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RESEARCH PAPER



Influence of porosity on thermal environment of functionally graded square (FGSP) and rectangular plates (FGRP)

Merdaci Slimane ^{*a*,*}, Hadj Mostefa Adda ^{*b*}, Beldjelili Youcef ^{*a*}, Bouchafa Ali ^{*a*}, Osama M.E.S. Khayal ^{*c*}

^a Civil Engineering and Public Works Department, Faculty of Technology, Djillali Liabes University, Sidi Bel Abbes 22000, Algeria

^b Department of Civil Engineering, Faculty of Science and Technology, University of Rélizane, Algeria

^c Department of Mechanical Engineering, Faculty of Engineering and Technology, Nile Valley University, Atbara,

Sudan

Abstract

This study examines the influence of porosity on the thermo-mechanical environment of functionally graded square (FGSP) and rectangular plates (FGRP). The current theory suggests that only four unknown functions are involved, compared to five in other shear deformation theories, and the boundary conditions on the upper and lower surfaces of the plate do not require shear correction factors. It is assumed that the material properties of this plate (FGSP) and (FGRP) vary continuously over the thickness of the plate according to a power law function in terms of the volume fractions of the constituents. The porosity distribution of the plates (FGSP) and (FGRP) is uniform over their cross-sections. Using the concept of virtual work, the equilibrium equations of a plate (FGSP) and (FGRP) are derived. Numerical results for the rectangular plates have been provided and compared to those found in the literature. The impact of aspect ratios and porosity volume on the bending and thermo-mechanical environment properties of the square (FGSP) and rectangular plates (FGRP) is examined.

Keywords: Functionally Graded(FG), thermo-mechanical environment, square plates (SP), rectangular plates (RP), porosity;

1. Introduction

A functionally graded material (FGM) is a heterogeneous composite material characterized by a continuous variation of mechanical properties from one point to another. This material is created by combining two or more substances with a graded distribution of their volume fractions. As discussed by Koizumi et al.[1], this type of material is suitable for various applications, including thermal barrier coatings for ceramic engines, electrical equipment, energy transformation, biomedical engineering, and optics, as presented in Refs [2-11].

Many experiments have been conducted to examine the vibration of functionally graded surfaces. Ferreira et al. [12] presented a three-dimensional specific solution for free and forced vibrations of simply supported, dynamically

E-mail address:slimanem2016@gmail.com

graded rectangular plates. Qian et al.[13] used a global collocation system along with first and third-order shear deformation plate theories to study the free vibrations of these plates. Recent experiments have utilized higher-order shear and standard deformable plate theory to investigate the static deformations and free vibration of dense rectangular plates and simply supported plates with dynamically graded elastic properties [14-17]. Matsunaga [18] presented a three-dimensional vibration approach for functionally graded rectangular plates. Rezaei et al. [19] analyzed natural frequencies and buckling stresses of plates made of dynamically graded materials, considering transverse shear, normal deformations, and rotational inertia effects. The impact of porosity on the natural frequency of thick porous cellular plates was studied by Askari et al. [20], Merdaci et al. [21]. Benachour et al. [22] investigated the impact of porosity on FGM sandwich plates using the principle of higher-order shear deformation. Recently, wave propagation in porosity-containing FG plates has been studied using various higher-order theories, as presented by Ebrahimi et al. [23] and Zhao et al. [24]. Analytical solutions for the vibration of FGM porosity plates were proposed by Merdaci et al. [25-28] and Zhu et al. [29].

In the manufacturing of FGMs, micro-porosities or voids may occur during the sintering phase due to the significant disparity in the solidification temperatures of the constituent materials. This phenomenon was studied by Wattanasakulpong et al. [30]. Merdaci [31] investigated porosities created by a sequential multi-step infiltration process within FGM specimens. The impact of porosity on the architecture of FGM structures under static loads was analyzed by Zenkour [32], Merdaci et al. [33], and Wattanasakulpong et al. [34]. Rezaei et al. [35] studied the effects of dynamic loads and emphasized that porosities should be taken into account. Consequently, increasing attention has been devoted in recent years to studies focused on the static and dynamic behavior of FGM material structures.

According to the literature, many researchers have discussed various shear deformation theories. These include first-order higher shear deformation theories Whitney et al. [36] and Nguyen et al. [37], third-order shear deformation theory studied by Reddy, [38], and sinusoidal shear deformation theory examined by Zenkour, [39]).

