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RESEARCH PAPER

Elastic analysis of FGM solid sphere with parabolic varying properties

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Abstract

Using plane elasticity theory (PET), elastic analysis for solid sphere made of functionally graded materials (FGMs) and subjected to constant pressure is investigated in this paper. The mechanical properties except Poisson's ratio are assumed to obey the parabolic variations in the radial direction. The emphasis of this article is to find an accurate solution for the analysis of the spherical dome structure in the case where the properties change based on a parabolic function. In this article, the constant inhomogeneity effect on elastic deformations as well as related stresses is investigated The displacement and stresses distributions are compared with the solutions of the finite element method (FEM) and good agreement are found.

Keywords: Solid Sphere; Functionally graded material (FGM); Finite element method (FEM); Parabolic.

1. Introduction

What has garnered significant attention in today's advanced industries is the field related to shells. These shells are generally used in modern industries as cylindrical, spherical, and conical shapes. Therefore, the stress analysis of these types of shells, especially spherical shells which have more applications, becomes even more critical. Panferov [1] utilized the successive approximations method to derive a solution to the problem of thermal loading on an elastic truncated conical pipe that has a uniform thickness. Hongjun et al. [2] investigated the elasticity of heterogeneous thick-walled cylinders using the Lamé method to analyze cases involving linear and exponential changes in properties within the thick-walled cylinders. Arefi and Rahimi [3] investigated the thermoelastic behavior of a cylinder made of functionally graded material (FGM) when subjected to both mechanical and thermal loads. Nejad and Rahimi [4] studied deformations and stresses in FGM pressurized thick hollow cylinder based on closed form solutions for one-dimensional steady-state thermal stresses. Xin et al. [5] introduced a thermo-elasticity solution for one-dimensional FG cylinders that is based on the constituents' volume fraction and accounts for both thermal and mechanical loads. Nejad et al. [6] presented on a consistent 3-D set of field equations developed by tensor analysis on an FGM thick shell with variable thickness. Yasinskyy and Tokova [7] conducted a study on a one-dimensional temperature transient analysis and thermal stress analysis in a FG cylinder. Ghannad and Nejad [8] presented an elastic analysis of axisymmetric thick-walled cylinders with different boundary conditions at the two ends based on first-order shear deformation theory (FSDT) and the virtual work principle. Kang [9] studied thick-walled shells for arbitrary geometry and thickness variations using three-dimensional elasticity theory. The equations are derived using the continuum mechanics in curvilinear coordinates. Nejad et al. [10] developed an elastic solution to determine displacements and stresses in a thick truncated conical shell under uniform pressure. The solution employed the MLM. Ghannad et al. [11] studied displacements and stresses in pressurized thick cylindrical shells with variable thickness using the perturbation technique. Sundarasivarao and Ganesan [12] investigated a conical shell by using the finite element technique that was exposed to pressure. Mirsky and Hermann [13] studied the solution for thick cylindrical shells made of isotropic materials and uniform by utilizing the FSDT. Witt [14] demonstrated axisymmetric temperature distributions in conical shells using a differential equation. To solve this equation, Witt proposed an assumption that the temperature distribution could be a combination of hyperbolic and cubic functions. Eipakchi et al. [15] have analyzed thick-walled cone structures subjected to varying internal pressure. In this study, equations are derived using second-order shear deformation theory, and solutions are obtained through the perturbation theory approach. Ghannad and Nejad [16] investigated elastic solution of pressurized clamped-clamped thick cylindrical shells made of FGMs based on the FSDT. Jane and Wu [17] presented research on the problem of thermoelasticity using the curvilinear circular conical coordinate system. Ghannad et al. [18] obtained an elastic solution for truncated conical shells characterized by their significant thickness and uniform thickness distribution. Obata and Noda [19] analyzed steady one-dimensional thermal stresses in hollow cylindrical and spherical objects using perturbation technique. Ghannad et al. [20] analyzed the