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RESEARCH PAPER

Moore-Gibson-Thompson Thermoelastic medium with Variable Thermal Conductivity

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Abstract

The present investigation deals with the effect of variable thermal conductivity in an isotropic, unbounded and homogeneous thermoelastic medium under Moore Gibson Thompson (MGT) thermoelasticity theory. The normal mode analysis technique is applied for obtaining the displacement, stress, and temperature field. The values of these components are obtained by simulation technique using MATLAB and are shown graphically. The results also depict the variations for different theories of thermoelasticity.

Keywords: Moore-Gibson-Thompson Thermoelasticity; Non-Fourier Heat Conduction; Variable Thermal Conductivity Effects; Analytical Thermomechanical Modeling; Generalized Heat Transport in Solids; MATLAB Simulation in Thermoelastic Media.

1. Introduction

The classical Fourier's law is widely employed by researchers to solve the conventional thermoelastic problems. But this law predicts infinite velocity of heat wave propagation which is practically impossible. The classical coupled thermoelasticity(CTE) theory was first introduced by Biot [1]. The first generalization was given by Lord and Shulman [2] by replacing the classical Fourier's law with the heat conduction equation

$$\left(1+\tau\frac{\partial}{\partial t}\right)\vec{q}=-k\nabla\theta. \tag{1}$$

Green and Lindsay [3] derived the second generalization by developing the thermoelastic theory of temperature rate dependent. Namely, Green and Naghdi [4-6] introduced three versions of thermoelasticity theory termed as GN-I, GN-II and GN-III. In context of GN-III model, the modified Fourier's law was defined as:

$$\vec{q}=-[k\nabla\theta+k^{*}\nabla v]. \tag{2}$$

But instantaneous propagation of heat waves still exists in this model. Consequently, a modification of this heat conduction equation was done by introducing a relaxation parameter as:

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$$\vec{q}+\tau_q\frac{\partial\vec{q}}{\partial t}=-[k\nabla\theta+k^*\nabla v]. \tag{3}$$

Above equation together with the equation of energy constitutes Moore Gibson Thompson (MGT) equation. A lot of work has been done in the past based on MGT theory. Authors have solved problems on MGT theory in the field of thermoelasticity [7-10], thermoelasticity with diffusion [11], non-local micropolar double porous thermoelastic medium [12], thermos-piezo-electric semiconducting solid [13] and microbeams resonators [14]. Some more contributors [15-18] in MGT theory of thermoelasticity are acknowledged in the section.

Thermoelastic medium under different theories has been explored by researchers during the last century. Huge number of problems has been solved and interesting results have been developed [19-22]. The thermal conductivity of an elastic material varies with temperature under exposure to high temperature. Therefore, the thermal conductivity cannot be treated as constant. So, it is necessary to take variable thermal conductivity into consideration. Various researchers are continuously working on this concept. The concept of variable thermal conductivity has attracted the attention of many researchers [23-35] in recent years.

The present paper deals with the effect of variable thermal conductivity in thermoelastic medium subjected to MGT theory. The authors have obtained the analytical expressions of various thermoelastic parameters using normal mode analysis. The results are depicted graphically in two and three dimensions to show the effect of varying thermal conductivity on all considered physical quantities for different theories of thermoelasticity.

2. Basic Equations

The equations governing isotropic and homogenous thermoelastic medium under Moore-Gibson-Thompson thermoelasticity theory in the absence of body forces and heat sources are given by [10]:

$$\sigma_{ij,j}=\rho\ddot{u}_i, \tag{4}$$

$$\sigma_{ij}=\lambda e\delta_{ij}+2\mu e_{ij}-\beta\theta\delta_{ij}, \tag{5}$$

$$\mu u_{i,jj}+(\lambda+\mu)u_{j,ij}-\beta\theta,i=\rho\ddot{u}_i, \tag{6}$$

$$\left(k^*+k\frac{\partial}{\partial t}\right)\nabla^2\theta=\left(\frac{\partial}{\partial t}+\tau_q\frac{\partial^2}{\partial t^2}\right)(\beta\theta_0 u_{i,i}+\rho c_v\ddot{\theta}). \tag{7}$$

From the above set of equations (6) and (7), we can extract the four different theories of thermoelasticity as follows:
(1) For Biot classical model $k^*=\tau_q=0$,

(2) For LS model $k^*=0$,

(3) For Green-Naghdi (Type III) model $\tau_q=0$,

(4) For MGT model $k^*\neq 0,\tau_q\neq 0$.

