Journal of Computational Applied Mechanics 2025, 56(2): 331-344

DOI: 10.22059/jcamech.2025.395439.1478

RESEARCH PAPER

Reflection of Harmonic Waves in a Nonlocal Rotating Micropolar Medium with Constant Magnetic Field under Three-phase-lag Theory with Temperature Dependent Elastic Model

Ambreen A. Khan [a], Sadiq M. Sait [b, c], R. Ellahi [a, d, *]

[a] *Department of Mathematics & Statistics, International Islamic University, Islamabad, Pakistan*
[b] *Department of Computer Engineering, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia*
[c] *Interdisciplinary Research Center for Smart Mobility and Logistics, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia*
[d] *Center for Modeling & Computer Simulation, Research Institute, King Fahd University of Petroleum & Minerals, Dhahran-31261, Saudi Arabia*

Abstract

The reflection of harmonic waves, like sound or light, has diverse applications, including sonar for object location, understanding seismic waves in geophysics, and even in the human eye's ability to see. The reflection of plane waves with constant material properties is available in existing literature, but very little attention has been given to the temperature-dependent modulus of elasticity. A novel model is proposed to study the propagation of harmonic plane waves through a nonlocal micropolar medium with temperature-dependent material properties. The influence of nonlocality, rotation effects, and the constant magnetic field is also taken into account. The precise formulations of the field quantities are presented and examined using the normal mode approach. The phase lag (TPL) theory is applied to model and solve the governing equations. The effects of rotation, temperature-dependent constants, and the nonlocality parameter on the different physical quantities have been examined and displayed graphically. Energy ratios are also computed by using the amplitude ratios. It is concluded that in a nonlocal, rotating, micropolar medium, reflection of harmonic waves provides four coupled quasi-waves, namely, quasi-transverse, quasi-longitudinal, quasi-micro rotational, and quasi-thermal, with different speeds, and the energy ratios and reflection coefficients are affected by nonlocal parameters, rotation, and micropolarity.

Keywords: MHD, Harmonic waves, non-local micropolar medium, rotations, energy ratios, temperature dependent elastic model, three phase lag theory

1. Introduction

The reflection of harmonic waves is very significant in seismology, earthquake engineering, etc. Many authors [1-3] have examined the reflection of plane waves with constant material properties. But little attention has been given to the temperature-dependent modulus of elasticity. Othman and Song [4]

[*] Corresponding author, (R. Ellahi): E-mails: rahmatellahi@yahoo.com

deliberated the reflection of P and SV waves with temperature-dependent material properties. Sheoran et al. [5] discussed the propagation of thermo-viscoelastic waves whose material properties are dependent on temperature. Ma and He [6] studied the non-local effect and variable properties of the thermoelastic wave. Khan et al. [7] observed the effect of temperature-dependent material parameters on the reflection and transmission of thermoelastic waves. Abo-Dhahab et al. [8] considered the reflection of plane wave with temperature-dependent physical parameters. Ailawalia and Sharma [9] used the numerical and analytical approach to discuss the solution of plane wave with temperature-dependent physical parameters.

In classical continuum mechanics, the elasticity of the medium is based on linear stress-strain law, which is unable to explain the microstructural behavior of the medium. Eringen [10] proposed a theory that is based on microstructural motion. Micropolar theory is more suitable to explain the behavior of fibreglass, polycrystalline material, solid propellant, etc. Eringen and Edelen [11] proposed the nonlocal elasticity theories, featuring nonlocality residuals of fields like internal energy, mass, entropy, etc. These residuals were determined, together with the constitutive laws, using appropriate thermodynamic limitations. They contributed to the development of these nonlocal elasticity theories. These theories are concerned with materials whose action at any given point is affected by the state of all points within the body [12, 13]. Kakkal et al. [14] explored the reflection of plane waves in a rotating transversely isotropic with nonlocal fiber-reinforced. Vin and Tuan [15] discussed the harmonic wave in a micropolar medium.

The classical theory of thermos-elasticity makes use of the thermal wave's infinite velocity. Although this assumption is an inadequate approximation in practice, it may be helpful for many technical challenges. Some investigations show that the thermal waves have a finite speed; hence, generalized thermo-elastic theories involve hyperbolic-type governing equations that are used to eliminate this discrepancy [16, 17]. The LS and GL theories have been used to understand the interaction between mechanical and thermal properties of elastic materials. In 1995, Tzou [18] introduced a dual-phase lag (DPL) theory in which a macroscopic delay response between the heat flux vector and the temperature gradient is possible. This model proposes to replace the Fourier's law with an approximation of a modified law that has two distinct translations for the temperature gradient and the heat flux vector. Further, Chandrasekharaiah [19] gave a brief review on different hyperbolic thermoelastic models. Khan and Tanveer [20] discussed the reflection and transmission of SV waves under DPL theory. Few core studies on rotation, micropolar medium, and temperature dependent elasticity and MHD can be found in [21-37].

Up until now, all of the studies have focused on the independent impacts of rotation, temperature-dependent elastic material, or nonlocality, or on only one or two of these factors at a time, but in this article, we take the combined effects to extend the previous studies. The current study accounts for temperature-dependent moduli implications on MHD nonlocal micropolar medium under the influence of TPL theory. The governing system of partial differential equations has been remodeled into ordinary differential equations by normal mode analysis.

2. Mathematical Modelling

Let us consider the Cartesian coordinate system (x, y, z) with x-axis is pointing vertically downward. Consider the isotropic, homogeneous, nonlocal micropolar thermoelastic medium which is rotating with angular velocity $\vec{\Omega} = (0, 0, \Omega)$. A constant magnetic field $\vec{H} = (0, 0, H_0)$, is applied in the z direction as shown in Fig. 1.

The basic equations of nonlocal micropolar thermos-elasticity under three phase lag theory is as follows:

$$\sigma_{ij,j} + F_i = \rho(1-\epsilon^2\nabla^2)\left[\vec{u}_{i,tt} + \{\vec{\Omega}\times(\vec{\Omega}\times\vec{u})\}_{,i} + (2\vec{\Omega}\times\vec{u}_{,t})_{,i}\right] \tag{1}$$

where F_i is the Lorentz force:

$$F_i = \mu(\vec{J}\times\vec{H})_{i}, \tag{2}$$

which is computed by using these as follow:

$$\vec{J} = \nabla\times\vec{h},\quad \vec{h} = curl(\vec{u}\times\vec{H}),\quad \vec{u} = (u, v, 0),\quad \vec{H} = (0, 0, H_0), \tag{3}$$

The component form of Eq. (2) is written as:

$$F_x = \mu_e\left(H_0^2\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right\}\right),\; F_y = \left(-H_0^2\left\{\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial x\partial y}\right\}\right),\; F_z = 0. \tag{4}$$