analytical solution of pressurized thick cylindrical shells with variable thickness based on the FSDT and using the asymptotic method (MAM) of the perturbation theory. Eipakchi [21] utilized a third-order shear deformation theory in conjunction with matched asymptotic expansion (MAE) from perturbation theory. This approach was applied to ascertain displacements and stresses within a thick conical shell. The shell's properties included homogeneity, isotropy, and axisymmetric, with non-uniform internal pressure leading to thickness variations. Kao et al. [22] provided an analytical solution for the buckling of cylindrical shell structures subjected to axial pressure. The symmetric and asymmetric axial loadings were solved using perturbation theory and Fourier series. Nejad et al. [23] presented semi-analytically of FG rotating thick hollow cylinder with variable thickness and clamped ends based on the FSDT using MLM. Civalek [24] utilized free vibration analysis in the of laminated conical shells. The author studied the cases of isotropic, orthotropic, and laminated materials and used the numerical solution of governing differential equations of motion based on transverse shear deformation theory to obtain the results. Nejad et al. [25] studied a mathematical solution that partially uses analysis and partially uses numerical methods to calculate the displacements and stresses in a cylindrical shell with varying thickness under a non-uniform pressure They systematically examined how the primary factors of the problem affect displacement and stress levels. Ray et al. [26] conducted an analysis of heat conduction through conical shells with varying inner radii and thicknesses. Nejad et al. [27] studied a semi-analytical solution in rotating thick truncated conical shells made of FGMs under non-uniform pressure using FSDT and MLM. Jabbari et al. [28] employed a research on the thermoelastic analysis of a thick constant thickness truncated conical shell that rotates, and is exposed to a temperature gradient and non-uniform internal pressure. Jabbari et al. [29] utilized a thermo-elastic analysis of a rotating truncated conical shell subjected to temperature, internal pressure, and external pressure by using the FSDT and MLM. They derived a solution for the problem by reducing it to a reverse thermo-elasticity problem. Hamzah et al. [30] investigated a comprehensive study of the vibration characteristics of cylindrical shells under different ambient temperatures by using (FEM). Kashkoli [31] studied a thermomechanical solution for creep analysis of FG thick cylindrical pressure vessels with variable thickness using the FSDT and MLM. Gharooni et al. [32] studied an analytical solution in axisymmetric clamped-clamped thick cylindrical shells made of FGM Using the third-order shear deformation theory. Jabbari et al. [33] analyzed Thermo-elastic analysis in rotating truncated conical shells with varying thickness made of FGMs higher-order shear deformation theory and MLM. Lai et al. [34] introduced an analytical method for the analysis of thick-walled cylindrical shells with circumferential corrugations. The formulation and solution involve shear deformation theory, by using eigenvectors and Fourier series. Nejad et al. [35] presented thermo-elastic analysis in a FG thick shell of revolution with arbitrary curvature and variable thickness based on higher-order shear deformation theory. Aghaienezhad et al. [36] studied GDQ method to analyze the behavior of spherical and cylindrical shells subjected to external pressure. Ifayefunmi and Ruan [37] analyzed a computational finite element study on the instability behavior of cylinder shells under axial compressive load. Ariatapeh et al. [38] presented stress and deformation in thick-walled cylindrical pressure vessels/pipes made of Mooney-Rivlin hyperelastic materials. They presented their distributions, and evaluates the effects of various factors. Mannani et al. [39] studied the mechanical stress, static strain, and deformation of a cylindrical pressure vessel under mechanical loads using higher-order sinusoidal shear deformation theory and thickness stretching formulation.

What is presented in this article is the elastic analysis of a solid spherical shell made of functionally graded material whose properties change parabolically and continuously.

2. Analytical Solution

A solid sphere made of parabolic functionally graded materials with radius b, is investigated. The sphere's material is graded through the radial direction. The sphere is assumed under the action of a constant pressure P_o, on the outer surface (Fig. 1).

Spherical coordinates (r,θ,ϕ), are considered are considered along the radial, circumferential and meridional directions, respectively.

