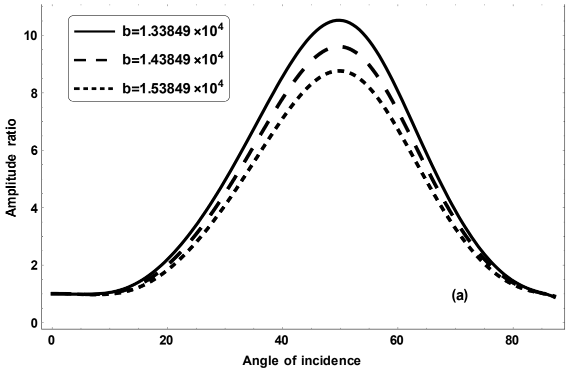

Fig. 3(a): $|Z_1|$ against angle of incidence.

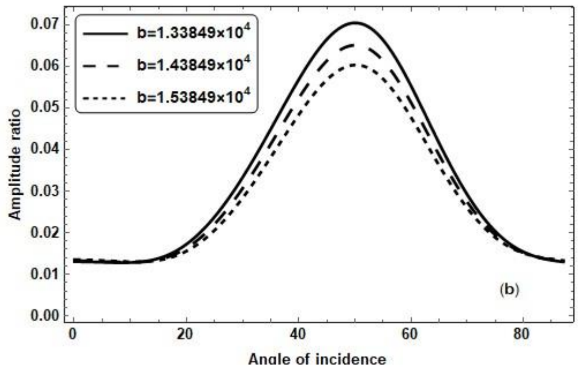

Fig. 3(b): $|Z_2|$ against angle of incidence.

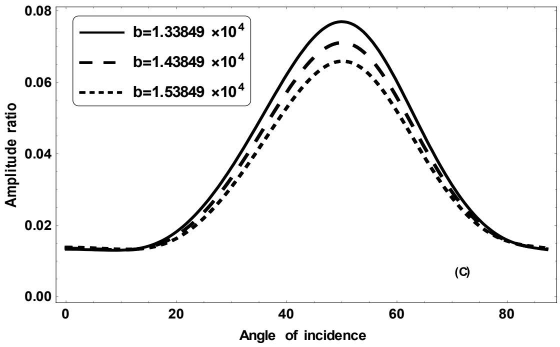

Fig. 3(c): $|Z_3|$ against angle of incidence.

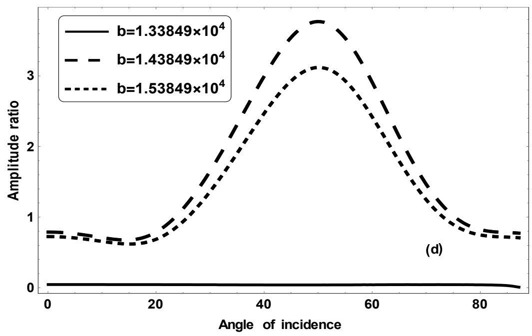

Fig. 3(d): $|Z_4|$ against angle of incidence.

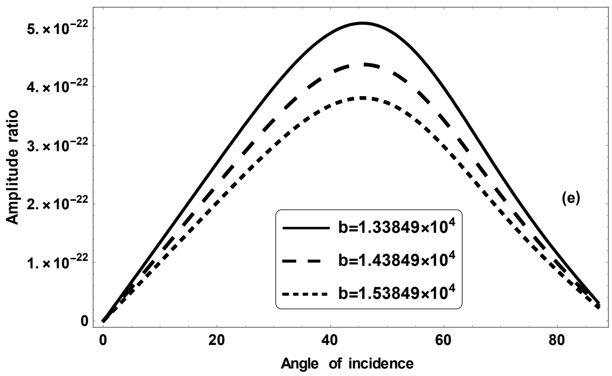

Fig. 3(e): $|Z_5|$ against angle of incidence.

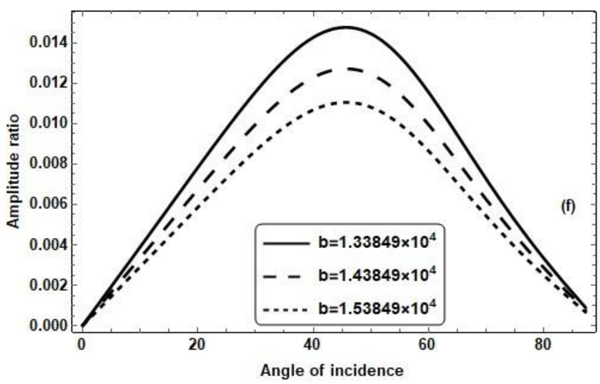

Fig. 3(f): $|Z_6|$ against angle of incidence.