3. Formulation of the problem Structure

We consider that medium to be parallel to $x-y$ plane with y axis pointing vertically downwards, so the displacement vector has two components for the plane strain problem i.e $\vec{u}=(u,v,0)$ where both u and v are functions of x, y, and t. Therefore, for two-dimensional problem above equations (4) to (7) can be expressed as:

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}-\beta\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \quad (8)$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+\mu\frac{\partial^2 v}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}-\beta\frac{\partial\theta}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2}, \quad (9)$$

$$\left(k^*+k\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta=\left(\frac{\partial}{\partial t}+\tau_q\frac{\partial^2}{\partial t^2}\right)\left[\beta\theta_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\rho c_v\frac{\partial^2\theta}{\partial t^2}\right]. \quad (10)$$

It is assumed that the thermal and mechanical properties vary with temperature. Therefore, thermal conductivity k may be treated as a function dependent on temperature [15] and is written as:

$$k(\theta)=k_0^*(1+k_1^*\theta). \quad (11)$$

We consider the following mapping [15]:

$$\hat{\theta}=\frac{1}{k_0^*}\int_0^\theta k(\xi)d\xi. \quad (12)$$

The dimensionless quantities defined below are introduced to felicitate the convenience of numerical calculations:

$$x'=\frac{x}{c_0t_0}, y'=\frac{y}{c_0t_0}, u'=\frac{u}{c_0t_0}, v'=\frac{v}{c_0t_0}, \quad t'=\frac{t}{t_0}, \quad \hat{\theta}'=\frac{\theta}{\theta_0}, \sigma'_{ij}=\frac{\sigma_{ij}}{\mu}, \tau'_q=\frac{\tau_q}{t_0}, \quad (13)$$

where, $c_0^2=\frac{\lambda+2\mu}{\rho}$.

Now using equations (11) to (13) in equations (8) to (10), the following equations in dimensionless form are obtained (on removing the primes):

$$\left(\frac{\partial^2}{\partial x^2}+A_1\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial t^2}\right)u+A_2\frac{\partial^2 v}{\partial x\partial y}-A_3\frac{\partial\hat{\theta}}{\partial x}=0, \quad (14)$$

$$\left(A_1\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial t^2}\right)v+A_2\frac{\partial^2 u}{\partial x\partial y}-A_3\frac{\partial\hat{\theta}}{\partial y}=0, \quad (15)$$

$$k_2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\hat{\theta}+k_1\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\hat{\theta}-\frac{\partial^3}{\partial t^3}\hat{\theta}-\tau_q\frac{\partial^4}{\partial t^4}\hat{\theta}=$$
$$a_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\tau_q a_2\frac{\partial^3}{\partial t^3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right). \quad (16)$$

where values of arbitrary constants are given in appendix.

4. Solution Methodology

Since we have considered the propagation of waves parallel to xy - plane therefore the solution of the variables may be considered in terms of modes and is written as:

$$\{u,v,\hat{\theta}\}=\{u^*,v^*,\hat{\theta}^*\}(y)e^{\omega t+iax}, \quad (17)$$

where ω is the complex time constant and a is the wave number in the x -direction.

Using (17) in (14) to (16), we get the following equations:

$$[A_1D^2 + B_1]u^* + iaA_2Dv^* - iaA_3\hat{\theta}^* = 0, \tag{18}$$
$$iaA_2Du^* + [D^2 + B_2]v^* - A_3D\hat{\theta}^* = 0, \tag{19}$$
$$iaB_5u^* + B_5Dv^* - [B_4D^2 - B_6]\hat{\theta}^* = 0. \tag{20}$$

Three coupled equations (18), (19) and (20) in terms of u^*, v^* and $\hat{\theta}$ are obtained. On solving these equations, we obtain a differential equation of order six given by,

$$[E_1D^6 + E_2D^4 + E_3D^2 + E_4]\hat{\theta}^* = 0. \tag{21}$$

The solution of above equations satisfying the radiation conditions $u^*, v^*, \hat{\theta}^* \to 0$ as $y \to \infty$ can be expressed as:

$$u^* = \sum_{j=1}^{3} F_j \exp\{-r_j y\}, \tag{22}$$
$$v^* = \sum_{j=1}^{3} G_j \exp\{-r_j y\}, \tag{23}$$
$$\hat{\theta}^* = \sum_{j=1}^{3} H_j \exp\{-r_j y\}, \tag{24}$$

where r_j^2 are roots of auxiliary equation of (21).