In this paper, it is assumed that the Poison's ratio υ, takes a constant value and the modulus of elasticity E, is assumed to vary radially according to parabolic form as follows [40]:

$$\begin{cases} E(r) = E_0\left(1 - nR^{\eta}\right) \\ n = 1 - \dfrac{E_{out}}{E_0} \quad , \quad R = \dfrac{r}{b} \end{cases} \tag{1}$$

where E_0 and E_{out} are modulus of elasticity in center and outer surface, respectively. Here, n and η are material parameters

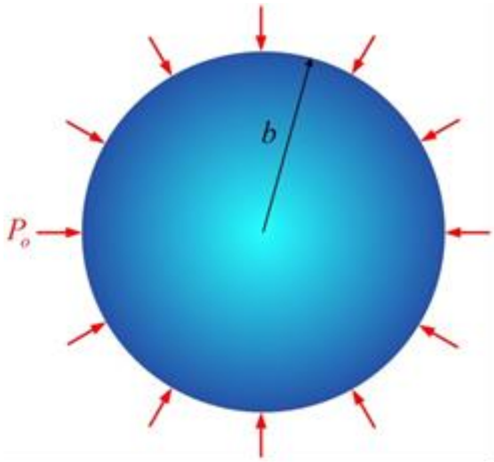

Fig. 1: FGM solid spherical shell with parabolic varying properties

The displacement in the r-direction is denoted by u. Three strain components can be expressed as

$$\varepsilon_r = \frac{du}{dr} \tag{2}$$

$$\varepsilon_\theta = \varepsilon_\phi = \frac{u}{r} \tag{3}$$

where ε_r and $\varepsilon_\theta = \varepsilon_\phi$ are radial and circumferential strains.

The Hooke's law is given by

$$\begin{bmatrix} \varepsilon_r \\ \varepsilon_\theta = \varepsilon_\phi \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -2\upsilon \\ -\upsilon & 1-\upsilon \end{bmatrix}\begin{bmatrix} \sigma_r \\ \sigma_\theta = \sigma_\phi \end{bmatrix} \tag{4}$$

where σ_r and $\sigma_\theta = \sigma_\phi$ are radial and circumferential stresses. Substituting Eqs. 2 and 3 into Eq. 4 yields

$$\begin{pmatrix}\sigma_r\\ \sigma_\theta\end{pmatrix}=\frac{E(R)}{b}\begin{pmatrix}A & 2B\\ B & A+B\end{pmatrix}\begin{pmatrix}\dfrac{du}{dR}\\ \dfrac{u}{R}\end{pmatrix} \tag{5}$$

where

$$\begin{cases}A=\dfrac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)}\\ B=\dfrac{\upsilon}{(1+\upsilon)(1-2\upsilon)}\\ \upsilon^*=\dfrac{B}{A}=\dfrac{\upsilon}{1-\upsilon}\end{cases} \tag{6}$$

The equilibrium equation of the FGM solid sphere, in the absence of body forces, is expressed as follows

$$\frac{d\sigma_r}{dR}+2\frac{\sigma_r-\sigma_\theta}{R}=0 \tag{7}$$

Substituting Eqs. 5, into Eq. 7, the equilibrium equation is expressed as

$$R^2\frac{d^2u}{dR^2}+R\left(2+\frac{RE'}{E}\right)\frac{du}{dR}-2\left(1-\upsilon^*\frac{RE'}{E}\right)u=0 \tag{8}$$

here, prime denotes differentiation with respect to. The general solution of Eq. 8 is as follows

$$u(R)=C_1G(R)+C_2H(R) \tag{9}$$

where C_1 and C_2 are arbitrary integration constants, and G and H are homogeneous solutions.
Substituting Eq. 9 into Eqs. 5, yields

$$\begin{pmatrix}\sigma_r\\ \sigma_\theta\end{pmatrix}=\frac{E(R)}{b}\begin{pmatrix}A & 2B\\ B & A+B\end{pmatrix}\begin{pmatrix}C_1G'+C_2H'\\ C_1\dfrac{G}{R}+C_2\dfrac{H}{R}\end{pmatrix} \tag{10}$$