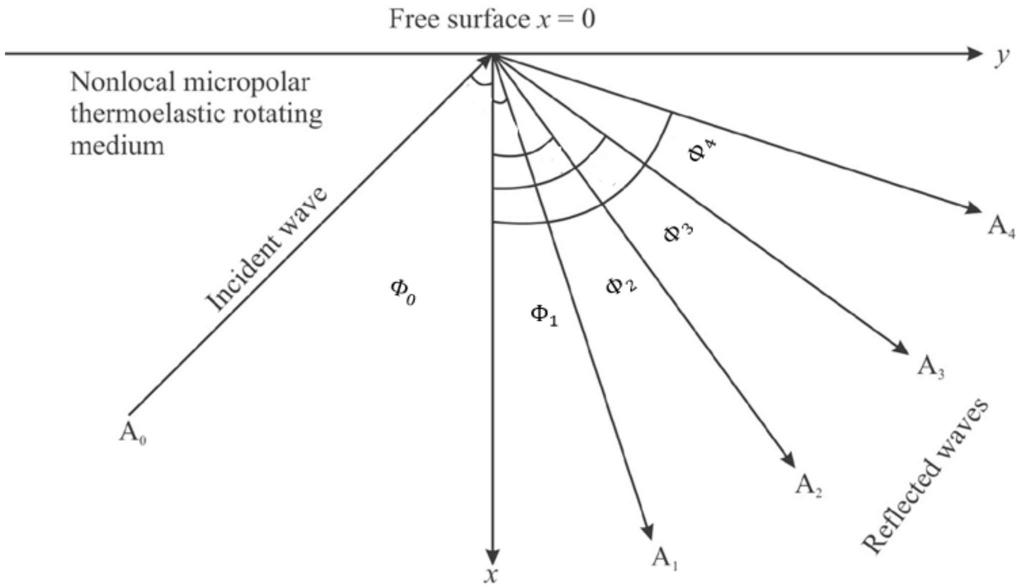


Fig. 1. Geometry of the problem

The couple stress equation of motion and the constitutive relations are given by:

$$(\alpha+\beta+\gamma)\nabla(\nabla.\vec{\Phi})-\gamma\nabla\times(\nabla\times\vec{\Phi})+K\nabla\times\vec{u}-2K\vec{\Phi}=(1-\nabla^2\epsilon^2)\rho_j\frac{\partial^2\vec{\Phi}}{\partial t^2}, \tag{5}$$

$$(1-\nabla^2\epsilon^2)t_{ij}=\sigma_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\Phi_r)-\beta_1\theta\delta_{ij}, \tag{6}$$

$$(1-\nabla^2\epsilon^2)\mu_{ij}=m_{ij}=\alpha\Phi_{r,r}\delta_{ij}+\beta\Phi_{i,j}+\gamma\Phi_{j,i}, \tag{7}$$

where $\vec{\Phi}=(0,0,\Phi_3)$ is the micro-rotation vector.

The heat equation under three phase lag theory is as follows:

$$(1-\nabla^2\epsilon^2)\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\vec{q}=-K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right)\nabla v-K^*\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla T, \tag{8}$$

where the thermal displacement gradient is ∇v.

$$\rho T_0\dot{\eta}=-\nabla.\vec{q}, \tag{9}$$

$$(1-\nabla^2\epsilon^2)\rho\eta T_0=\beta_1 T_0 e+\rho C_E(T-T_0), \tag{10}$$

where η the specific entropy and $\vec{q}$ the heat flux vector.

Using Eq. (8) through Eq. (10) one can get:

$$\left[K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right)-K^*\frac{\partial}{\partial t}\left(1+\tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2 T=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\rho C_E\ddot{T}+\beta_1 T_0\ddot{e} \tag{11}$$

where $\beta_1=(3\lambda+K+2\mu)\alpha_{t1},\ K_1^*=C_E\frac{(K+2\mu)}{4},$

K^* the thermal conductivity, β_1 the thermal expansion coefficient, K_1^* the material characteristic, C_E the specific heat, τ_v,τ_T,τ_q are three phase-lag parameters.

We assume that all material parameters are function of temperature as follows:

$$(\lambda,\mu,\alpha,\beta,\gamma,K,\beta_1,\beta_0) = (\lambda',\mu',\alpha',\beta',\gamma',K',\beta_1',\beta_0')f(T_0), \tag{12}$$

where $f(T_0) = (1-\alpha^2 T_0)$

$f(T_0)$ is a non-dimensional function of temperature and is the empirical material constant. When $f(T_0) = 1$, the material properties are independent of temperature.

The stress component can be represented by considering constitutive relations as:

$$(1-\nabla^2\epsilon^2)t_{xx} = \sigma_{xx} = \alpha_1\left[(\lambda',2\mu',K')\frac{\partial u}{\partial x} + \lambda'\frac{\partial v}{\partial y} - {\beta'}_1\theta\right], \tag{13}$$

$$(1-\nabla^2\epsilon^2)t_{yy} = \sigma_{yy} = \alpha_1\left[(\lambda',2\mu',K')\frac{\partial v}{\partial y} + \lambda'\frac{\partial u}{\partial x} - {\beta'}_1\theta\right], \tag{14}$$

$$(1-\nabla^2\epsilon^2)t_{xy} = \sigma_{xy} = \alpha_1\left[\mu'\frac{\partial u}{\partial y} + (\mu'+K')\frac{\partial v}{\partial x} - K'\Phi_3\right], \tag{15}$$

$$(1-\nabla^2\epsilon^2)\mu_{xz} = m_{xz} = \alpha_1\left[\gamma'\frac{\partial\Phi_3}{\partial x}\right], \tag{16}$$

$$(1-\nabla^2\epsilon^2)\mu_{yz} = m_{yz} = \alpha_1\left[\gamma'\frac{\partial\Phi_3}{\partial y}\right]. \tag{17}$$

The component form of Eqs. (1), (5) and (11) are

$$(\lambda'+\mu')\frac{\partial e}{\partial x} + (\mu'+K')\nabla^2 u + K'\frac{\partial\Phi_3}{\partial y} - {\beta'}_1\frac{\partial\theta}{\partial x} + F_x = (1-\epsilon^2\nabla^2)\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t}\right], \tag{18}$$

$$(\lambda'+\mu')\frac{\partial e}{\partial y} + (\mu'+K')\nabla^2 v + K'\frac{\partial\Phi_3}{\partial x} - {\beta'}_1\frac{\partial\theta}{\partial y} + F_y = (1-\epsilon^2\nabla^2)\rho\left[\frac{\partial^2 v}{\partial t^2} - \Omega^2 v - 2\Omega\frac{\partial u}{\partial t}\right], \tag{19}$$

$$\gamma'\nabla^2\Phi_3 + K'\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2K'\Phi_3 = (1-\epsilon^2\nabla^2)\rho j\frac{\partial^2\Phi_3}{\partial t^2}, \tag{20}$$

$$\left[K_1^*\left(1+\tau_v\frac{\partial}{\partial t}\right) + K^*\frac{\partial}{\partial t}\left(1+\tau_T\frac{\partial}{\partial t}\right)\right]\nabla^2 T = \left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E\ddot{T} + {\beta'}_1 T_0\ddot{e}\right). \tag{21}$$

The following non dimensional variables are introduced as follow:

$$(\tilde{x},\tilde{y},\tilde{\epsilon}) = \frac{\omega^*}{c_o}(x,y,\epsilon),\ \omega^* = \frac{\rho C_E c_o^2}{K^*},\ \tilde{\theta} = \frac{\theta}{T_o},(\tilde{t},\tilde{\tau}_o) = \omega^*(t,\tau_o),\ (\tilde{u},\tilde{v}) = \frac{\rho\omega^* c_o}{{\beta'}_1 T_o}(u,v),$$
$$c_o^2 = \frac{\lambda'+2\mu'+K'}{\rho},\ \tilde{\Omega} = \frac{\Omega}{\omega^*},\ (\tilde{\sigma}_{ij},\tilde{t}_{ij}) = \frac{1}{\beta' T_o}(\sigma_{ij},t_{ij}),\ \tilde{m}_{ij} = \frac{\omega^*}{c_o\beta' T_o}m_{ij},\ \tilde{\Phi}_3 = \frac{\rho c_o^2}{\beta' T_o}\Phi_3. \tag{22}$$

The potential functions q and Ψ are defined as:

$$\tilde{u} = grad\, q + curl\,\Psi,\ \tilde{v} = grad\, q - curl\,\Psi, \tag{23}$$

where q and Ψ are scalar and vector potential functions to explain the dilatational and transverse pat of the displacement vector.