Further, the constants are related by the following relations:

$F_j = X_jH_j, G_j = Y_jH_j, (j = 1,2,3)$.

Also, the values of X_j, Y_j are mentioned in Appendix.

5. Boundary Conditions

For obtaining the values of constants H_j, the following boundary conditions are used:

1. A mechanical force $F\exp\{\omega t + iax\}$ is applied at the free surface $x = 0$ along the normal direction.

$$\sigma_{yy} = -Fe^{\omega t + iax}. \tag{25}$$

2. Further the free surface $x = 0$ is free from tangential stress.

$$\sigma_{yx} = 0\,. \tag{26}$$

3. The insulated free surface is considered.

$$\frac{\partial \theta}{\partial y} = 0. \tag{27}$$

Using (17), (22)-(24) in the above boundary conditions, we get a non-homogenous system of three equations. This system is solved by simulation technique using MATLAB software to find the values of unknown parameters.

6. Numerical Computation and Discussion

For proving the effect of thermal conductivity on various physical quantities, we take the example of copper [10]. The physical constants involved in this problem can be taken as follows: $\lambda = 7.76 \times 10^{10}\text{kgm}^{-1}\,\text{s}^{-2}, \mu = 3.86 \times 10^{10}\text{kgm}^{-1}\,\text{s}^{-2}, \rho = 8954\text{Kgm}^{-3}, C_v = 383.1\text{JKg}^{-1}\,\text{K}^{-1}, k_0^* = 386\text{Wm}^{-1}\,\text{K}^{-1}, k^* = 5930.38 \times 10^{10}\text{Wm}^{-1}\,\text{K}^{-1}s^{-1}$, $k_1 = 1, k_2 = 1, \beta = (3\lambda + 2\mu)\alpha, \alpha = 1.78 \times 10^{-5}K^{-1}, \theta_0 = 293K, \tau_q = 0.01$

All calculations are performed for $t = 1, \omega_0 = -2.5, \xi = 1$, where $\omega = \omega_0 + i\xi$ at the surface $x = 1$. The variations in considered physical quantities are studied for the above numerical values against horizontal distance y under Biot model, LS model, GN-III model and MGT Model depicted graphically in two and three dimensions.

Figure 1 shows variation of tangential displacement with horizontal distance. The values of tangential displacement coincide for all models discussed above subjected to constant thermal conductivity $K_1^s = 0$. But when thermal conductivity is varied to $K_1^s = -2$, the variation of tangential displacement coincides for MGT and LS model and attains its maximum value near the interface $y = 0$. Also, the value of tangential displacement for both the models decreases as horizontal distance increases. But reverse pattern is observed in case of Biot model. Further in case of GN-III theory, the value of tangential displacement does not vary with horizontal distance.

Figure 2 shows variation of normal displacement with horizontal distance. The values of normal displacement increase exponentially for all models subjected to constant thermal conductivity $K_1^s = 0$. But when thermal conductivity is varied to $K_1^s = -2$, the normal displacement for MGT and LS model first decreases in range $0 \leq y \leq 0.5$ and then becomes constant. The value of normal displacement does not vary with horizontal distance for Biot model and GN-III model for $K_1^s = -2$.

Figure 3 shows variation of tangential stress with horizontal distance. The values of tangential stress coincide for all models in case of constant thermal conductivity $K_1^s = 0$ and GN-III model in case $K_1^s = -2$. But the tangential stress follows a bell shape variation in case $K_1^s = -2$ for MGT and LS model.Further in case of Biot model, the value of tangential stress first decreases when horizontal distance lies in range $0 \leq y \leq 0.25$ and then increases sharply as horizontal distance increases.

Figure 4 shows variation of normal stress with horizontal distance. The values of normal stress first decrease in range $0 \leq y \leq 0.5$ for all models and then became constant in case of constant thermal conductivity $K_1^s = 0$. In case of variable thermal conductivity, the behaviour of normal stress coincides for MGT and LS model. There is no variation in normal stress with horizontal distance in case of Biot model and GN-III model.

Figure 5 shows variation of temperature with horizontal distance. The values of temperature increase exponentially for Biot and LS model for $K_1^s = 0$. Also, the values of temperature increase with horizontal distance for MGT and GN-III model. But when thermal conductivity is varied to $K_1^s = -2$, the variation in temperature coincides for all model and is constant with horizontal distance for $K_1^s = -2$.