The bending response of FG plates with porosities resting on elastic foundations in a hydrothermal environment has been examined by Zenkour et al. [40]. The impact of an initial defect on the hygro-thermo-mechanical behaviour of FG plates on an elastic foundation was studied by Belkorissat et al. [41]. The thermomechanical bending response of thick FGM plates on Winkler-Pasternak elastic foundations was investigated by Bouderba et al. [42]. Using high-order theory, the response of perfect and imperfect thermomechanical bent rectangular plates made of hybrid ceramic and metal materials (FGP) was examined by Zerrouki et al. [43].

This work models the thermo-mechanical behavior of the bending response of both perfect and imperfect square plates (FGSP) and rectangular plates (FGRP) using a higher-order normal and shear deformation plate theory. The material properties of porous square and rectangular plates are influenced by variations in temperature loads. The analysis of perfect and imperfect square and rectangular plates is derived using the principle of virtual work based on the present theory, incorporating porosity and thermal effects. The effects of the porosity factor and other parameters are thoroughly investigated. The results obtained are compared with those from other studies.

2. Mathematical model

In this study, two types of square plate (FGSP) and rectangular plate (FGRP) models are used to study the influence of porosity. The porosities are assumed to be uniformly distributed over the plate section (FGSP & FGRP), which is proposed as follows:

$$E(zz) = E_{mm} - \frac{P(\chi)}{2} \left(E_{cc} + E_{mm} \right) + \left(E_{cc} - E_{mm} \right) \left(\frac{1}{2} + \frac{z}{h} \right)^{k}$$
(1)

Where: E_{cc} and E_{mm} are the corresponding properties of ceramics and metal, respectively. k is the volume fraction exponent that takes values greater than or equal to zero. The above power-law assumption reflects a simple rule of mixtures used to obtain the effective properties of the ceramic-metal plate.

The displacement field of a material point situated at (xx, yy, zz) in the FG plate is represented as:

$$u(xx, yy, zz) = u_0 - z\varphi_1 + f(zz)\varphi_2,$$

$$v(xx, yy, zz) = v_0 - z\varphi_3 + f(zz)\varphi_4,$$

$$w(xx, yy, zz) = w_{bb} + w_{ss}$$
(2)



Rectangular plate (FGRP)

Square plate (FGSP)

Fig. 1. Configuration of (FGSP) and (FGRP) porous plate.

where:

$$\varphi_1 = \frac{\partial w_{bb}}{\partial x}, \ \varphi_2 = \frac{\partial w_{ss}}{\partial x}, \ \varphi_3 = \frac{\partial w_{bb}}{\partial y}, \ \varphi_4 = \frac{\partial w_{ss}}{\partial y}.$$
 (3)

In this study the new function f(zz) is presented in the following form:

$$f(zz) = z - \frac{z}{2} \left(\frac{h^2}{4} - \frac{z^2}{3} \right), \quad g(zz) = 1 - f'(zz)$$
(4)

The static equations can be obtained by using the principle of virtual displacements. It can be stated in its analytical form as:

$$\int (\delta U + \delta V) = 0 \tag{5}$$

Where δU is the variation of the strain energy; δV is the variation of the potential energy. Can be seen that the displacement field in Eq. (2) uses only four unknowns (u_0 , v_0 , w_{ss} and w_{bb}). Linear deformations can be obtained in the form of:

$$\begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{cases} = \begin{cases} \frac{\partial u_{0}}{\partial x} \\ \frac{\partial v_{0}}{\partial y} \\ \frac{\partial u_{0}}{\partial y} + \frac{\partial v_{0}}{\partial x} \end{cases} - z \begin{cases} \frac{\partial^{2} w_{b}}{\partial x^{2}} \\ \frac{\partial^{2} w_{b}}{\partial y^{2}} \\ 2 \frac{\partial^{2} w_{b}}{\partial x \partial y} \end{cases} + f(z) \begin{cases} \frac{\partial^{2} w_{s}}{\partial x^{2}} \\ \frac{\partial^{2} w_{s}}{\partial y^{2}} \\ 2 \frac{\partial^{2} w_{s}}{\partial x \partial y} \end{cases}, \quad \begin{cases} \gamma_{yz} \\ \gamma_{xz} \end{cases} = \frac{df(z)}{dz} \begin{cases} \frac{\partial w_{s}}{\partial y} \\ \frac{\partial w_{s}}{\partial x} \end{cases}, \quad \varepsilon_{z} = 0$$
(6)

Eq. (7) may be used to write the component relations of a FG plate.