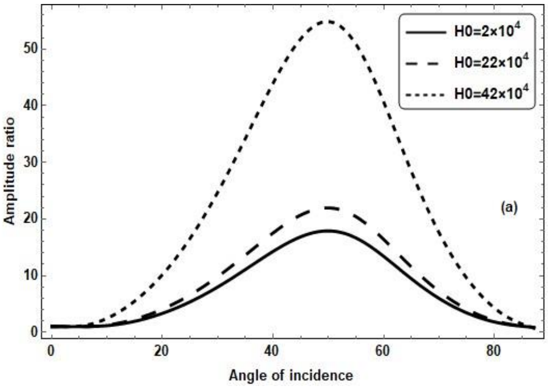

Fig. 4(a): $|Z_1|$ against angle of incidence.

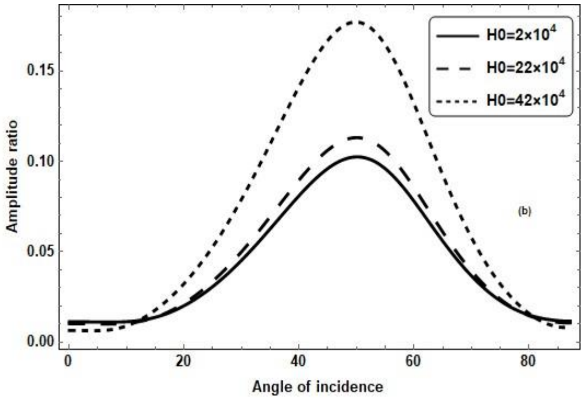

Fig. 4(b): $|Z_2|$ against angle of incidence.

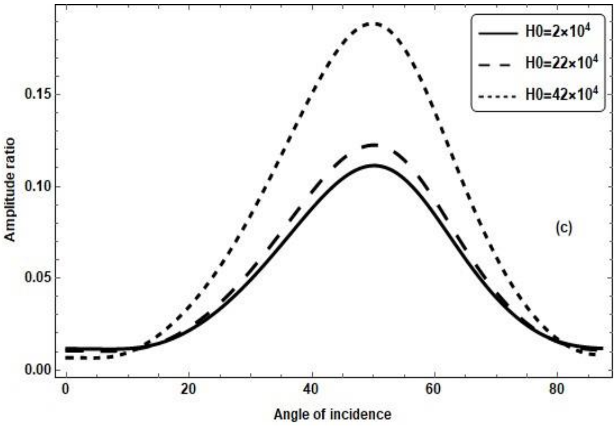

Fig. 4(c): $|Z_3|$ against angle of incidence.

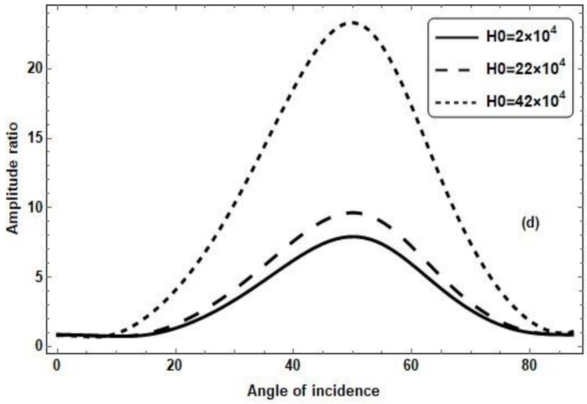

Fig. 4(d): $|Z_4|$ against angle of incidence.

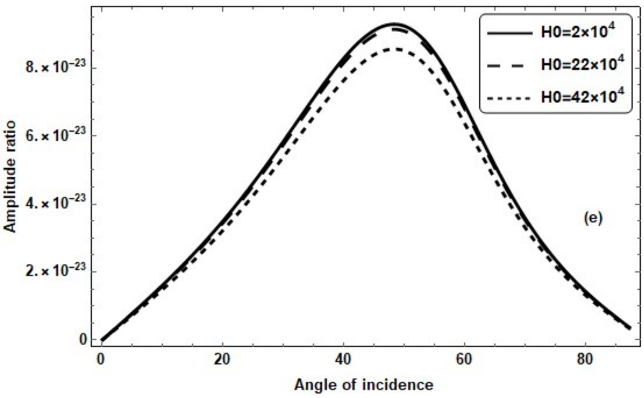

Fig. 4(e): $|Z_5|$ against angle of incidence.

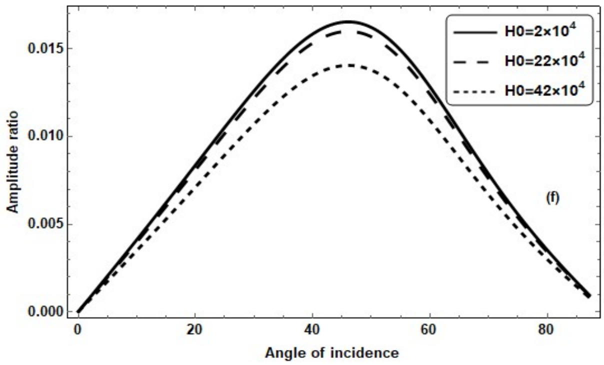

Fig. 4(f): $|Z_6|$ against angle of incidence.