Figures 6 to 10 gives the three-dimensional views of all field quantities for $K_1^s = -2$. These figures are helpful in finding the exact values of all quantities for different values of x and y coordinates.

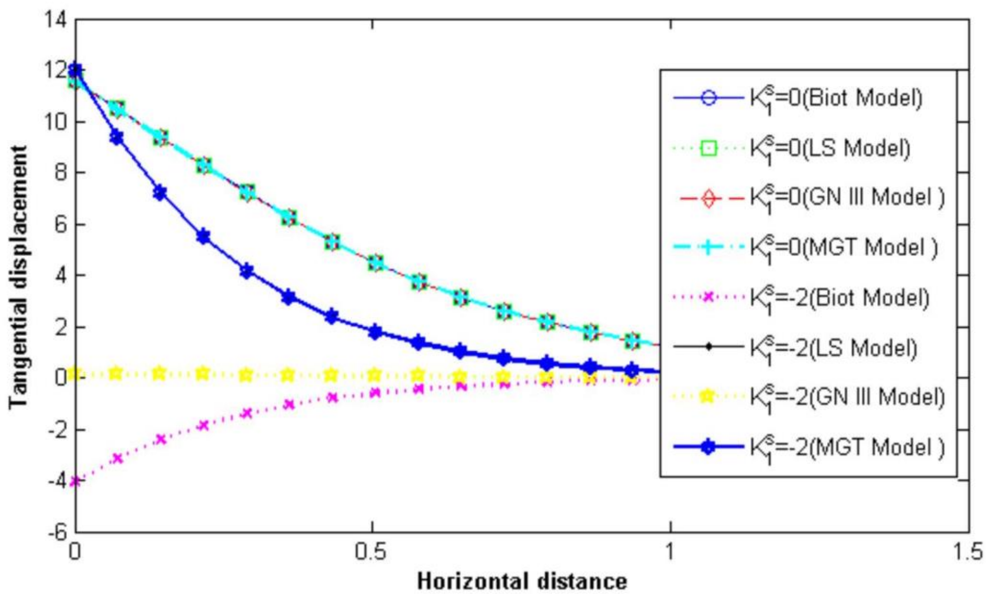

Figure 1: u **vs** y

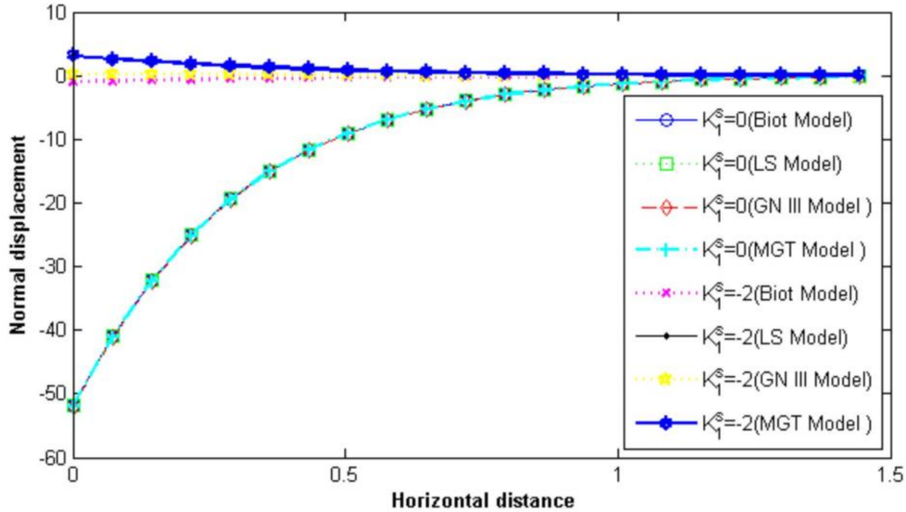

Figure 2: v **vs** y

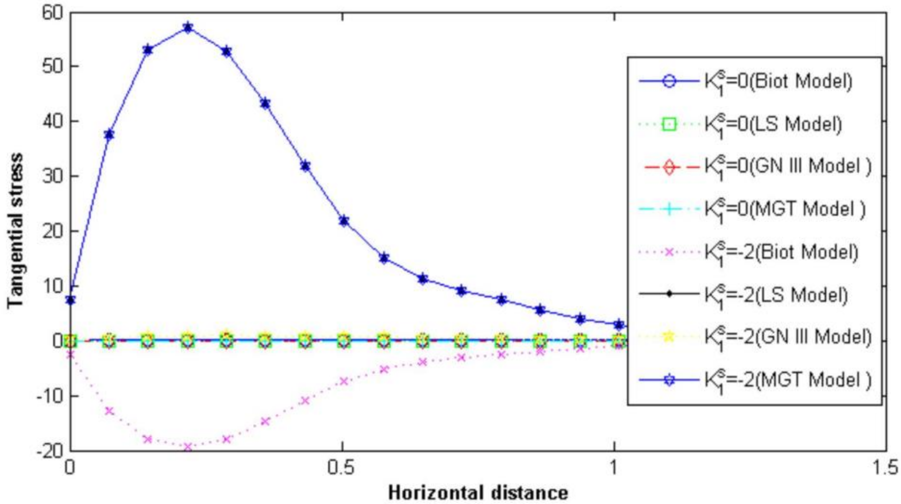

Figure 3: σ_{xy} **vs** y

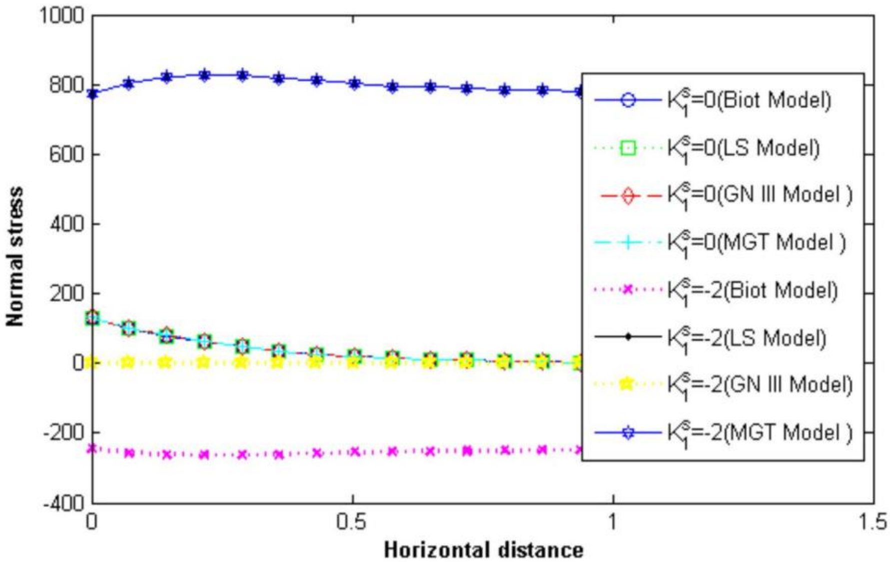

Figure 4: σ_{yy} **vs** y

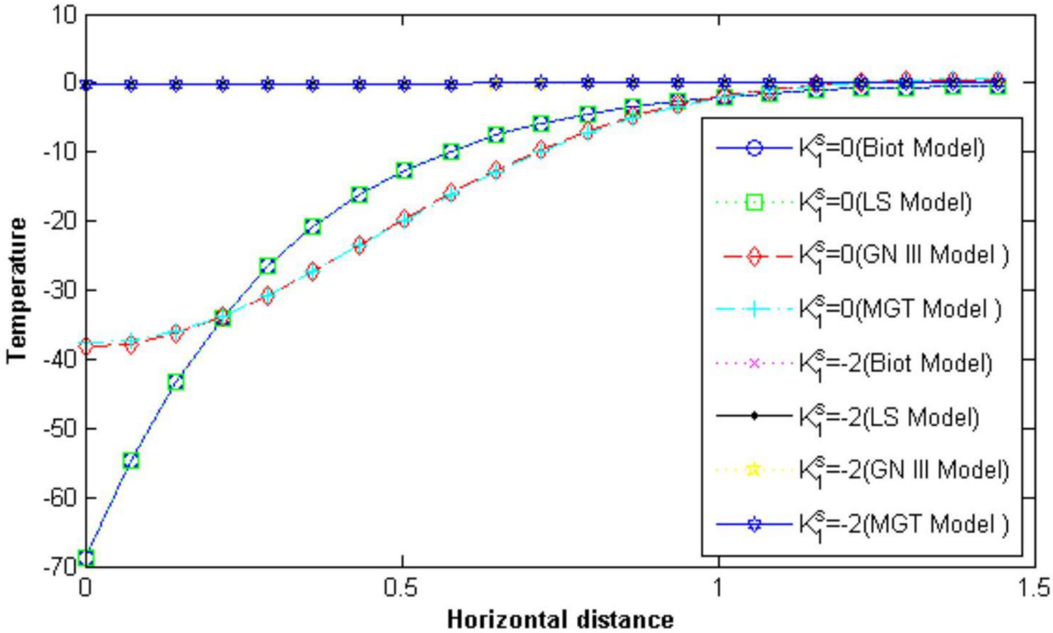

Figure 5: θ_{yy} **vs** y

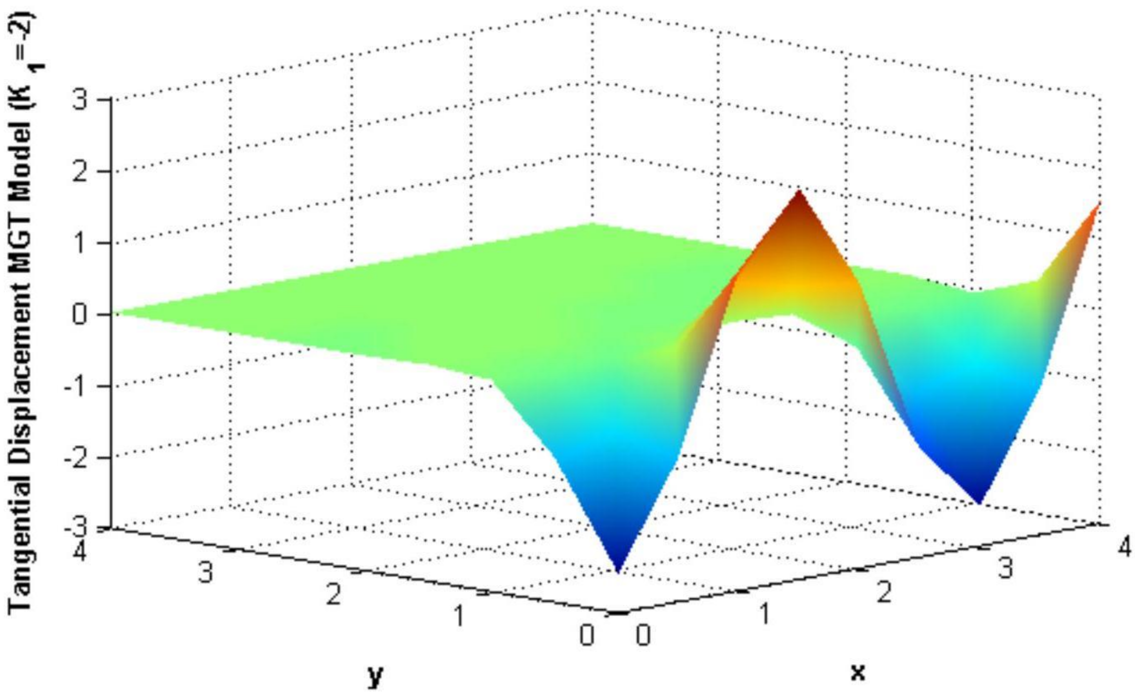