Using Eqs. (22) and (23) into Eqs. (13) to (21), we get the following set of coupled partial differential equations:

$$(1-\epsilon^2\nabla^2)t_{xx} = \sigma_{xx} = \alpha_1\left[\frac{\partial^2 q}{\partial x^2} + C_6\frac{\partial^2 q}{\partial y^2} + C_7\frac{\partial^2\Psi}{\partial x\partial y} - \theta\right], \tag{24}$$

$$(1-\epsilon^2\nabla^2)t_{xy} = \sigma_{xy} = \alpha_1\left[C_7\frac{\partial^2 q}{\partial x\partial y} - \frac{1}{C a_2}\frac{\partial^2\Psi}{\partial x^2} + C_8\frac{\partial^2\Psi}{\partial y^2} - C_9\Phi_3\right], \tag{25}$$

$$(1-\nabla^2\epsilon^2)\mu_{xz} = m_{xz} = \alpha_1\left[d_1\frac{\partial\Phi_3}{\partial x}\right], \tag{26}$$

$$\left[\nabla^2 - c_o(1-\epsilon^2\nabla^2)\left(\frac{\partial^2}{\partial t^2} - \Omega^2\right) + R_H\nabla^2\right]q - 2C_o(1-\epsilon^2\nabla^2)\Omega\frac{\partial\Psi}{\partial t} - \theta, \tag{27}$$

$$2C_2(1-\epsilon^2\nabla^2)\Omega\frac{\partial q}{\partial t} + \left[\nabla^2 - c_2(1-\epsilon^2\nabla^2)\left(\frac{\partial^2}{\partial t^2} - \Omega^2\right) + R_H\nabla^2\right]\Psi + C_1\Phi_3 = 0, \tag{28}$$

$$\left[\nabla^2 - 2C_3 - C_4(1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\right]\Phi_3 - C_3\nabla^2\Psi, \tag{29}$$

$$\left[a_{10}\left(1+\tau_v\frac{\partial}{\partial t}\right)+\frac{\partial}{\partial t}\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla^2-\frac{\partial^2}{\partial t^2}\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\right]\theta=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\frac{\partial^2}{\partial t^2}(\delta_o\nabla^2 q), \tag{30}$$

Where

$C_o=\frac{1}{\alpha_1}$, $C_1=\frac{K'}{\mu'+K'}$, $C_2=\frac{a_0\rho c_0^2}{\mu'+K'}$, $C_3=\frac{K'c_0^2}{\gamma'\omega^*}$, $C_4=\frac{a_0\rho j c_0^2}{\gamma'}$, $C_5=\frac{{\beta'_1}^2 T_0}{\rho K^*\omega^*}$, $C_6=\frac{\lambda'}{\rho c_0^2}$, $C_7=\frac{2\mu'+K'}{\rho c_0^2}$, $C_8=\frac{\mu'}{\rho c_0^2}$, $C_9=\frac{K'}{\rho c_0^2}$, $C_{10}=\frac{K_1^*}{K^*\omega^*}$, $\delta_0=\frac{{\beta'_1}^2\alpha_1 T_0}{\rho K^*\omega^*}$, $d_1=\frac{\gamma'{\omega^*}^2}{\rho c_0^4}$, $R_H=\frac{c_\alpha}{c_0^2}$, $c_\alpha=\frac{{\mu'}_e}{\rho}$

3. Solutions of the problem

A harmonic wave is propagated in the xy plane making an angle Φ with x-axis. We consider the solution of Eqs. (27)-(30) as follows:

$$[q,\Psi,\Phi_3,\theta](x,y,t)=[q^*,\Psi^*,{\Phi_3}^*,\theta^*]exp[ik(-x\cos\Phi+y\sin\Phi)-\iota\gamma t], \tag{31}$$

where γ is the angular frequency, defined as $\gamma=kv$, k and v are the wave number and phase velocity respectively.

Using Eq. (31) into Eqs. (27)-(30), we get system of four coupled homogeneous equations

$$(D_1+D_2k^2)q^*+(D_3+D_4k^2)\Psi^*-\theta^*=0, \tag{32}$$

$$(D_1a_2+D_5k^2)\Psi^*+C_1{\Phi_3}^*-C_2(D_3+D_4k^2)q^*=0, \tag{33}$$

$$C_3k^2\Psi^*+(D_7+D_6k^2){\Phi_3}^*=0, \tag{34}$$

$$(D_8k^2+D_9)\theta-D_{10}k^2q=0, \tag{35}$$

where

$$D_1=\gamma^2+\Omega^2, D_2=D_1\epsilon^2-(1+R_H), D_3=2_\iota\Omega\gamma, D_4=D_3\epsilon^2, D_5=a_2D_1\epsilon^2-1,$$

$$D_6=a_4\beta^2\epsilon^2-1, D_7=a_4\gamma^2-2a_{3,}D_8=\left(a_{10}(\iota\tau_v\gamma-1)+\gamma(\iota+\tau_T\gamma)\right),$$

$$D_9=\gamma^2-\iota\tau_q\gamma^3-\frac{\tau_q^2}{2}\gamma^4,\ D_{10}=\delta_0D_9.$$

The nontrivial solution for the system of Eqs. (32)-(35) is

$$v^8+Lv^6+Mv^4+Nv^2+O=0, \tag{36}$$

where

$$L=\gamma^2\left[\frac{-D_1D_9C_1C_3+D_1D_2D_7D_9C_2+2D_3D_4D_7D_9C_2+D_1^2D_6D_9D_2+D_1^2D_7D_8D_2+D_3^2D_8D_9C_2+D_3^2D_7D_8D_2+D_1D_5D_7D_9-D_1D_7D_{10}C_2}{D_1^2D_7D_9C_2+D_3^2D_7D_9C_2}\right],$$

$$M=\gamma^4\left[\frac{+D_1D_2D_7D_8C_2+2D_3D_4D_6D_9C_2+2D_3D_4D_7D_8C_2-D_1D_6D_{10}C_2-D_1D_8C_1C_3+D_1D_2D_6D_9C_2-D_{10}C_1C_3-D_5D_7D_{10}+D_1^2D_6D_8C_2+D_3^2D_6D_8C_2+D_4^2D_7D_9C_2+D_1D_5D_6D_9+D_1D_5D_7D_8+D_2D_5D_7D_9+D_2D_9C_1C_3}{D_1^2D_7D_9C_2+D_3^2D_7D_9C_2}\right],$$

$$N=\gamma^6\left[\frac{-D_2D_8C_1C_3+D_1D_2D_6D_8C_2+2D_3D_4D_6D_8C_2-D_5D_6D_{10}+D_4^2D_6D_9a_2+D_1D_5D_6D_8+D_2D_5D_6D_9+D_2D_5D_7D_8+D_4^2D_7D_8C_2}{D_1^2D_7D_9C_2+D_3^2D_7D_9C_2}\right],$$

$$O=\gamma^8\left[\frac{D_4^2D_6D_8C_2+D_2D_5D_6D_8}{D_1^2D_7D_9C_2+D_3^2D_7D_9C_2}\right],$$

Eq. (36) with complex coefficient is biquadratic in v^2. The roots are complex in nature so the propagation velocity is $v_n = v_{nR} + v_{nI}$ where in the subscript R and I represent the real and imaginary components respectively.

4. Results and discussion

This section consists of further three subsections namely to describe (i) Reflection phenomenon, (ii) energy partition and (iii) graphical illustrations.