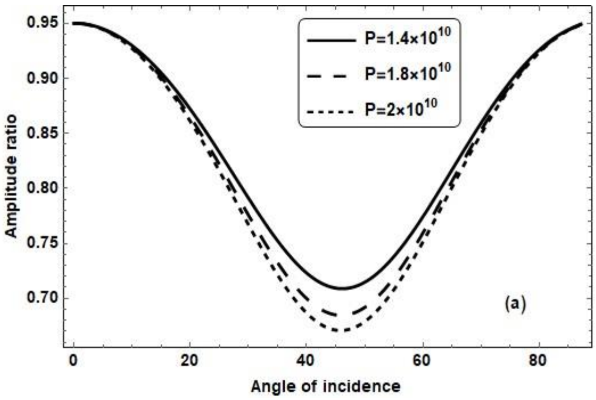

Fig. 5(a): $|Z_1|$ against angle of incidence.

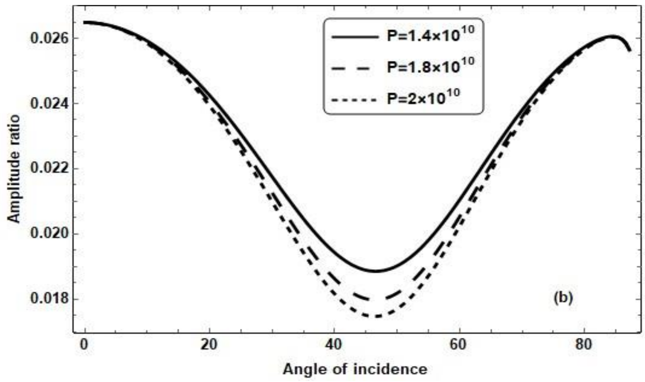

Fig. 5(b): $|Z_2|$ against angle of incidence.

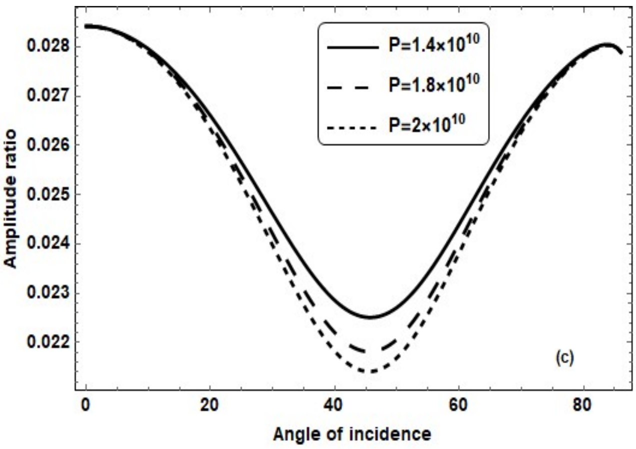

Fig. 5(c): $|Z_3|$ against angle of incidence.

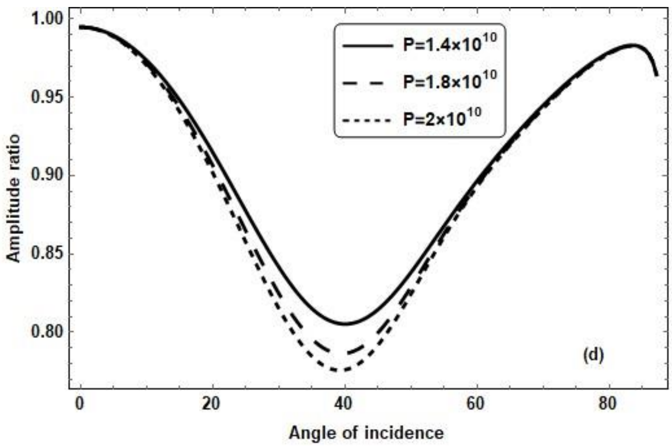

Fig. 5(d): $|Z_4|$ against angle of incidence.

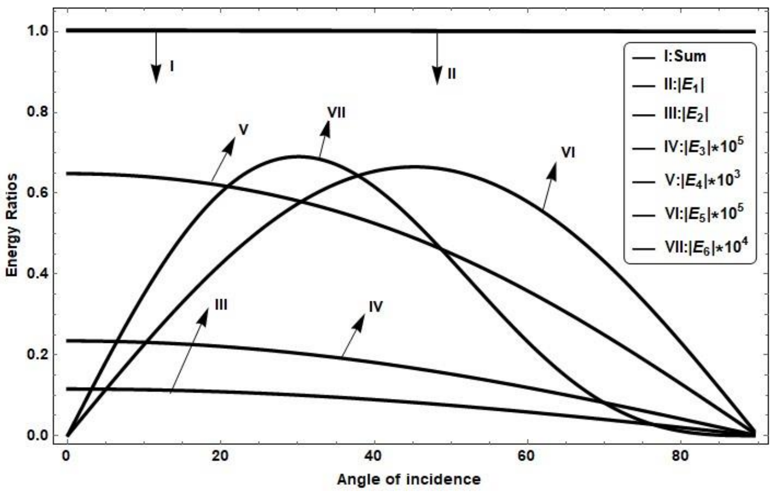

Fig.6: Energy Ratios versus angle of incidence.

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