The forms of G and H will be determined next. Substituting Eq. 1 into Eq. 8, the governing differential equation is as follows

$$R^2\left(1-nR^\eta\right)\frac{d^2u}{dR^2}+R\left[2-(2+\eta)nR^\eta\right]\frac{du}{dR}-2\left[1-\left(1-\upsilon^*\eta\right)nR^\eta\right]u=0 \tag{11}$$

Equation 11 is a homogeneous hypergeometric differential equation. Using a new variable $x=nR^\eta$ and applying the transformation $u(R)=Ry(x)$, the result Eq. 11 is

$$x(1-x)\frac{d^2y}{dx^2}+\left[1+\frac{3}{\eta}-\left(2+\frac{3}{\eta}\right)x\right]\frac{dy}{dx}-\frac{1+2\upsilon^*}{\eta}y=0 \tag{12}$$

The solution of Eq. 12 is given as

$$y(x) = C_1 F_H(\alpha, \beta, \delta; x) + \bar{C}_2 x^{-3/\eta} F_H(\alpha - \delta + 1, \beta - \delta + 1, 2 - \delta; x) \tag{13}$$

With $F_H(\alpha, \beta, \delta; x)$ being the hypergeometric function defined by Abramowitz and Stegun [41],

$$F_H(\alpha, \beta, \delta; x) = 1 + \sum_{k=1}^{\infty} \frac{(\alpha)_k (\beta)_k}{(\delta)_k} \frac{x^k}{k!} \tag{14}$$

where

$$(\alpha)_k = \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+k-1) \tag{15}$$

Thus

$$F_H(\alpha, \beta, \delta; x) = 1 + \frac{\alpha\beta}{\delta}\frac{x}{1!} + \frac{\alpha(\alpha+1)\beta(\beta+1)}{\delta(\delta+1)}\frac{x^2}{2!}$$

$$+ \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{\delta(\delta+1)(\delta+2)}\frac{x^3}{3!} + \cdots \tag{16}$$

The arguments α, β, and δ of F_H in Eq. 16 are determined as

$$\begin{cases} \alpha = \dfrac{2(1+2\upsilon^*)}{(3+\eta) + \sqrt{(3+\eta)^2 - 4\eta(1+2\upsilon^*)}} \\ \beta = \dfrac{2(1+2\upsilon^*)}{(3+\eta) - \sqrt{(3+\eta)^2 - 4\eta(1+2\upsilon^*)}} \\ \delta = 1 + \dfrac{3}{\eta} \end{cases} \tag{17}$$

From $u(R) = Ry(nR^\eta)$, the homogeneous solutions G and H are found in the form

$$\begin{cases} G(R) = RF_H(\alpha, \beta, \delta; nR^\eta) \\ H(R) = \dfrac{1}{R^2} F_H(\alpha - \delta + 1, \beta - \delta + 1, 2 - \delta; nR^\eta) \end{cases} \tag{18}$$

The Eqs. 9 and 10 may be rewritten with nondimensional parameters as

$$U(R) = C_3 G(R) + C_4 H(R) \tag{19}$$

$$\begin{pmatrix} \bar{\sigma}_r \\ \bar{\sigma}_\theta \end{pmatrix} = (1 - nR^\eta) \begin{pmatrix} A & 2B \\ B & A+B \end{pmatrix} \begin{pmatrix} C_3 G' + C_4 H' \\ C_3 \dfrac{G}{R} + C_4 \dfrac{H}{R} \end{pmatrix} \tag{20}$$

where

$$\begin{cases} U = \dfrac{uE_0}{bP_o} \\ \bar{\sigma} = \dfrac{\sigma}{P_o} \\ \dfrac{C_3}{C_1} = \dfrac{C_4}{C_1} = \dfrac{E_0}{bP_o} \end{cases} \tag{21}$$

Integration constants C_3 and C_4 are determined by using the following boundary conditions

$$\begin{cases} U(R=0) = 0 \\ \bar{\sigma}_r(R=1) = -1 \end{cases} \tag{22}$$

Thus

$$\begin{cases} C_3 = -\dfrac{1}{(1-n)\left[AG'(1) + 2BG(1)\right]} \\ C_4 = 0 \end{cases} \tag{23}$$