4.1 Reflection phenomenon

In this section, we will examine the reflection phenomena that occurs when a longitudinal wave moving with velocity v_1 making an angle Φ_0 with normal. The incoming coupled longitudinal wave generates four reflected coupled plane waves with speeds V_1, V_2, V_3 and V_4 at angles Φ_1, Φ_2, Φ_3 and Φ_4 respectively, with the normal.

The incident and reflected waves, may be expressed as:

$$(q, \Psi, \theta, \Phi_3) = (1, \eta_1, \tau_1, \xi_1) A_0 M_0^- + \sum_{n=1}^{4} (1, \eta_n, \tau_n, \xi_n) A_n M_n^+, \tag{37}$$

where corresponding to amplitude A_0, the phase factor at angle Φ_0 of the incident wave is

$$M_0^- = \exp[\iota k_1(-x\cos\Phi_0 + y\sin\Phi_0) - \iota\gamma_1 t], \tag{38}$$

and with the amplitude A_n, the phase factor at angle Φ_0 of the reflected wave is

$$M_n^- = exp[\iota k_n(x\ \cos\Phi_n + y\ \sin\Phi_n) - \iota\gamma_n t], \quad n = 1, \ldots, 4. \tag{39}$$

The coupling parameters η_n, τ_n, and ξ_n are given below

$$\eta_n = \frac{C_5 k_n^4 + C_6 k_n^2 + C_7}{C_8 k_n^4 + C_9 k_n^2 + C_{10}}, \quad \tau_n = \frac{B_{10} k_n^2}{k_n^2 B_8 + B_9}, \quad \xi_n = -\frac{a_8 k_n^2 \eta_n}{B_7 + B_6 k_n^2} \tag{40}$$

The following are the boundary conditions defined at the free surface:

Normal stress, tangential stress, coupled stress, and temperature is zero at free surface

$$\sigma_{xx}(0, y, t) = 0, \ \sigma_{xy}(0, y, t) = 0, \ m_{xz}(0, y, t) = 0, \ \theta(0, y, t) = 0. \tag{41}$$

If and only if $\gamma = \gamma_1 = \gamma_3 = \gamma_2 = \gamma_4$, and Snell's law holds, then these boundary conditions are identically satisfied.

$$k_1 sin\Phi_0 = k_2 sin\Phi_2 = k_1 sin\Phi_1 = k_3 sin\Phi_3 = k_4 sin\Phi_4. \tag{42}$$

It is also known as (modified Snell's law).

$$\frac{sin\Phi_0}{V_1} = \frac{sin\Phi_2}{V_2} = \frac{sin\Phi_1}{V_1} = \frac{sin\Phi_3}{V_3} = \frac{sin\Phi_4}{V_4} \tag{43}$$

We observe from Snell's law (42) that $\Phi_0 = \Phi_1$, the additional reflection angles are dependent on phase velocities $V_1, V_3 . V_2$, and V_4.

Using Eq. (37) into boundary conditions (38)-(41), we get the four nonhomogeneous system of equations.

$$\sum_{m=1}^{4} b_{nm} X_m = Y_n, \tag{44}$$

Where $X_m = \frac{A_m}{A_0}, (m = 1, \ldots, 4)$ indicate reflection coefficients (ratio of the reflected wave's amplitude to the incident wave's amplitude). The expressions for b_{nm}, and Y_n are as follows:

$$b_1 m = [C_6 + (1 - C_6)cos^2\Phi_m + C_7 \eta_m cos\Phi_m sin\Phi_m] k_m^2 + \tau_m,$$

$$b_2m = \left[C_7 cos\Phi_m sin\Phi_m - \left(\frac{1}{C_2} cos^2\Phi_m - C_8 sin^2\Phi_m\right)\eta_m\right]k_m^2 + C_9\xi_m,$$
$$b_3m = \tau_m b_4 m = k_m\xi_m cos\Phi_m, (m = 1,2,3,4).$$
$$Y_1 = -[C_6 + (1 - C_6)cos^2\Phi_0 + C_7\eta_1 cos\Phi_0 sin\Phi_0]k_1^2 + \tau_1,$$
$$Y_2 = \left[C_7 cos\Phi_0 sin\Phi_0 + \left(\frac{1}{C_2} cos^2\Phi_2 - C_8 sin^2\Phi_0\right)\eta_1\right]k_1^2 + C_9\xi_1,$$
$$Y_3 = -b_{31}, Y_4 = b_{41}.$$

4.2 Energy partition

The energy partitioning is used to verify the analytical expression of amplitude ratios. The energy flux is denoted by P can be calculated. The energy transmission is given by

$$P = \sigma_{xx}\frac{\partial u}{\partial t} + \sigma_{xy}\frac{\partial v}{\partial t} + m_{xz}\frac{\partial \Phi_3}{\partial t}. \tag{45}$$

The average energy transmission per unit surface area per unit time is symbolized by P, which stands for the time average of P over a period. The energy ratios S_n of the different reflected waves are determined by dividing the energy of the incident wave by the energy of the reflected coupled waves.

For reflected waves, the energy ratios S_n are defined as below:

$$S_n = \frac{P_n}{P_0}, (n = 1,2,3,4), \tag{46}$$

Where

$$S_n = \frac{P_n}{P_0}, (n = 1,2,3,4), \tag{47}$$

$$P_n = \begin{bmatrix} -(C6 + (1 - C6)cos^2\Phi_n + C_7\eta_n cos\Phi_n sin\Phi_n + \frac{\tau_n}{k_n^2})cos\Phi_n + \eta_n sin\Phi_n \\ + \left\{C_7 cos\Phi_n sin\Phi_n - \left(\frac{cos^2\Phi_n}{C_2} - C_8 sin^2\Phi_n\right)\eta_n = \frac{C_9\xi_n}{k_n^2}\right\}(\eta_n cos\theta_n - sin\theta_n) \\ + \frac{d_1\xi_n^2}{k_n^2}cos\theta_n \end{bmatrix}\omega_1 k_1^3 A_n^2,$$

$$(n = 1,2,3,4),$$

$$P_0 = \begin{bmatrix} -(C6 + (1 - C6)cos^2\Phi_0 - C_7\eta_1 cos\Phi_0 sin\Phi_n + \frac{\tau_n}{k_1^2})cos\Phi_0 + \eta_1 sin\Phi_0 \\ + \left\{C_7 cos\Phi_0 sin\Phi_0 - \left(\frac{cos^2\Phi_0}{C_2} - C_8 sin^2\Phi_0\right)\eta_1 = \frac{C_9\xi_1}{k_1^2}\right\}(sin\Phi_0 + \eta_1 cos\Phi_0) \\ + \frac{d_1\xi_1^2}{k_1^2}cos\Phi_0 \end{bmatrix}\omega_1 k_1^3 A_0^2.$$