Figure 6: u **for** $K_1 = -2$

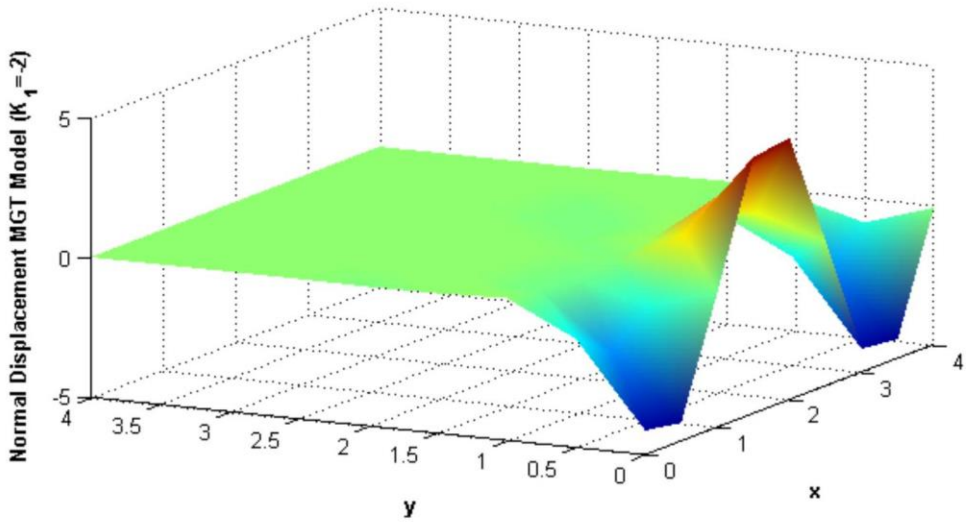

Figure 7: v **for** $K_1 = -2$

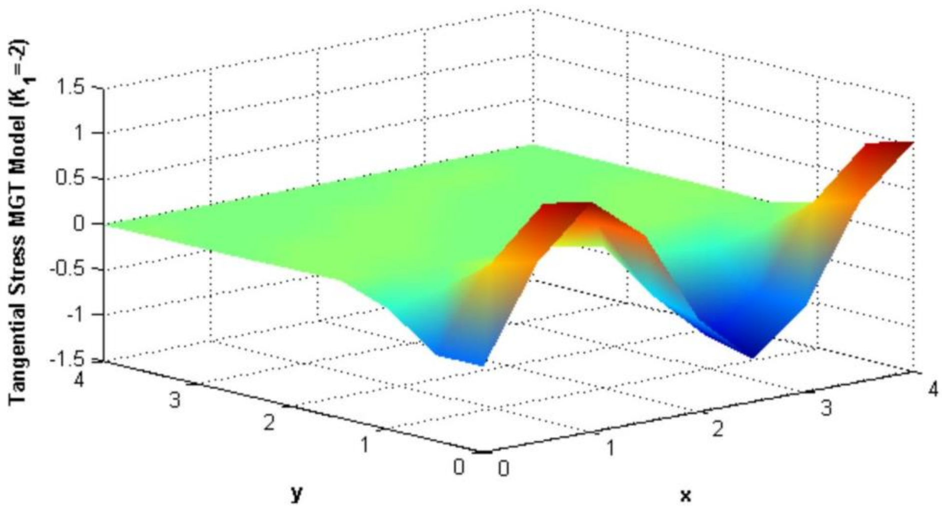

Figure 8: σ_{xy} **for** $K_1 = -2$

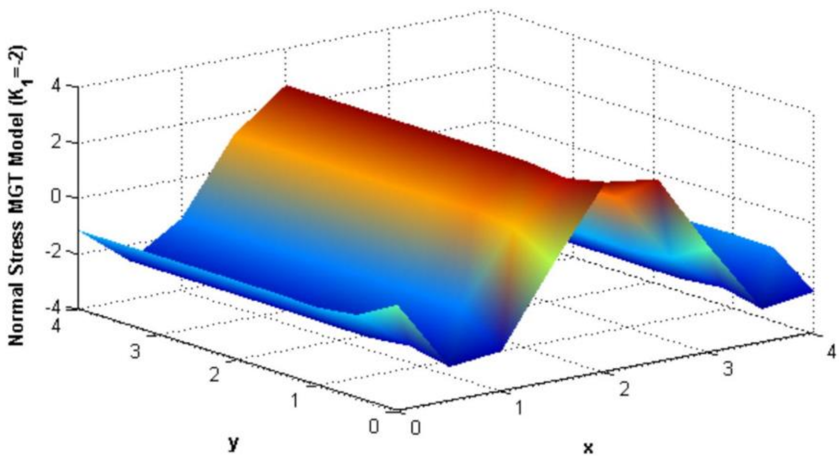

Figure 9: σ_{yy} **for** $K_1 = -2$

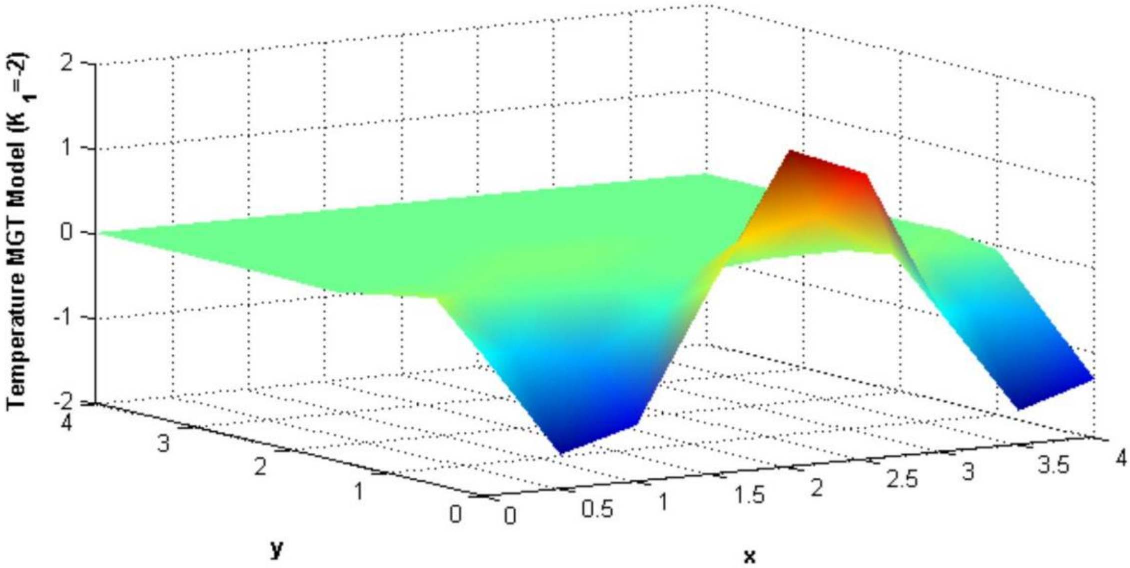

Figure 10: θ **for** $K_1 = -2$

7. Conclusions

On the basis of the above numerical discussion, the following conclusions can be drawn:

1. There is an appreciable effect of varying thermal conductivity on all field quantities.