Hence, non-dimensional radial displacement, radial stress and circumferential stress are found as follows

$$U = -\frac{G(R)}{(1-n)\left[AG'(1) + 2BG(1)\right]} \tag{24}$$

$$\bar{\sigma}_r = -\left[1 + \frac{n\left(1-R^{\eta}\right)}{1-n}\right]\left[\frac{AG'(R) + 2B\dfrac{G(R)}{R}}{AG'(1) + 2BG(1)}\right] \tag{25}$$

$$\bar{\sigma}_\theta = -\left[1 + \frac{n\left(1-R^{\eta}\right)}{1-n}\right]\left[\frac{BG'(R) + (A+B)\dfrac{G(R)}{R}}{AG'(1) + 2BG(1)}\right] \tag{26}$$

3. Numerical Analysis

In the field of the infinitesimal theory of elasticity, an axisymmetric pressurized parabolic FGM solid sphere is studied. In order to numerical analysis of problem, an axisymmetric element has been applied for modeling and meshing.

For modeling of FGM solid sphere, an innovative application for multilayering of wall thickness in the radial direction has been performed. In this approach, homogenous layers which are of identical thickness and step-variable elasticity modulus have been formed.

4. Results and Discussion

In this section, consider a solid sphere with an arbitrary radius of b, subjected to an arbitrary constant uniform pressure, P_0. It is assumed that the Poisson's ratio, υ, has a constant value of 0.3. For the presentation of the results it is convenient to use the following dimensionless and normalized variables.

In Fig. 2, for different values of n and η, dimensionless modulus of elasticity along through the radial direction is plotted. It is apparent from the curve that at the same position $(0 < R < 1)$, for $n = -0.5$, dimensionless modulus of elasticity increases as η decreases, while for $n = +0.5$, of elasticity increases as η increases.

Distribution of the radial displacement and the radial stress along the radial direction for different values of n and $\eta = 0.9$ are shown in Figs. 3 and 4. According to these figures, at the same position $(0 < R < 1)$, for higher values of n, radial displacement and radial stress increase.

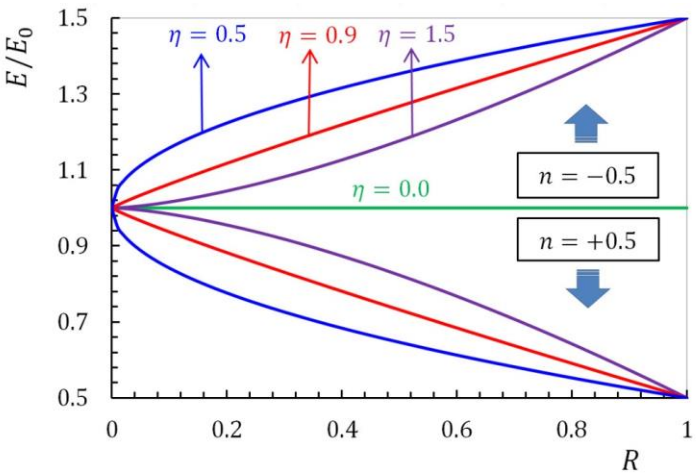

Fig. 2: Radial distribution of Modulus of elasticity

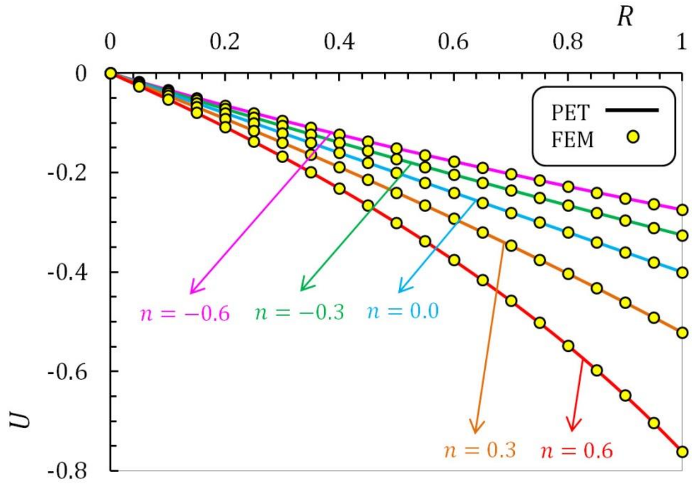

Fig. 3: Radial distribution of radial displacement ($\eta = 0.9$)