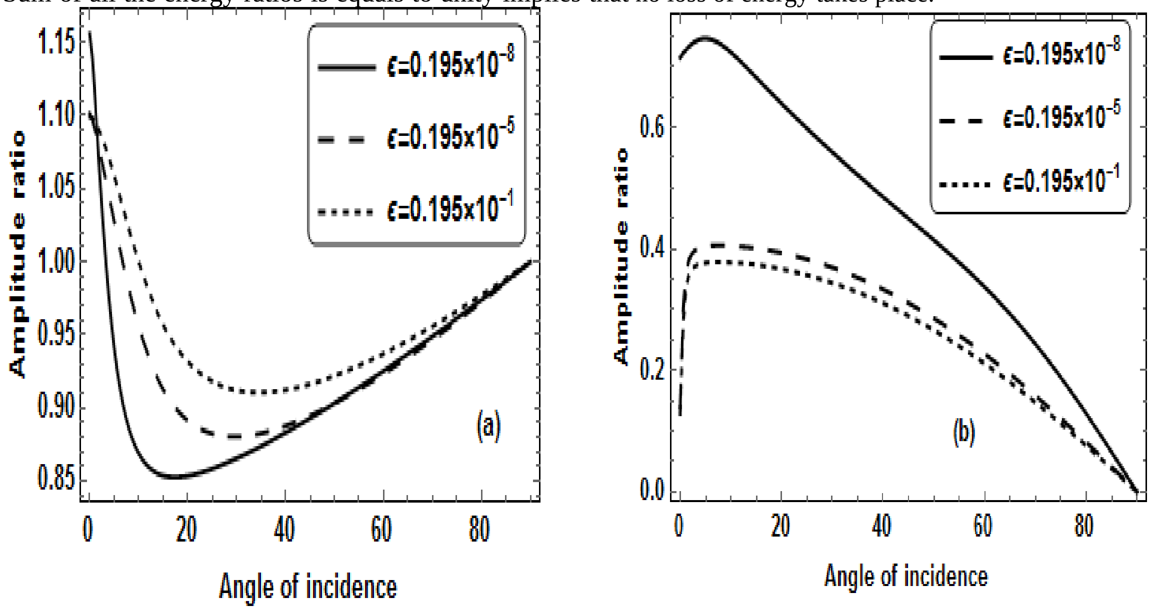
4.3 Graphical illustrations

For graphical simulation, the following numerical values of various physical constants as given in [38, 39] are utilized.

$K^* = 2.510Wm^{-1}K^{-1}, j = 0.2 \times 10^{-19}m^2,\ \tau_0 = 0.02s,\ K = 1.0 \times 10^{10}kgm^{-1}s^{-2},$
$r = 0.779 \times 10^9 kgms^{-2}, a_t = 2.36 \times 10^{-5}K^{-1}, T_0 = 2p93K, \alpha = 2.0, e_0\, 0.39, a_d = 0.5 \times 10^{-9}m, C_E = 9.623 \times 10^2 JKg^{-1}k^{-1}, \epsilon = 0.195 \times 10^{-9}m, \lambda = 9.4 \times 10^{10}kgm^{-2}s^{-2},$

$\rho = 1.74 \times 10^3 kgm^{-3}, \mu = 4.0 \times 10^{10}kgm^{-1}s^{-2}.$

We have find the energy ratios and reflection coefficients corresponding to the angle of incidence. Fig. (2) analyze the effect of nonlocal material. The amplitude ratio of $|Z_2|$ to $|Z_4|$ shows decrement as increasing the values ϵ of while the amplitude ratio of $|Z_1|$ depicts increment. For all values of ϵ, the reflection coefficients shows the regular pattern of fluctuation. Fig. (3) shows the behavior of angular velocity Ω. It demonstrates that $|Z_1|$ to $|Z_3|$ decreases as increasing the values of Ω, while $|Z_4|$ shows increment before $\theta = 20^o$ after $\theta = 20^o$ no noticeable change is observed. The pattern of fluctuation is apparent in all reflection coefficients. Fig. (4) demonstrates the effect of micropolar parameter. The amplitude ratio of $|Z_1|$ shows increment as increasing the values of K while the amplitude ratio of $|Z_2|$ and $|Z_3|$ depicts decrement. No noticeable impact is seen on $|Z_4|$. Fig. (5) demonstrates the behavior of temperature dependence α_2. The profile of amplitude ratios Z_i (i= 1, 2, 3, 4) are decreasing by increasing in the temperature dependence parameter α_2. Fig. (6) demonstrates energy ratios S_n (n = 1, 2, 3, 4) of several waves and their sum at speed V_1. I and II shows value of sum and S_1, both are nearly equals to 1. The curve of energy ratios III, IV and V are shown in the Fig. (6) after multiplying by 10^5, 10^3 and 10^3 respectively. The amplitude ratio of $|Z_1|$, $|Z_2|$ and $|Z_3|$ are very small, that's why the profile of energy ratios is also very small. Sum of all the energy ratios is equals to unity implies that no loss of energy takes place.



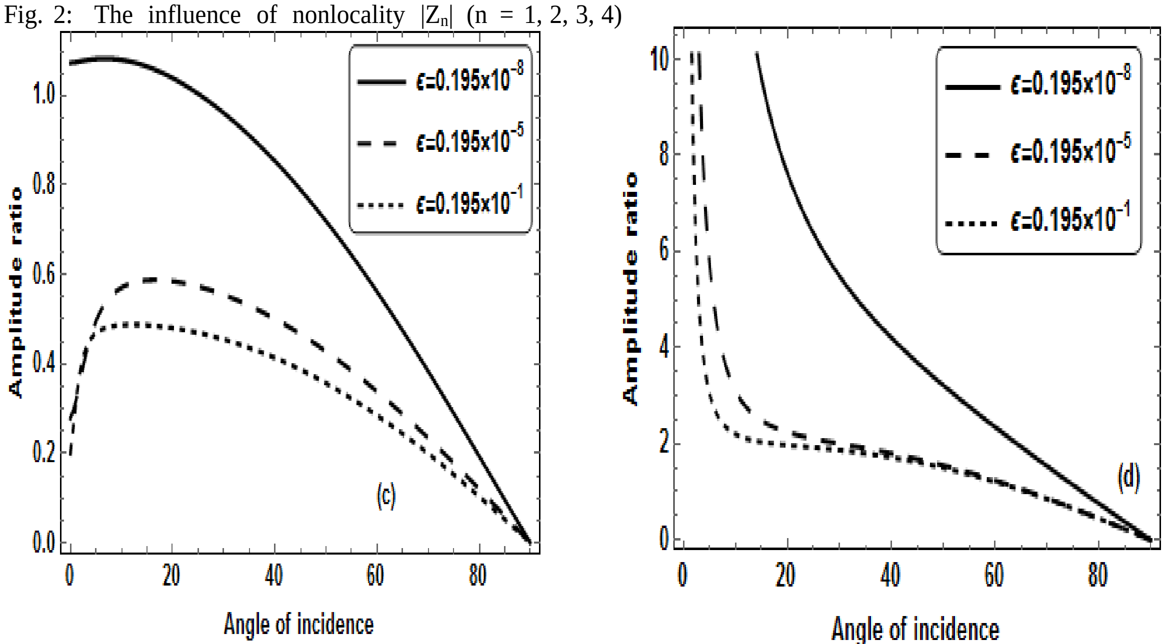


Fig. 2: The influence of nonlocality $|Z_n|$ (n = 1, 2, 3, 4) with angle of incidence.