2. For constant thermal conductivity, tangential displacement is identical across all models. For variable thermal conductivity, MGT and LS models show coinciding tangential displacement, peaking near the interface, and decreasing with horizontal distance.
3. The variation of all considered physical quantities coincides for MGT and LS model in constant as well as variable therrmal conductivity.
4. Normal displacement increases exponentially for all models in case of constant thermal conductivity.
5. Normal force stress shows identical behavior for MGT and LS models whereas there is no variation for Biot and GN-III models in case of variable conductivity.

Appendix

$$c_0^2 = \frac{\lambda+2\mu}{\rho}, k_1 = \frac{k_0^*}{\rho C_v c_0^2 t_0}, k_2 = \frac{k^*}{\rho C_v c_0^2}, a_2 = \frac{\beta}{\rho C_v}, A_1 = \frac{\mu}{\rho c_0^2}, A_2 = \frac{\lambda+\mu}{\rho c_0^2},$$
$$A_3 = \frac{\beta\theta_0}{\rho c_0^2}, B_1 = -(a^2+\omega^2), B_2 = -(a^2 A_1 + \omega^2), B_3 = 1+\tau_q\omega, B_4 = k_2 + k_1\omega,$$
$$B_5 = a_2\omega^2 B_3, B_6 = B_4 a^2 + B_3\omega^3, C_1 = -a^2 A_2^2 - B_1 - B_2 A_1, C_2 = a^2 A_2 A_3 + A_3 B_1,$$
$$C_3 = B_5 - B_5 A_2, C_4 = A_3 B_5 + A_2 B_6, E_1 = A_1 A_2 B_4, E_2 = C_3 A_1 A_3 - A_1 C_4 - A_2 C_1 B_4,$$
$$E_3 = C_2 C_3 + B_2 B_5 A_1 A_3 + C_1 C_4 + B_1 B_2 A_2 B_4, E_4 = B_2 B_5 C_2 - B_1 B_2 C_4, L_1 = A_2 B_4$$
$$L_2 = B_2 B_5, L_3 = iaA_2^2 B_4, L_4 = ia(A_3 C_3 - A_2 C_4), L_5 = iaA_3 B_2 B_5, L_6 = A_1 C_3,$$
$$L_7 = B_1 C_3 + A_1 B_2 B_5, L_8 = B_1 B_2 B_5,$$
$$X_j = \frac{L_3 r_j^4 + L_4 r_j^2 + L_5}{L_6 r_j^4 + L_7 r_j^2 + L_8}, Y_j = \frac{C_4 r_j - L_1 r_j^3}{C_3 r_j^2 + L_2}, (j = 1,2,3)$$

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