The circumferential stress along the radial direction for different values of and is plotted in Fig. 5. It must be noted from this figure that at the same position, almost for $R < 0.65$, there is an increase in the value of the circumferential stress as increases, whereas for $R > 0.65$ this situation was reversed. Besides, along the radial direction for the positive magnitudes of the circumferential stress decreases, while for negative magnitude of n, the circumferential stress increases.

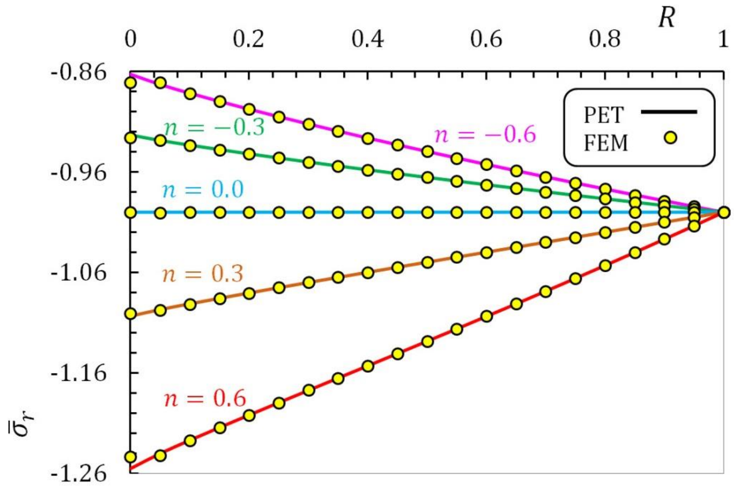

Fig. 4: Radial distribution of radial stress ($\eta = 0.9$)

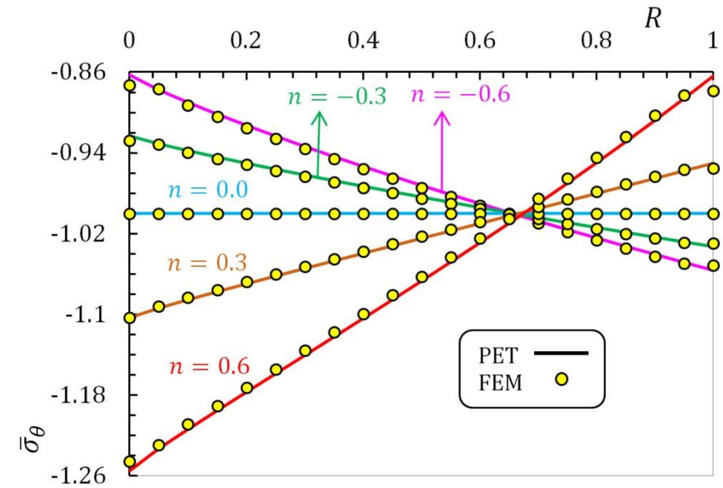

Fig. 5: Radial distribution of circumferential stress ($\eta = 0.9$)

5. Conclusion

In the present study, analytical and numerical solutions for radial and circumferential stresses, and radial displacement in functionally graded solid sphere subjected to external pressure are obtained. The material is assumed to be isotropic with constant Poisson's ratio. The modulus of elasticity is considered to be parabolic-varying in the radial direction.

To show the effect of inhomogeneity on the stress distributions, different values were considered for material parameter n. The maximum of radial and circumferential stresses for negative values of material parameter n, occur on the external surface, whereas for positive values of n, this situation was reversed. The maximum radial displacements in all the areas of the solid sphere occur on the external surface. Good agreement was found between the analytical solutions and the solutions carried out through the FEM. Results show that value of n has a great effect on the deformations and stresses and is a useful parameter from a design point of view in that it can be tailored to specific applications to control the stress and displacements.

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