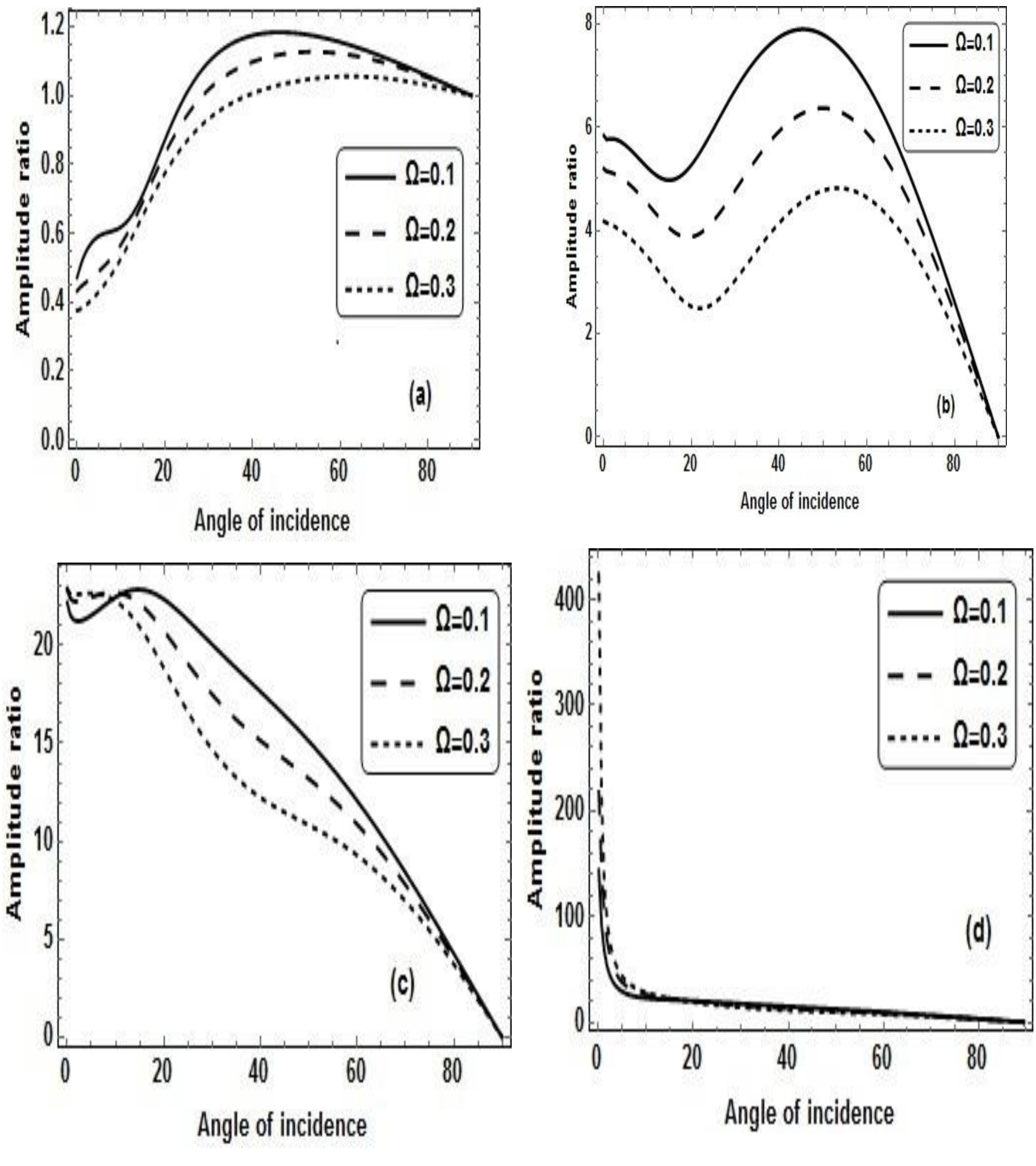


Fig. 3: The influence of rotation $|Z_n|$ ($n = 1, 2, 3, 4$) with angle of incidence.

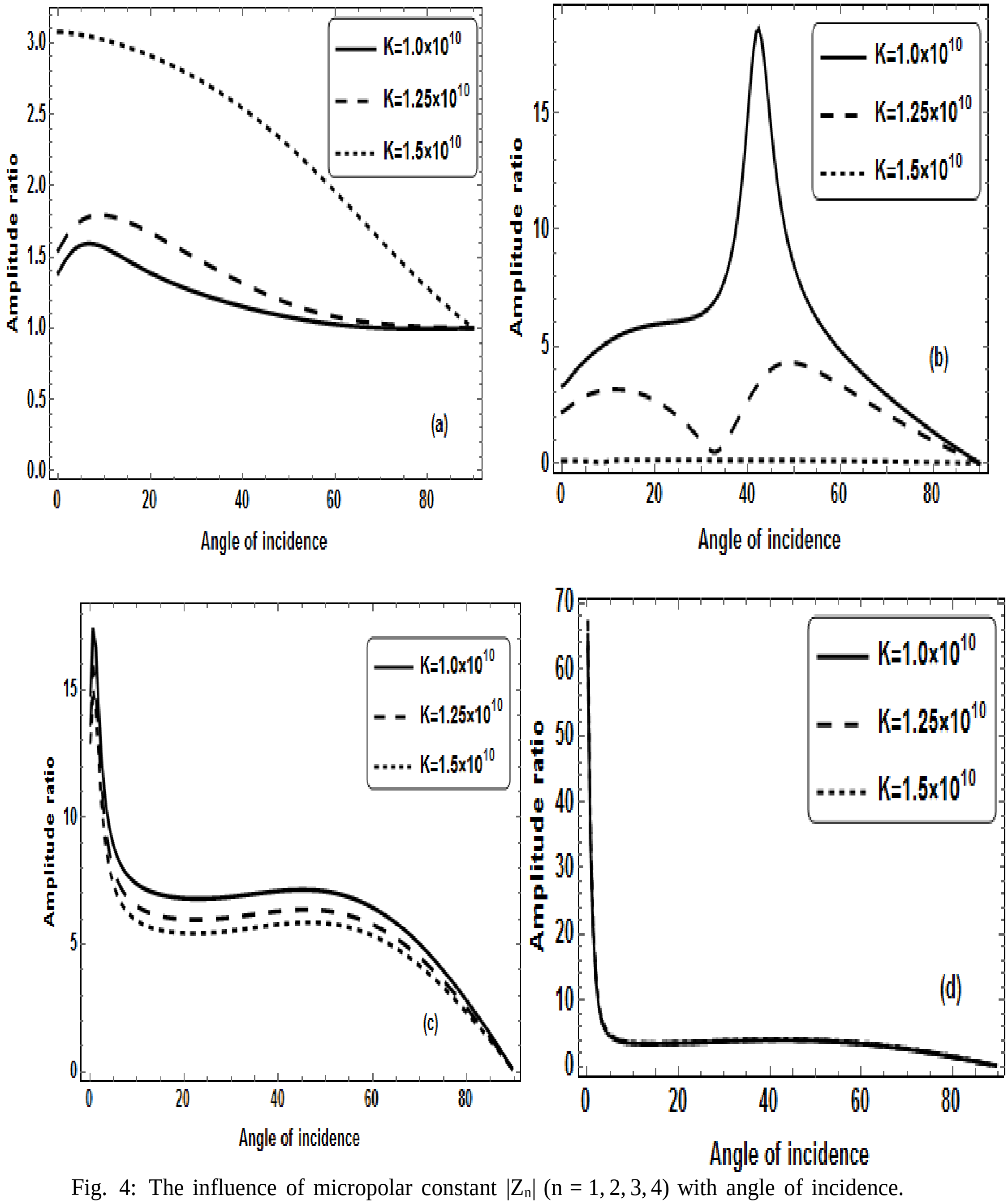


Fig. 4: The influence of micropolar constant $|Z_n|$ (n = 1, 2, 3, 4) with angle of incidence.

1. Conclusion

Above examination concludes that the angular velocity decreases the amplitude ratios of $|Z_1|$, $|Z_2|$ and $|Z_3|$ while increase the amplitude ratio of $|Z_4|$. All amplitude ratios are significantly impacted by the micropolar constant. The temperature dependent constant decreases the amplitude ratios of $|Z_1|$, $|Z_2|$ and $|Z_3|$ while increase the amplitude ratio of $|Z_4|$. The majority of incident energy waves travel along the reflected wave with reflection coefficient $|Z_1|$. According to the above analysis the fastest wave is couple longitudinal wave. Above investigation propose that there is no energy loss as the sum of all the energy ratios is equals to unity. This demonstrates that lack of energy dissipation at free surface during reflection phenomenon. Hence at each angle of incidence law of energy balance has been confirmed.