$$\begin{cases} \sigma_{xx} \\ \sigma_{yy} \end{cases} = \frac{E(z)}{1 - v^2} \begin{bmatrix} 1 & v \\ v & 1 \end{bmatrix} \begin{cases} \varepsilon_{xx} - \alpha \ \Delta T \\ \varepsilon_{yy} - \alpha \ \Delta T \end{cases},$$
(7a)

$$\left\{\tau_{xy_{,1}}, \tau_{yz_{,2}}, \tau_{zx_{,3}}\right\} = \frac{E(z)}{2(1+\nu)} \left\{\gamma_{xy_{,1}}, \gamma_{yz_{,2}}, \gamma_{zx_{,3}}\right\}$$
(7b)

where $(\sigma_{xx}, \sigma_{yy}, \tau_{xy_{1}}, \tau_{yz_{2}}, \tau_{zx_{3}})$ and $(\mathcal{E}_{xx}, \mathcal{E}_{yy}, \gamma_{xy_{1}}, \gamma_{yz_{2}}, \gamma_{zx_{3}})$ are the stress and strain components, respectively. Where $\Delta T = T - T_0$ in which T_0 is the reference temperature.

The distribution of applied temperature T(xx, yy, zz) through the thickness of the FG plate are:

$$T(xx, yy, zz) = T_{11}(xx, yy) + \frac{zz}{h}T_{22}(xx, yy) + \frac{\psi(zz)}{h}T_{33}(xx, yy)$$
(8)

Where T_{11} , T_{22} , and T_{33} are thermal loads.

3. Numerical results and discussions

The results were presented using dimensionless expressions for deflections and stresses.

$$\overline{w} = \frac{10^2 hD}{a^4 q_0} w \left(\frac{a}{2}, \frac{b}{2}\right), \quad \overline{\sigma}_x = \frac{1}{a^2 q_0} \sigma_x \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right), \quad \overline{\tau}_{xy} = \frac{1}{10q_0} \tau_{xy} \left(0, 0, \frac{-h}{3}\right), \quad \overline{\tau}_{xz} = -\frac{h}{10q_0} \tau_{xz} \left(0, \frac{b}{2}, 0\right) \tag{9}$$

Where $D = \frac{h^3 E_c}{12(1-v^2)}$ and Thickness coordinates: $\overline{z} = z / h$.

The materials considered were Titanium and Zirconia. For all figures, Young's modulus, Poisson's ratio, and thermal expansion coefficients are taken as 66.2 GPa, 0.33, and $10.3 \times (10^{-6})^{\circ}$ C) for Titanium (Ti-6Al-4V), and 117 GPa, 0.33, and $7.11 \times (10^{-6})^{\circ}$ C) for Zirconia (ZrO₂), respectively. The values for the porosity parameter were taken as 0, 0.1, 0.2, and 0.3.

The current deflection results for the perfect FG square and rectangular plate are very similar to the results of other theories (Zenkour et al.[40], and Belkorissat et al.[41]) present in the Table 1. It is shown that the deflections values and rise with increasing heat loads (t₂₂).

Theory	$t_{22} = 0$		$t_{22} = 100$	
Theory	a=b	3a=b	a=b	3a=b
Zenkour et al.[40]	0.3729	1.1820	7.2408	13.4032
Belkorissat et al.[41]	0.3781	1.1958	6.9668	13.0560
Present	0,37959	1.19845	6.96730	13.0577

 Table 1: The deflection of Perfect square and rectangular plates (a/h =10, q_0=100, k=2 t₃₃=0)

From Table 2, again, the deflection and stresses results of FGRP subjected to a mechanical load compare very well with the theory solutions (FSDT(Whitney et al. [36]), TSDT(Reddy, [38]) and SSDT(Zenkour, [39])) for (FGRP) are consistent, it demonstrates the present model's validity. It is clear that the deflection and stresses increase as the porosity values $P(\chi)$ rises.

Table 2: Comparisons of deflections and stresses	of perfect and imperfect rectangular plate (FGRP).
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Theory	w	σ_{xx}	$\bar{\tau}_{\mathrm{xz,3}}$	$\overline{\tau}_{xy,1}$
Whitney et al. [36]	0,85892	0,51065	0,72949	-0,34377
Reddy [38]	0,85891	0,51545	0,72797	-0,42956
Zenkour [39]	0,85887	0,51362	0,72784	-0,44327
Present (P(χ)=0.0)	0,85889	0,51346	0,72796	-0,42955
Present (P(χ)=0.1)	0,93185	0,513457	0,727954	-0,429549
Present (P(χ)=0.2)	1,01835	0,513459	0,727961	-0,42955

From Table 3, again, the deflection results compare very well with the theory solutions (Bouderba et al. [42]) for the perfect square plate (FGSP) are consistent, it demonstrates the present model's validity. It is clear that the deflection reduces as the side-to-thickness ratio (a/h) rises.