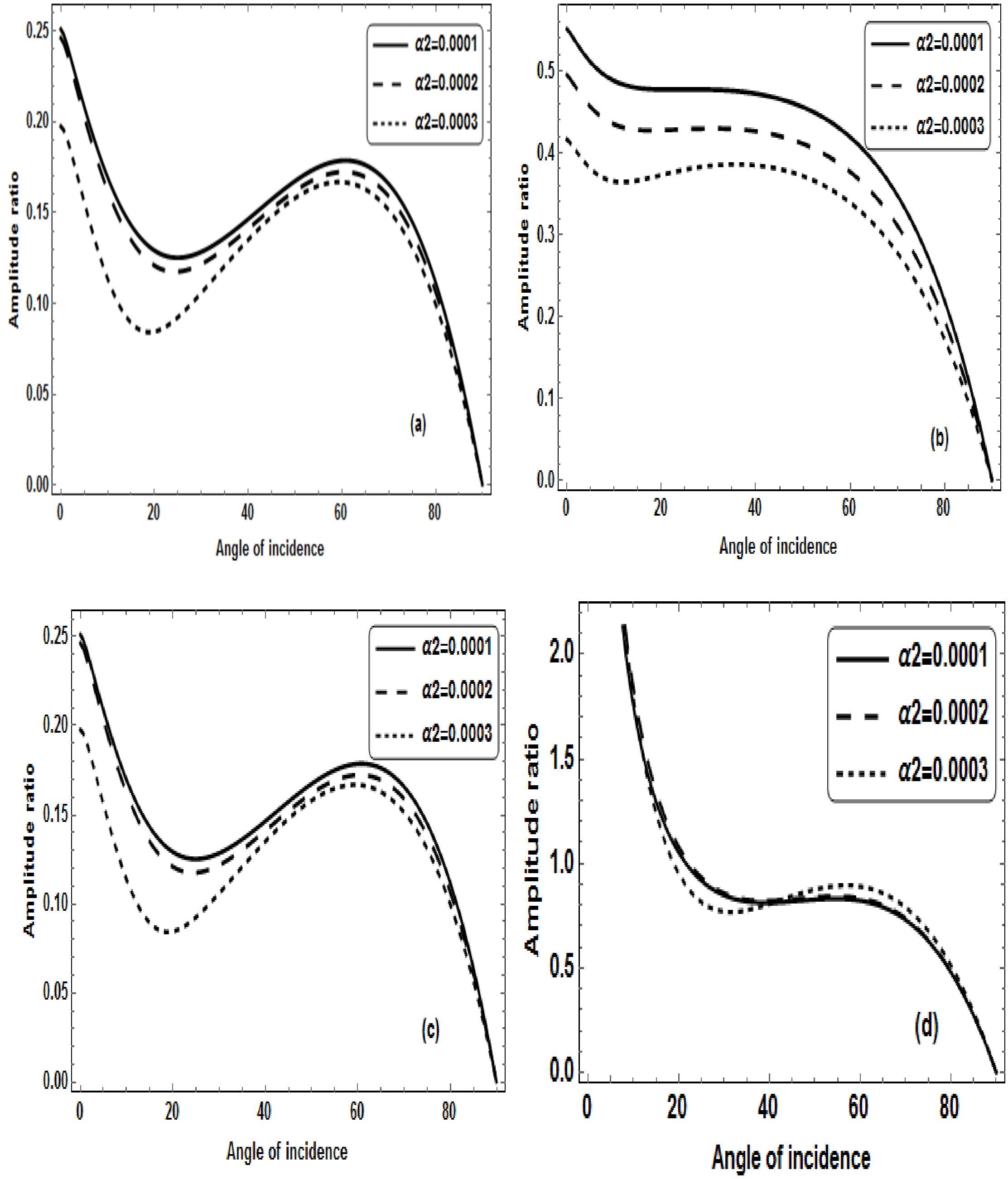


Fig. 5: The influence of temperature dependent constant $|Z_n|$ ($n = 1, 2, 3, 4$) with angle ofincidence.

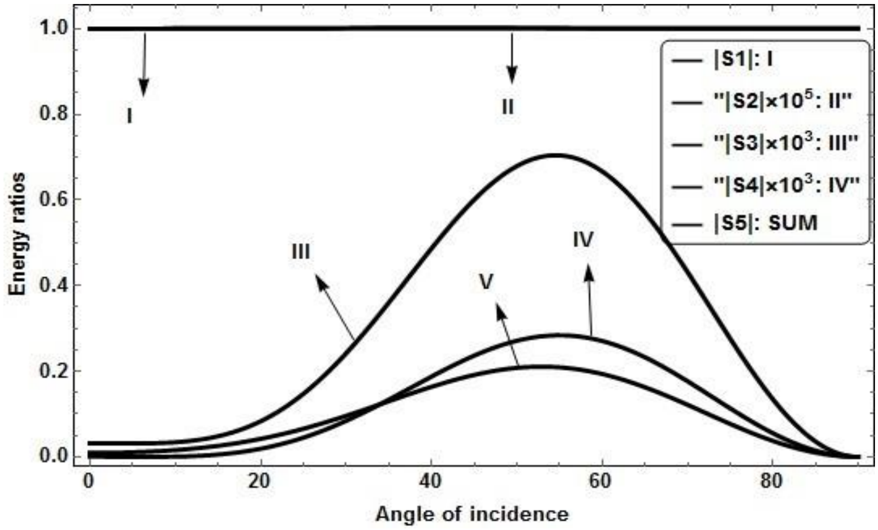


Fig. 6: Energy ratios profile against angle of incidence.

References

[1] J. Cooper, Henry F., E. L. Reiss, Reflection of Plane Viscoelastic Waves from Plane Boundaries, *The Journal of the Acoustical Society of America*, Vol. 39, No. 6, pp. 1133-1138, 1966.

[2] A. Singh, S. Guha, Reflection of plane waves from the surface of a piezothermoelastic fiber-reinforced composite half-space, *Mechanics of Advanced Materials and Structures*, Vol. 28, pp. 1-13, 03/20, 2020.

[3] C.-H. Lin, V. W. Lee, M. D. Trifunac, The reflection of plane waves in a poroelastic half-space saturated with inviscid fluid, *Soil Dynamics and Earthquake Engineering*, Vol. 25, No. 3, pp. 205-223, 2005/04/01/, 2005.

[4] M. I. A. Othman, Y. Song, Reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli, *Applied Mathematical Modelling*, Vol. 32, No. 4, pp. 483-500, 2008/04/01/, 2008.

[5] S. S. Sheoran, K. K. Kumar, S. and Deswal, Fractional order thermo-viscoelastic problem with temperature dependent modulus of elasticity, *Mechanics of Advanced Materials and Structures*, Vol. 23, No. 4, pp. 407-414, 2016/04/02, 2016.

[6] J.-T. Ma, T.-H. and He, Investigation on the dynamic responses of a generalized thermoelastic problem with variable properties and nonlocal effect, *Journal of Thermal Stresses*, Vol. 42, No. 4, pp. 426-439, 2019/04/03, 2019.

[7] A. A. Khan, A. Zaman, S. Yaseen, Impact of two relaxation times on thermal, P and SV waves at interface with magnetic field and temperature dependent elastic moduli, *Results in Physics*, Vol. 8, pp. 324-335, 2018/03/01/, 2018.

[8] S. Abo-Dahab, A. Abd-Alla, M. Othman, Reflection of plane waves on generalized thermoelastic medium under effect of temperature dependent properties and initial stress with three-phase-lag model, *Mechanics Based Design of Structures and Machines*, Vol. 50, pp. 1-14, 04/24, 2020.

[9] P. Ailawalia, V. Sharma, Plane waves in temperature dependent hygrothermoelastic medium under the influence of gravity—Analytical and numerical approach, *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 103, No. 10, pp. e202200293, 2023.

[10] A. C. Eringen, Theory of Micropolar Fluids, *Journal of Mathematics and Mechanics*, Vol. 16, No. 1, pp. 1-18, 1966.

[11] A. C. Eringen, D. Edelen, On nonlocal elasticity, *International journal of engineering science*, Vol. 10, No. 3, pp. 233-248, 1972.

[12] A. C. Eringen, On nonlocal fluid mechanics, *International Journal of Engineering Science*, Vol. 10, No. 6, pp. 561-575, 1972/06/01/, 1972.

[13] A. C. Eringen, Nonlocal Continuum Theory of Liquid Crystals, *Molecular Crystals and Liquid Crystals*, Vol. 75, No. 1, pp. 321-343, 1981/10/01, 1981.

[14] R. Poonia, K. Kalkal, S. Deswal, Reflection of plane waves in a rotating nonlocal fiber-reinforced transversely isotropic thermoelastic medium, *Journal of Thermal Stresses,* Vol. 46, 02/16, 2023.

[15] P. Vinh, T. Tran, Harmonic plane waves in isotropic micropolar medium based on two-parameter nonlocal theory, *Archive of Applied Mechanics,* Vol. 93, pp. 1-19, 05/24, 2023.

[16] H. W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, *Journal of the Mechanics and Physics of Solids,* Vol. 15, No. 5, pp. 299-309, 1967.

[17] A. E. Green, K. Lindsay, Thermoelasticity, *Journal of elasticity,* Vol. 2, No. 1, pp. 1-7, 1972.

[18] D. Y. Tzou, A Unified Field Approach for Heat Conduction From Macro- to Micro-Scales, *Journal of Heat Transfer-transactions of The Asme,* Vol. 117, pp. 8-16, 1995.

[19] D. Chandrasekharaiah, Hyperbolic thermoelasticity: a review of recent literature, 1998.

[20] A. A. Khan, S. Tanveer, Transmission and reflection of SV waves at micropolar solid–liquid interface with dual-phase lag theory, *Indian Journal of Physics,* Vol. 96, No. 4, pp. 1153-1165, 2022/03/01, 2022.

[21] M. Marin, R. Ellahi, A. Chirilă, On solutions of Saint-Venant's problem for elastic dipolar bodies with voids, *Carpathian Journal of Mathematics,* Vol. 33, No. 2, pp. 219-232, 2017.

[22] M. Marin, S. Vlase, R. Ellahi, M. M. Bhatti, On the Partition of Energies for the Backward in Time Problem of Thermoelastic Materials with a Dipolar Structure, *Symmetry,* Vol. 11, No. 7, pp. 863, 2019.

[23] M. Fekry, M. I. A. Othman, Three-phase lag model for thermal conductivity of a thermo-viscoelastic porous medium, *Chinese Journal of Physics,* Vol. 92, pp. 1253-1266, 2024/12/01/, 2024.

[24] M. I. A. Othman, B. Singh, The effect of rotation on generalized micropolar thermoelasticity for a half-space under five theories, *International Journal of Solids and Structures,* Vol. 44, No. 9, pp. 2748-2762, 2007/05/01/, 2007.

[25] M. Marin, E. Rahmat, V. Sorin, M. M. and Bhatti, On the decay of exponential type for the solutions in a dipolar elastic body, *Journal of Taibah University for Science,* Vol. 14, No. 1, pp. 534-540, 2020/01/01, 2020.

[26] M. Marin, A. Öchsner, R. Ellahi, M. M. Bhatti, A semigroup of contractions in elasticity of porous bodies, *Continuum Mechanics and Thermodynamics,* Vol. 33, No. 5, pp. 2027-2037, 2021/09/01, 2021.

[27] S. M. Said, Influence of gravitational on a rotating nonlocal thermoelastic medium with thermal variable conductivity, *Journal of Computational and Applied Mathematics,* Vol. 454, pp. 116180, 2025/01/15/, 2025.

[28] M. M. Bhatti, R. Ellahi, S. Sait, R. Ullah, Exact solitary wave solutions of time fractional nonlinear evolution models: a hybrid analytic approach, *Bulletin of the Transilvania University of Brasov. Series III: Mathematics and Computer Science*, pp. 83-98, 09/03, 2024.

[29] I. Kaur, P. Lata, K. Singh, Reflection of plane harmonic wave in rotating media with fractional order heat transfer and two temperature, *Partial Differential Equations in Applied Mathematics,* Vol. 4, pp. 100049, 2021/12/01/, 2021.

[30] R. Ellahi, A. Zeeshan, F. Hussain, T. Abbas, Two-Phase Couette Flow of Couple Stress Fluid with Temperature Dependent Viscosity Thermally Affected by Magnetized Moving Surface, *Symmetry,* Vol. 11, No. 5, pp. 647, 2019.

[31] R. Ellahi, The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe: Analytical solutions, *Applied Mathematical Modelling,* Vol. 37, No. 3, pp. 1451-1467, 2013/02/01/, 2013.

[32] A. Jafarimoghaddam, M. Turkyilmazoglu, A. V. Roşca, I. Pop, Complete theory of the elastic wall jet: A new flow geometry with revisited two-phase nanofluids, *European Journal of Mechanics - B/Fluids,* Vol. 86, pp. 25-36, 2021/03/01/, 2021.

[33] F. He, A. A. Delouei, R. Ellahi, S. Z. Alamri, A. Emamian, S. Ghorbani, Unsteady temperature distribution in a cylinder made of functionally graded materials under circumferentially-varying convective heat transfer boundary conditions, *Zeitschrift für Naturforschung A,* Vol. 78, No. 10, pp. 893-906, 2023.

[34] J. Mehboob, R. Ellahi, S. M. Sait, N. S. Akbar, Optimizing bioconvective heat transfer with MHD Eyring–Powell nanofluids containing motile microorganisms with viscosity variability and porous media in ciliated microchannels, *International Journal of Numerical Methods for Heat & Fluid Flow,* Vol. 35, No. 2, pp. 825-846, 2025.

[35] U. Khan, R. Ellahi, R. A. Khan, S. T. Mohyud-Din, Extracting new solitary wave solutions of Benny–Luke equation and Phi-4 equation of fractional order by using (G′/G)-expansion method, *Optical and Quantum Electronics,* Vol. 49, pp. 1-14, 2017.

[36] Rahmatullah, R. Ellahi, S. T. Mohyud-Din, U. Khan, Exact traveling wave solutions of fractional order Boussinesq-like equations by applying Exp-function method, *Results in Physics,* Vol. 8, pp. 114-120, 2018/03/01/, 2018.

[37] R. Ellahi, S. Alamri, A. Majeed, Effects of MHD and slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation, *Journal of Taibah University for Science*, Vol. 12, pp. 1-7, 06/14, 2018.

[38] A. C. Eringen, Plane waves in nonlocal micropolar elasticity, *International Journal of Engineering Science,* Vol. 22, No. 8, pp. 1113-1121, 1984/01/01/, 1984.

[39] R. Poonia, S. Deswal, K. K. Kalkal, Propagation of plane waves in a nonlocal transversely isotropic thermoelastic medium with voids and rotation, *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik,* Vol. 103, No. 9, pp. e202200493, 2023.