Figure 2. illustrates how the deflection changes for various side-to-thickness ratios and porosity coefficient of perfect and imperfect and with/without a thermal load for square plate (FGSP) and rectangular plate (FGRP). For the (FGSP) and (FGRP), the deflection of the (FGSP) & (FGRP) decreases as the (a/h) increases. When the porosity parameter is increased under a thermal load, deflection decreases for some side-to-thickness ratio values.

For both perfect and imperfect FG plates, as well as with and without thermal loads, we investigate dimensionless deflection variation as a function of the geometric ratio (a/b) for a ratio of equal thickness (a/h = 10) and a power law index (k = 2), as shown in Figure 3. The results show that for both perfect and defective FG plates, the deflection reduces as the aspect ratio rises. We can see in this figure which effects thermal load has on the different aspect ratios (a/b) and which effects porosity has on those ratios. The thermal load has the most significant effects, while the porosity effects are not as significant.

square plate (FGSP) and $(q_0=100, t_{11}=0, t_{22}=t_{33}=10)$							
k	Theory	a/h= 5	a/h=10	a/h=20	a/h=50		
0	Bouderba et al. [42]	4,04970	1,20607	0,49406	0,29463		
	Present	4,05037	1,20606	0,49407	0,29464		
1	Bouderba et al. [42]	4,92170	1,48440	0,62354	0,38238		
	Present	4,92266	1,48447	0,62354	0,38238		
2	Bouderba et al. [42]	5,10720	1,54760	0,65537	0,40539		
	Present	5,10855	1,54764	0,65537	0,40539		
3	Bouderba et al. [42]	5,20060	1,57840	0,67020	0,41571		
	Present	5,20218	1,57851	0,67020	0,41570		
4	Bouderba et al. [42]	5,26690	1,60000	0,68046	0,42274		
	Present	5,26858	1,60015	0,68045	0,42275		
5	Bouderba et al. [42]	5,32010	1,61750	0,68878	0,42850		
	Present	5,32180	1,61754	0,68876	0,42851		

Table 3: Effect of the side-to-thickness ratio (a/h) and the volume fraction exponent on the dimensionless of square plate (FGSP) and ($q_0=100$, $t_{11}=0$, $t_{22}=t_{33}=10$)





Fig 2: Variation of deflection versus (a/h) for rectangular plate (FGRP) & square plate (FGSP).



Fig 3: Variation of deflection versus aspect ratio for plate (FG).

Figure 4. Illustrates how decreasing the dimensionless deflections of both perfect and imperfect rectangular plate (FGRP) and square plate (FGSP) is possible by raising the porosity parameter and thermo-mechanic loading. Furthermore, for the same value of the porosity parameter, the dimensionless deflection in the case of thermal load $(t_{22}=t_{33}=20)$ is larger than that in the cases of $t_{22}=t_{33}=5$, 10 and 15. This is mostly because of where the porosity is and how important thermal loading is.

As can be seen in Figure 5 and 6, rectangular plate (FGRP) and square plate (FGSP) that are perfect or imperfect have tensile stresses on their top surface, compressive stresses on their bottom side, and zero values on their middle axis. In contrast, rectangular plate (FGRP) & square plate (FGSP) that have porosity had the lowest values because plate material characteristics vary. It is evident that as the porosity coefficient rises, so do the shear stresses. A position above the mid-plane of the perfect and imperfect rectangular plate (FGRP) experiences the highest value of stresses.





Fig 4: Variation of deflection versus of the porosity coefficient and different thermo-mechanic loading for rectangular plate (FGRP) & square plate (FGSP).

4. Conclusion

In this study, the analysis of perfect and imperfect functionally graded rectangular plates (FGRP) and square plate (FGSP) in a thermal environment with different qualities, porosity factors, and other characteristics is based on the higher-order shear deformation theory. The results for porous functionally graded rectangular plates (FGRP) and square plate (FGSP), particularly how temperature affects the mechanical response of the material, have been presented. The results also show that thermal parameters and the porosity factor are important in porous rectangular plates (FGRP) and square plate (FGSP). The present method is quite effective for accurate bending analysis of (FGRP) & (FGSP), according to all comparison experiments. Finally, the study of the thermo-mechanical behavior of perfect and defective rectangular plates (FGRP) and square plate (FGSP) and square plate (FGSP).



Fig 5: Variation of axial stress across the thickness of rectangular plate (FGRP) & square plate (FGSP).



Fig 6: Variation of shear stress across the thickness of rectangular plate (FGRP) & square plate (FGSP).

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